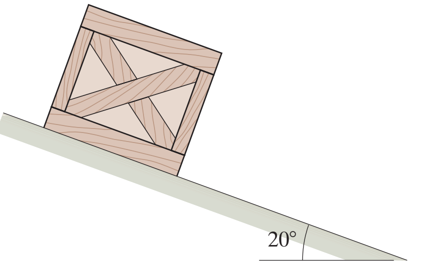
Problem 14.1 In Active Example 14.1, suppose that the coefficient of kinetic friction between the crate and the inclined surface is $\mu_k = 0.12$. Determine the distance the crate has moved down the inclined surface at $t = 1$ s.



Solution: There are three unknowns in this problem: N, f, and a. We will first assume that the crate does not slip. The governing equations are

$$\Sigma F \searrow: (445\ \text{N})\sin 20^\circ - f$$

$$= \left(\frac{445\ \text{N}}{9.81\ \text{m/s}^2}\right) a$$

$$\Sigma F \nearrow: N - (445\ \text{N})\cos 20^\circ = 0$$

No slip: $a = 0$

Solving, we find that $N = 418.2$ N, $f = 152.2$ N, $a = 0$.

To check the no slip assumption, we calculate the maximum friction force

$$f_{\text{max}} = \mu_s N = 0.2(418.2\ \text{N}) = 83.64\ \text{N}.$$

Since $f > f_{\text{max}}$, we conclude that our no slip assumption is false. The governing equations are now known to be

$$\Sigma F \searrow: (445\ \text{N})\sin 20^\circ - f = \left(\frac{445\ \text{N}}{9.81\ \text{m/s}^2}\right) a$$

$$\Sigma F \nearrow: N - (445\ \text{N})\cos 20^\circ = 0$$

Slip: $f = 0.12\ N$

Solving we have $N = 418.2$ N, $f = 50.2$ N, $a = 2.25\ \text{m/s}^2$.

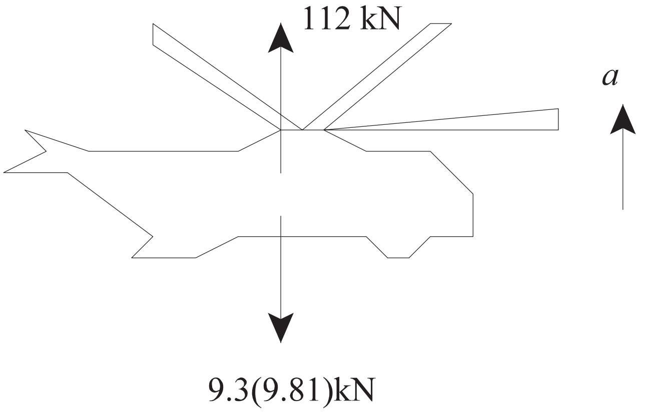
To find the distance we have $d = \frac{1}{2}at^2 = \frac{1}{2}(2.25\ \text{m/s}^2)\ (1\ \text{s})^2 = 1.13$ m.

$$\boxed{d = 1.13\ \text{m.}}$$

Problem 14.2 The mass of the Sikorsky UH-60A helicopter is 9300 kg. It takes off vertically with its rotor exerting a constant upward thrust of 112 kN.

(a) How fast is the helicopter rising 3 s after it takes off?
(b) How high has it risen 3 s after it takes off?

Strategy: Be sure to draw the free-body diagram of the helicopter.



Solution: The equation of motion is

$$\Sigma F : 112 \text{ kN} - 9.3(9.81) \text{ kN}$$

$$= (9{,}300 \text{ kg})a$$

Solving, we find that

$$a = 2.23 \text{ m/s}^2.$$

Using kinematics we can answer the questions

$$a = 2.23 \text{ m/s}^2,$$

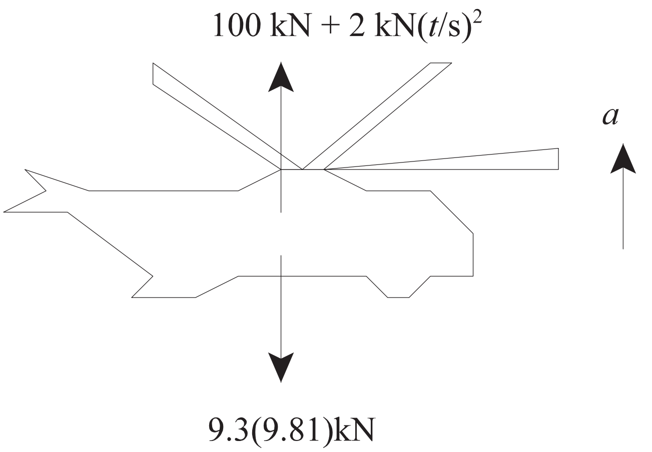
$$v = at = (2.23 \text{ m/s}^2)(3 \text{ s}) = 6.70 \text{ m/s},$$

$$h = \frac{1}{2}at^2 = \frac{1}{2}(2.23 \text{ m/s}^2)(3 \text{ s})^2 = 10.0 \text{ m}.$$

$$\boxed{(a)\ 6.70 \text{ m/s}, \quad (b)\ 10.0 \text{ m}.}$$

Problem 14.3 The mass of the Sikorsky UH-60A helicopter is 9,300 kg. It takes off vertically at $t = 0$. The pilot advances the throttle so that the upward thrust of its engine (in kN) is given as a function of time in seconds by $T = 100 + 2t^2$.

(a) How fast is the helicopter rising 3 s after it takes off?
(b) How high has it risen 3 s after it takes off?



Solution: The equation of motion is

$$\Sigma F : 100 \text{ kN} + 2 \text{ kN}\left(\frac{\text{t}}{\text{s}}\right)^2$$

$$-9.3(9.81) \text{ kN} = (9,300 \text{ kg})a$$

Solving, we find that

$$a = (0.943 \text{ m/s}^2) + (0.215 \text{ m/s}^4)t^2.$$

Using kinematics we can answer the questions

$$a = (0.943 \text{ m/s}^2) + (0.215 \text{ m/s}^4)t^2$$

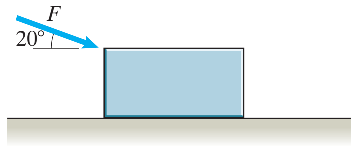
$$v = (0.943 \text{ m/s}^2)t + \frac{1}{3}(0.215 \text{ m/s}^4)t^3$$

$$h = \frac{1}{2}(0.943 \text{ m/s}^2)t^2 + \frac{1}{12}(0.215 \text{ m/s}^4)t^4$$

Evaluating these expressions at $t = 3$ s,

$$\boxed{(a)\ v = 4.76 \text{ m/s}, \quad (b)\ d = 5.69 \text{ m}.}$$

Problem 14.4 The horizontal surface is smooth. The 30-N box is at rest when the constant force F is applied. Two seconds later, the box is moving to the right at 20 m/s. Determine F.

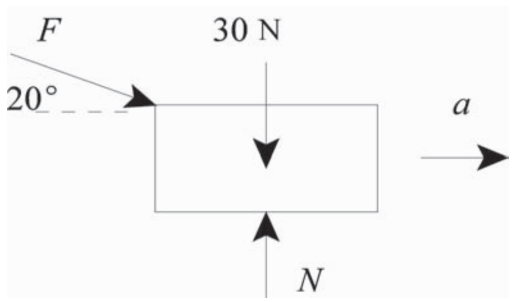


Solution: We use one governing equation and one kinematic relation

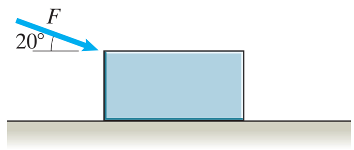
$$\Sigma F_x : F\cos 20^\circ = \left(\frac{30\ \text{N}}{9.81\ \text{m/s}^2}\right) a,$$

$$v = (20\ \text{m/s}) = a(2\ \text{s}).$$

Solving, we find $a = 10\ \text{m/s}^2$, $\boxed{F = 32.5\ \text{N.}}$



Problem 14.5 The coefficient of kinetic friction between the 30-N box and the horizontal surface is $\mu_k = 0.1$. The box is at rest when the constant force F is applied. Two seconds later, the box is moving to the right at 20 m/s. Determine F.



Solution: We use two governing equations, one slip equation, and one kinematic relation

$$\Sigma F_x : F\cos 20^\circ - f = \left(\frac{30\ \text{N}}{9.81\ \text{m/s}^2}\right) a,$$

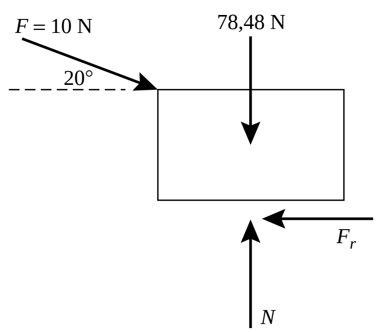
$$\Sigma F_y : N - F\sin 20^\circ - 30\ \text{N} = 0,$$

$$f = (0.1)N,$$

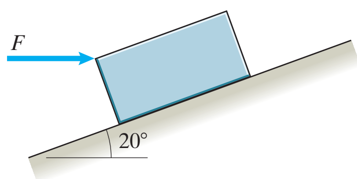
$$v = (20\ \text{m/s}) = a(2\ \text{s}).$$

Solving, we find

$$a = 10\ \text{m/s}^2,\ N = 42.7\ \text{N},\ f = 4.27\ \text{N},\ \boxed{F = 37.1\ \text{N.}}$$



Problem 14.6 The inclined surface is smooth. The velocity of the 114-kg box is zero when it is subjected to a constant horizontal force $F = 20$ N. What is the velocity of the box two seconds later?



Solution: From the governing equation we can find the acceleration of the box (assumed to be down the incline).

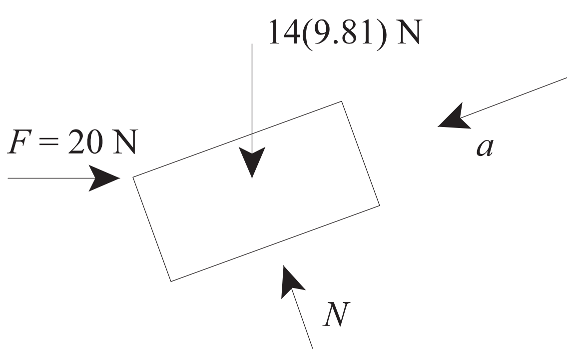
$$\Sigma F \swarrow : 14(9.81)\ \text{N}\ \sin 20^\circ$$

$$-(20\ \text{N})\cos 20^\circ = (14\ \text{kg})a$$

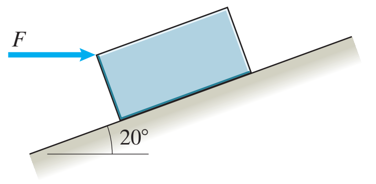
Solving, we have $a = 2.01\ \text{m/s}^2$.
Since $a > 0$, our assumption is correct.
Using kinematics,

$$v = at = (2.01\ \text{m/s}^2)(2\ \text{s}) = 4.03\ \text{m/s}.$$

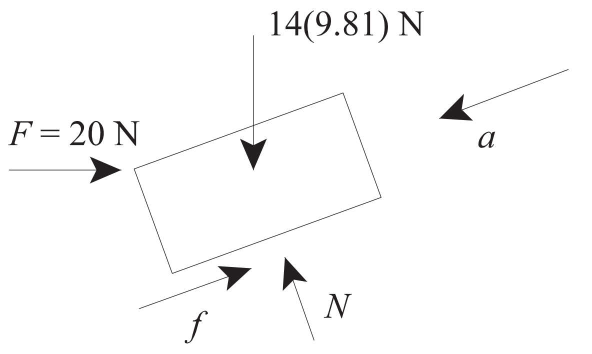
$$\boxed{v = 4.03\ \text{m/s down the incline.}}$$



Problem 14.7 The coefficient of kinetic friction between the 14-kg box and the inclined surface is $\mu_k = 0.1$. The velocity of the box is zero when it is subjected to a constant horizontal force $F = 20$ N. What is the velocity of the box two seconds later?



Solution: From the governing equations and the slip equation, we can find the acceleration of the box (assumed to be down the incline).



$$\Sigma F \swarrow: 14(9.81)\ \text{N} \sin 20^\circ - f$$

$$-(20\ \text{N}) \cos 20^\circ = (14\ \text{kg})a,$$

$$\Sigma F \nwarrow: N - 14(9.81)\ \text{N} \cos 20^\circ$$

$$-(20\ \text{N}) \sin 20^\circ = 0.$$

$$\text{Slip: } f = (0.1)N.$$

Solving, we have

$$a = 1.04\ \text{m/s}^2,\ N = 136\ \text{N},\ f = 13.6\ \text{N}.$$

Since $a > 0$, our assumption is correct.

Using kinematics,

$$v = at = (1.04\ \text{m/s}^2)(2\ \text{s}) = 2.08\ \text{m/s}.$$

$$\boxed{v = 2.08\ \text{m/s down the incline.}}$$

Problem 14.8 The 700 N skier is schussing on a 25° slope. At the instant shown, he is moving at 20 m/s. The kinetic coefficient of friction between his skis and the snow is $\mu_k = 0.08$. If he makes no attempt to check his speed, how long does it take for it to increase to 30 m/s?

Solution: The governing equations and the slip equation are used to find the acceleration

$$\Sigma F \nearrow: N - (700 \text{ N}) \cos 25^\circ = 0,$$

$$\Sigma F \searrow: (700 \text{ N}) \sin 25^\circ - f$$

$$= \left(\frac{700 \text{ N}}{9.81 \text{ m/s}^2}\right) a.$$

$$\text{Slip: } f = (0.08)N.$$

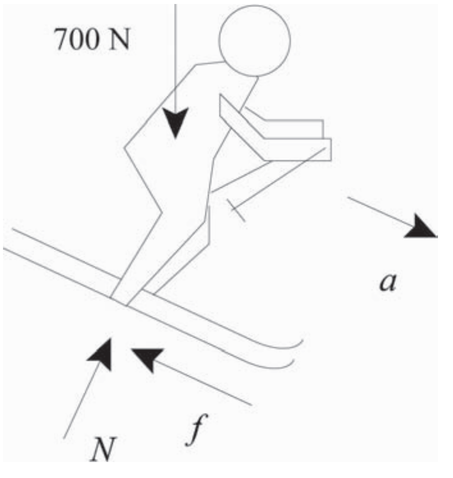
Solving yields

$$a = 3.43 \text{ m/s}^2, N = 634.4 \text{ N}, \ f = 50.8 \text{ N}.$$

Using kinematics, we find

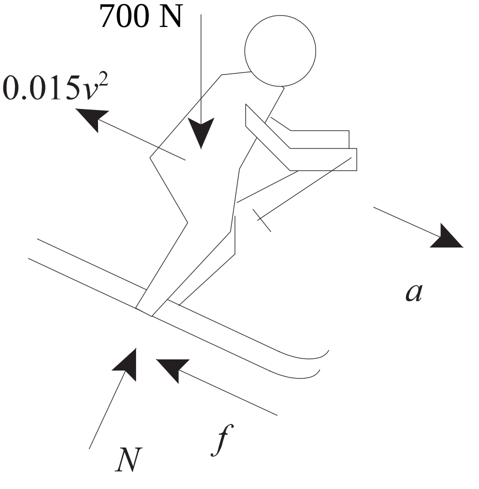
$$30 \text{ m/s} = 20 \text{ m/s} + (3.43 \text{ m/s}^2)t$$

Solving yields $\boxed{t = 2.92 \text{ s.}}$



Problem 14.9 The 700 N skier is schussing on a 25° slope. At the instant shown, he is moving at 20 m/s. The kinetic coefficient of friction between his skis and the snow is $\mu_k = 0.08$. Aerodynamic drag exerts a resisting force on him of magnitude $0.015v^2$, where v is the magnitude of his velocity. If he makes no attempt to check his speed, how long does it take for it to increase to 60 m/s?

Solution: The governing equations and the slip equation are used to find the acceleration



$$\Sigma F \nearrow: N - (700\text{ N})\cos 25^\circ = 0,$$

$$\Sigma F \searrow: (700\text{ N})\sin 25^\circ - f$$

$$-(0.015)v^2$$

$$= \left(\frac{700\text{ N}}{9.81\text{ m/s}^2}\right)a.$$

$$\text{Slip: } f = (0.08)N.$$

Solving yields

$$N = 634.4\text{ N}, \ f = 50.8\text{ N},$$

$$a = (3.43\text{ m/s}^2) - (0.00021\text{ m}^{-1})v^2.$$

Using kinematics, we find

$$a = \frac{dv}{dt} = (3.43\text{ m/s}^2) - (0.00021\text{ m}^{-1})v^2$$

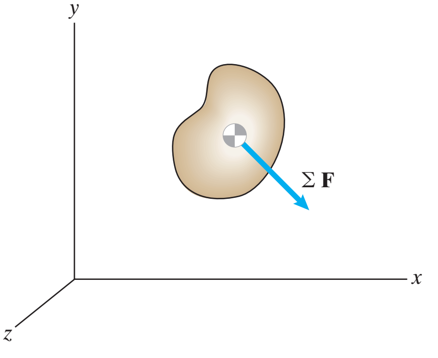
$$\int_{20\text{ m/s}}^{60\text{ m/s}} \frac{dv}{(3.43\text{ m/s}^2) - (0.00021\text{ m}^{-1})v^2} = \int_0^t dt = t$$

Performing the integration, we find

$$t = \frac{1}{2(0.0268)} \ln\left[\frac{3.43 + x(0.0268)}{3.43 - x(0.0268)}\right]_{20}^{60}$$

Solving yields $\boxed{t = 13.1\text{ s.}}$

Problem 14.10 The total external force on the 10-kg object is constant and equal to $\Sigma\mathbf{F} = 90\mathbf{i} - 60\mathbf{j} + 20\mathbf{k}$ (N). At time $t = 0$, its velocity is $\mathbf{v} = -14\mathbf{i} + 26\mathbf{j} + 32\mathbf{k}$ (m/s). What is its velocity at $t = 4$ s? (See Active Example 14.2.)



Solution:

$$\mathbf{a} = \frac{\mathbf{F}}{m} = \frac{(90\mathbf{i} - 60\mathbf{j} + 20\mathbf{k})\text{ N}}{10\text{ kg}} = (9\mathbf{i} - 6\mathbf{j} + 2\mathbf{k})\text{ m/s}^2.$$

$$\mathbf{v} = \mathbf{a}t + \mathbf{v}_0 = [(9\mathbf{i} - 6\mathbf{j} + 2\mathbf{k})\text{ m/s}^2](4\text{s}) + (-14\mathbf{i} + 26\mathbf{j} + 32\mathbf{k})\text{ m/s}.$$

$$\boxed{\mathbf{v} = (22\mathbf{i} + 2\mathbf{j} + 40\mathbf{k})\text{ m/s.}}$$

Problem 14.11 The total external force on the 10-kg object shown in Problem 14.10 is given as a function of time by $\Sigma\mathbf{F} = (-20t + 90)\mathbf{i} - 60\mathbf{j} + (10t + 40)\mathbf{k}$ (N). At time $t = 0$, its position is $\mathbf{r} = 40\mathbf{i} + 30\mathbf{j} - 360\mathbf{k}$ (m) and its velocity is $\mathbf{v} = -14\mathbf{i} + 26\mathbf{j} + 32\mathbf{k}$ (m/s). What is its position at $t = 4$ s?

Solution:

$$\mathbf{a} = \frac{1}{(10\text{ kg})}[(-20t + 90)\mathbf{i} - 60\mathbf{j} + (10t + 40)\mathbf{k}]N$$

$$\mathbf{a} = [(-2t + 9)\mathbf{i} - 6\mathbf{j} + (t + 4)\mathbf{k}]\text{ m/s}^2$$

Integrate to get the velocity

$$\mathbf{v} = \int \mathbf{a}\,dt + \mathbf{v}_0$$

$$\mathbf{v} = \left[(-t^2 + 9t - 14)\mathbf{i} + (-6t + 26)\mathbf{j} + \left(\frac{1}{2}t^2 + 4t + 32\right)\mathbf{k}\right]\text{ m/s}$$

Integrate again to get the position

$$\mathbf{r} = \int \mathbf{v}\,dt + \mathbf{r}_0$$

$$\mathbf{r} = \left[\left(-\frac{1}{3}t^3 + \frac{9}{2}t^2 - 14t + 40\right)\mathbf{i} + (-3t^2 + 26t + 30)\mathbf{j}\right.$$

$$\left. + \left(\frac{1}{6}t^3 + 2t^2 + 32t - 360\right)\mathbf{k}\right]\text{ m}$$

At the time indicated ($t = 4$ s) we have

$$\boxed{\mathbf{r} = [34.7\mathbf{i} + 86\mathbf{j} - 189.3\mathbf{k}]\text{ m}}$$

Problem 14.12 The position of the 10-kg object shown in Problem 14.10 is given as a function of time by $\mathbf{r} = (20t^3 - 300)\mathbf{i} + 60t^2\mathbf{j} + (6t^4 - 40t^2)\mathbf{k}$ (m). What is the total external force on the object at $t = 2$ s?

Solution:

$$\mathbf{r} = [(20t^3 - 300)\mathbf{i} + (60t^2)\mathbf{j} + (6t^4 - 40t^2)\mathbf{k}]\text{ m}$$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = [(60t^2)\mathbf{i} + (120t)\mathbf{j} + (24t^3 - 80t)\mathbf{k}]\text{ m/s}$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = [(120t)\mathbf{i} + (120)\mathbf{j} + (72t^2 - 80)\mathbf{k}]\text{ m/s}^2$$

$$\mathbf{F} = m\mathbf{a} = (10\text{ kg})[(120t)\mathbf{i} + (120)\mathbf{j} + (72t^2 - 80)\mathbf{k}]\text{ m/s}^2$$

$$\mathbf{F} = [(1200t)\mathbf{i} + (1200)\mathbf{j} + (720t^2 - 800)\mathbf{k}]\text{ N}$$

At the time $t = 2$ s,

$$\boxed{\mathbf{F} = [2.40\mathbf{i} + 1.20\mathbf{j} + 2.08\mathbf{k}]\text{ kN}}$$

Problem 14.13 The total force exerted on the 80,000-N launch vehicle by the thrust of its engine, its weight, and aerodynamic forces during the interval of time from $t = 2$ s to $t = 4$ s is given as a function of time by $\Sigma\mathbf{F} = (2000 - 400t^2)\mathbf{i} + (5200 + 440t)\mathbf{j} + (800 + 60t^2)\mathbf{k}$ (N). At $t = 2$ s, its velocity is $\mathbf{v} = 12\mathbf{i} + 220\mathbf{j} - 30\mathbf{k}$ (m/s). What is its velocity at $t = 4$ s?

Solution: Working in components we have

$$a_x = \frac{F_x}{m} = \frac{(2000 - 400t^2)\text{ N}}{\left(\dfrac{80{,}000\text{ N}}{9.81\text{ m/s}^2}\right)} = (0.245 - 0.05t^2)\text{ m/s}^2,$$

$$a_y = \frac{F_y}{m} = \frac{(5200 + 440t)\text{ N}}{\left(\dfrac{80{,}000\text{ N}}{9.81\text{ m/s}^2}\right)} = (0.638 + 0.054t)\text{ m/s}^2,$$

$$a_z = \frac{F_z}{m} = \frac{(800 + 60t^2)\text{ N}}{\left(\dfrac{80{,}000\text{ N}}{9.81\text{ m/s}^2}\right)} = (0.098 + 0.0074t^2)\text{ m/s}^2$$

We find the velocity at $t = 4$ s, by integrating: $\mathbf{v} = \int_{2\text{ s}}^{4\text{ s}} \mathbf{a}dt + \mathbf{v}_0$. In components this is

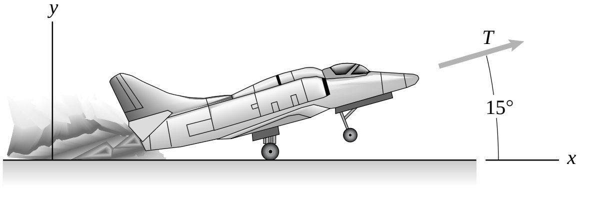
$$v_x = \left([0.245][4 - 2] - \frac{1}{3}[0.05][4^3 - 2^3] + 12\right)\text{ m/s} = 11.56\text{ m/s},$$

$$v_y = \left([0.638][4 - 2] + \frac{1}{2}[0.054][4^2 - 2^2] + 220\right)\text{ m/s} = 222.8\text{ m/s},$$

$$v_z = \left([0.098][4 - 2] + \frac{1}{3}[0.0074][4^3 - 2^3] - 30\right)\text{ m/s} = -29.7\text{ m/s},$$

Thus $\boxed{\mathbf{v} = (11.56\mathbf{i} + 222.8\mathbf{j} - 29.7\mathbf{k})\text{ m/s.}}$

Problem 14.14 At the instant shown, the horizontal component of acceleration of the 115.6 kN airplane due to the sum of the external forces acting on it is 14 m/s^2. If the pilot suddenly increases the magnitude of the thrust T by 17.8 kN, what is the horizontal component of the plane's acceleration immediately afterward?



Solution: Before

$$\sum F_x : F_x = \left(\frac{115600\text{ N}}{9.81\text{ m/s}^2}\right)(14\text{ m/s}^2) = 164975\text{ N}$$

After

$$\sum F_x : 164975\text{ N} + (17800\text{ N})\cos 15^\circ = \left(\frac{11560\text{ N}}{9.81\text{ m/s}^2}\right)a$$

$$\Rightarrow \boxed{a = 15.46\text{ m/s}^2}$$

Problem 14.15 At the instant shown, the rocket is traveling straight up at 100 m/s. Its mass is 90,000 kg and the thrust of its engine is 2400 kN. Aerodynamic drag exerts a resisting force (in newtons) of magnitude $0.8v^2$, where v is the magnitude of the velocity. How long does it take for the rocket's velocity to increase to 200 m/s?

Solution: The equation of motion is

$$\Sigma F : (2400 \text{ kN}) - (90{,}000 \text{ kg})(9.81 \text{ m/s}^2)$$

$$-(0.8 \text{ kg/m})v^2 = (90{,}000 \text{ kg})a$$

Solving for the acceleration we have

$$a = \frac{dv}{dt} = (16.9 \text{ m/s}^2) - (8.89\times10^{-6} \text{ m}^{-1})v^2$$

$$\int_{100 \text{ m/s}}^{200 \text{ m/s}} \frac{dv}{(16.9 \text{ m/s}^2) - (8.89\times10^{-6} \text{ m}^{-1})v^2} = \int_0^t dt = t$$

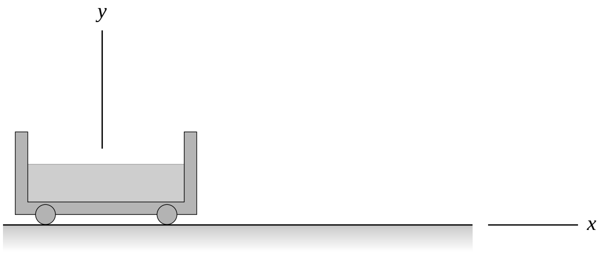
Carrying out the integration, we find

$$t = (81.7 \text{ s})(\tanh^{-1}[(0.000726)(200)] - \tanh^{-1}[(0.000726)(100)])$$

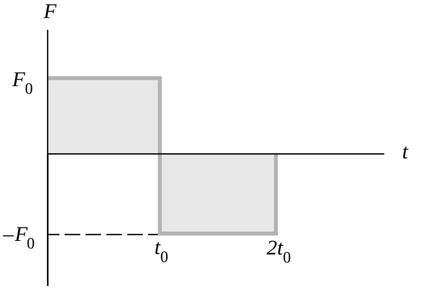
$$\boxed{t = 6.01 \text{ s}}$$

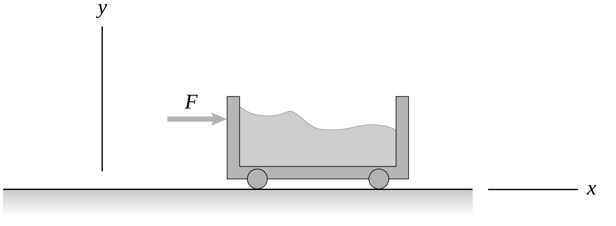
Problem 14.16 A 2-kg cart containing 8 kg of water is initially stationary (Fig. a). The center of mass of the "object" consisting of the cart and water is at $x = 0$. The cart is subjected to the time-dependent force shown in Fig. b, where $F_0 = 5$ N and $t_0 = 2$ s. Assume that no water spills out of the cart and that the horizontal forces exerted on the wheels by the floor are negligible.

(a) Do you know the acceleration of the cart during the period $0 < t < t_0$?
(b) Do you know the acceleration of the center of mass of the "object" consisting of the cart and water during the period $0 < t < t_0$?
(c) What is the x-coordinate of the center of mass of the "object" when $t > 2t_0$?



(a)





(b)

Solution:

(a) No, the internal dynamics make it impossible to determine the acceleration of just the cart.
(b) Yes, the entire system (cart + water) obeys Newton's 2nd Law.

$$\boxed{\sum F : (5 \text{ N}) = (10 \text{ kg})a \ \Rightarrow\ a = \frac{5 \text{ N}}{10 \text{ kg}} = 0.5 \text{ m/s}^2}$$

(c) The center of mass moves as a "super particle".

For $0 < t < t_0$

$$5 \text{ N} = (10 \text{ kg})a \Rightarrow a = \frac{5 \text{ N}}{10 \text{ kg}} = 0.5 \text{ m/s}^2$$

$v = (0.5 \text{ m/s}^2)t, \quad s = (0.25 \text{ m/s}^2)t^2$

At $t = t_0 = 2$ s, $v = 1.0$ m/s, $s = 1.0$ m

For $t_0 < t < 2t_0$,

$-5 \text{ N} = (10 \text{ kg})a, \ a = -0.5 \text{ m/s}^2, \ v = -(0.5 \text{ m/s}^2)(t - t_0) + 1.0 \text{ m/s}$

$s = -(0.25 \text{ m/s}^2)(t - t_0)^2 + (1.0 \text{ m/s})(t - t_0) + 1.0 \text{ m}$

For $t \geq 2t_0$, $\boxed{a = v = 0, \ s = 2.0 \text{ m}}$

Problem 14.17 The combined weight of the motorcycle and rider is 1601 N. The coefficient of kinetic friction between the tires and the road is $\mu_k = 0.8$. The rider starts from rest, spinning the rear wheel. Neglect the horizontal force exerted on the front wheel by the road. In two seconds, the motorcycle moves 10.67 m. What was the normal force between the rear wheel and the road?

Solution: Kinematics

$a =$ constant, $v = at$, $s = \frac{1}{2}at^2$, $10.67 \text{ m} = \frac{1}{2}a(2 \text{ s})^2 \Rightarrow a = 5.33 \text{ m/s}^2$

Dynamics: $F_r = \left(\frac{1601 \text{ N}}{9.81 \text{ m/s}^2}\right)(5.33 \text{ m/s}^2) = 870 \text{ N}$

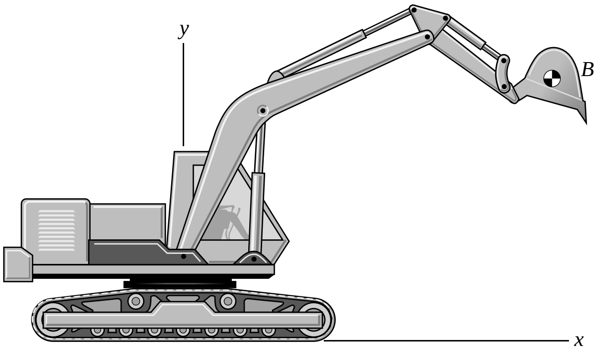
Friction: $\boxed{F_r = (0.8)N \Rightarrow N = \frac{870 \text{ N}}{0.8} = 1089.8 \text{ N}}$

Problem 14.18 The mass of the bucket B is 180 kg. From $t = 0$ to $t = 2$ s, the x and y coordinates of the center of mass of the bucket are

$x = -0.2t^3 + 0.05t^2 + 10$ m,

$y = 0.1t^2 + 0.4t + 6$ m.

Determine the x and y components of the force exerted on the bucket by its supports at $t = 1$ s.



Solution:

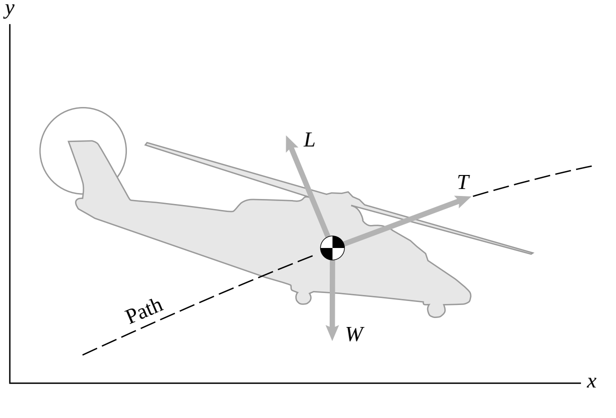
$x = -0.2t^3 + 0.05t^2 + 10, \quad y = 0.1t^2 + 0.4t + 6$

$v_x = -0.6t^2 + 0.1t, \quad v_y = 0.2t + 0.4$

$a_x = -1.2t + 0.1, \quad a_y = 0.2$

$$\boxed{\begin{aligned} F_x &= ma_x = (180 \text{ kg})(-1.2[1 \text{ s}] + 0.1) \text{ m/s}^2 = -198 \text{ N} \\ F_y &= ma_y + mg = (180 \text{ kg})(0.2) \text{ m/s}^2 + (180 \text{ kg})(9.81 \text{ m/s}^2) \\ &= 1800 \text{ N} \end{aligned}}$$

Problem 14.19 During a test flight in which a 9000-kg helicopter starts from rest at $t = 0$, the acceleration of its center of mass from $t = 0$ to $t = 10$ s is $\mathbf{a} = (0.6t)\mathbf{i} + (1.8 - 0.36t)\mathbf{j}$ m/s^2. What is the magnitude of the total external force on the helicopter (including its weight) at $t = 6$ s?



Solution: From Newton's second law: $\sum \mathbf{F} = m\mathbf{a}$. The sum of the external forces is $\sum \mathbf{F} = \mathbf{F} - \mathbf{W} = 9000[(0.6t)\mathbf{i} + (1.8 - 0.36t)\mathbf{j}]_{t=6} = 32400\mathbf{i} - 3240\mathbf{j}$, from which the magnitude is

$$\boxed{\left|\sum \mathbf{F}\right| = \sqrt{32400^2 + 3240^2} = 32562 \text{ (N)}.}$$

Problem 14.20 The engineers conducting the test described in Problem 14.19 want to express the total force on the helicopter at $t = 6$ s in terms of three forces: the weight W, a component T tangent to the path, and a component L normal to the path. What are the values of W, T, and L?

Solution: Integrate the acceleration: $\mathbf{v} = (0.3t^2)\mathbf{i} + (1.8t - 0.18t^2)\mathbf{j}$, since the helicopter starts from rest. The instantaneous flight path angle is $\tan\beta = \dfrac{dy}{dx} = \left(\dfrac{dy}{dt}\right)\left(\dfrac{dx}{dt}\right)^{-1} = \dfrac{(1.8t - 0.18t^2)}{(0.3t^2)}$. At $t = 6$ s, $\beta_{t=6} = \tan^{-1}\left(\dfrac{(1.8(6) - 0.18(6)^2)}{0.3(6)^2}\right) = 21.8^\circ$. A unit vector tangent to this path is $\mathbf{e}_t = \mathbf{i}\cos\beta + \mathbf{j}\sin\beta$. A unit vector normal to this path $\mathbf{e}_n = -\mathbf{i}\sin\beta + \mathbf{j}\cos\beta$. The weight acts downward:

$$\boxed{\mathbf{W} = -\mathbf{j}(9000)(9.81) = -88.29\mathbf{j}\ \text{(kN)}.}$$

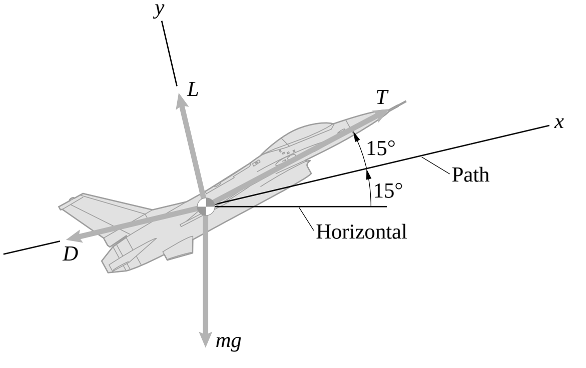
From Newton's second law, $\mathbf{F} - \mathbf{W} = m\mathbf{a}$, from which $\mathbf{F} = \mathbf{W} + m\mathbf{a} = 32400\mathbf{i} + 85050\mathbf{j}$ (N). The component tangent to the path is

$$\boxed{T = \mathbf{F}\cdot\mathbf{e}_t = 32400\cos\beta + 85050\sin\beta = 61669.4\ \text{(N)}}$$

The component normal to the path is

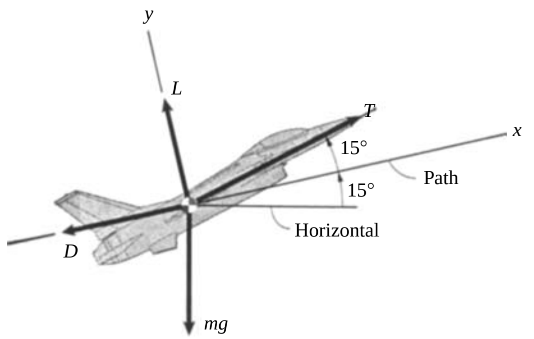
$$\boxed{L = \mathbf{F}\cdot\mathbf{e}_n = -32400\sin\beta + 85050\cos\beta = 66934\ \text{(N)}}$$

Problem 14.21 At the instant shown, the 11,000-kg airplane's velocity is $\mathbf{v} = 270\ \mathbf{i}$ m/s. The forces acting on the plane are its weight, the thrust $T = 110$ kN, the lift $L = 260$ kN, and the drag $D = 34$ kN. (The x-axis is parallel to the airplane's path.) Determine the magnitude of the airplane's acceleration.



Solution: Let us sum forces and write the acceleration components along the x and y axes as shown. After the acceleration components are known, we can determine its magnitude. The equations of motion, in the coordinate directions, are $\sum F_x = T\cos 15^\circ - D - W\sin 15^\circ = ma_x$, and $\sum F_y = L + T\sin 15^\circ - W\cos 15^\circ = ma_y$. Substituting in the given values for the force magnitudes, we get $a_x = 4.03$ m/s^2 and $a_y = 16.75$ m/s^2. The magnitude of the acceleration is $|\mathbf{a}| = \sqrt{a_x^2 + a_y^2} = 17.23$ m/s^2

Problem 14.22 At the instant shown, the 11,000-kg airplane's velocity is $\mathbf{v} = 300\mathbf{i}$ (m/s). The rate of change of the magnitude of the velocity is $dv/dt = 5$ m/s^2. The radius of curvature of the airplane's path is 4500 m, and the y axis points toward the concave side of the path. The thrust is $T = 120{,}000$ N. Determine the lift L and drag D.



Solution:

$$a_x = 5 \text{ m/s}^2$$

$$m = 11000 \text{ kg}$$

$$g = 9.81 \text{ m/s}^2$$

$$|\mathbf{V}| = 300 \text{ m/s}$$

$$T = 120000 \text{ N}$$

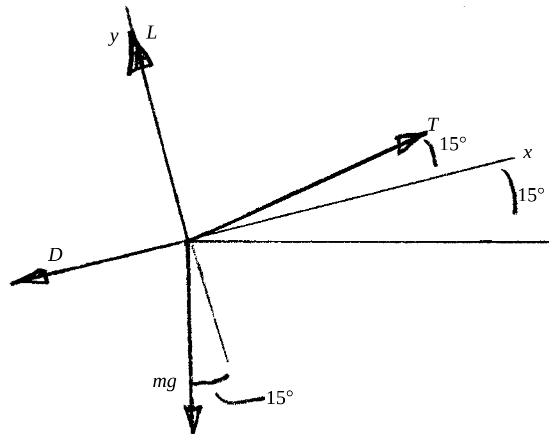
$$\rho = 4500 \text{ m}$$

$$\sum F_x: \quad T\cos 15^\circ - D - mg\sin 15^\circ = ma_x$$

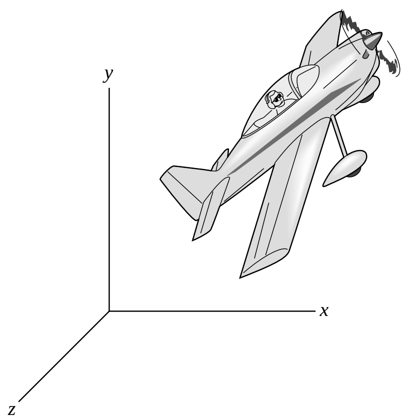
$$\sum F_y: \quad L + T\sin 15^\circ - mg\cos 15^\circ = ma_y$$

$$a_y = V^2/\rho$$

Solving, $\underline{D = 33.0 \text{ kN}, \quad L = 293 \text{ kN}}$



Problem 14.23 The coordinates in meters of the 360-kg sport plane's center of mass relative to an earth-fixed reference frame during an interval of time are $x = 20t - 1.63t^2$, $y = 35t - 0.15t^3$, and $z = -20t + 1.38t^2$, where t is the time in seconds. The y- axis points upward. The forces exerted on the plane are its weight, the thrust vector $\mathbf{T}$ exerted by its engine, the lift force vector $\mathbf{L}$, and the drag force vector $\mathbf{D}$. At $t = 4$ s, determine $\mathbf{T} + \mathbf{L} + \mathbf{D}$.



Solution: There are four forces acting on the airplane. Newton's second law, in vector form, given $\mathbf{T} + \mathbf{L} + \mathbf{D} + \mathbf{W} = (\mathbf{T} + \mathbf{L} + \mathbf{D}) - mg\mathbf{j} = m\mathbf{a}$. Since we know the weight of the airplane and can evaluate the total acceleration of the airplane from the information given, we can evaluate the $(\mathbf{T} + \mathbf{L} + \mathbf{D})$ (but we cannot evaluate these forces separately without more information. Differentiating the position equations twice and evaluating at $t = 4.0$ s, we get $a_X = -3.26$ m/s^2, $a_Y = -3.60$ m/s^2, and $a_Z = 2.76$ m/s^2. (Note that the acceleration components are constant over this time interval. Substituting into the equation for acceleration, we get $(\mathbf{T} + \mathbf{D} + \mathbf{L}) = m\mathbf{a} + mg\mathbf{j}$. The mass of the airplane is 360 kg. Thus, $(\mathbf{T} + \mathbf{D} + \mathbf{L}) = -1174\mathbf{i} + 2236\mathbf{j} + 994\mathbf{k}$ (N)

Problem 14.24 The force in newtons exerted on the 360-kg sport plane in Problem 14.23 by its engine, the lift force, and the drag force during an interval of time is $\mathbf{T}+\mathbf{L}+\mathbf{D} = (-1000+280t)\,\mathbf{i} + (4000-430t)\,\mathbf{j} + (720+200t)\,\mathbf{k}$, where t is the time in seconds. If the coordinates of the plane's center of mass are (0, 0, 0) and its velocity is $20\mathbf{i}+35\mathbf{j}-20\mathbf{k}$ (m/s) at $t=0$, what are the coordinates of the center of mass at $t=4$ s?

Solution: Since we are working in nonrotating rectangular Cartesian coordinates, we can consider the motion in each axis separately. From Problem 14.23, we have $(\mathbf{T}+\mathbf{D}+\mathbf{L}) = m\mathbf{a}+mg\mathbf{j}$. Separating the information for each axis, we have $ma_X = -1000+280t$, $ma_Y = 4000-430t-mg$, and $ma_Z = 720+200t$

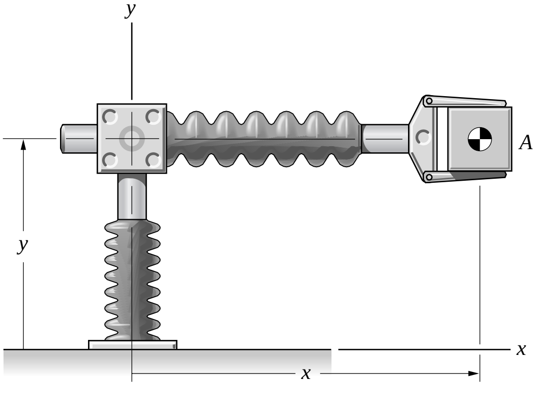
Integrating the x equation, we get $v_x = v_{x0} + (1/m)(-1000t + 280t^2/2)$ and $x = v_{X0}t + (1/m)(-1000t^2/2 + 280t^3/6)$.

Integrating the y equation, we get $v_Y = v_{Y0} + (1/m)((4000-mg)t - 430t^2/2)$ and $y = v_{Y0}t + (1/m)((4000-mg)t^2/2 - 430t^3/6)$

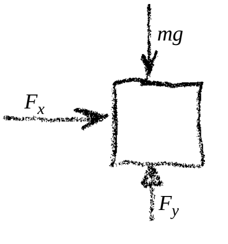
Integrating the z equation, we get $v_Z = v_{Z0} + (1/m)(720t + 200t^2/2)$ and $z = v_{Z0}t + (1/m)(720t^2/2 + 200t^3/6)$.

Evaluating at $t=4$ s we find the aircraft at $(66.1, 137.7, -58.1)m$ relative to its initial position at $t=0$.

Problem 14.25 The robot manipulator is programmed so that $x = 40+24t^2$ mm, $y = 4t^3$ mm, and $z=0$ during the interval of time from $t=0$ to $t=4$ s. The y axis points upward. What are the x and y components of the total force exerted by the jaws of the manipulator on the 2-kg widget A at $t=3$ s?



Solution:



$x = 40+24t^2$ mm $\qquad y = 4t^3$ mm

$v_x = 48t$ mm/s $\qquad v_y = 12t^2$ mm/s

$a_x = 48$ mm/s^2 $\qquad a_y = 24t$ mm/s^2

At $t=3$ s

$a_x = 48\times10^{-3}$ m/s^2, $\qquad a_y = 72\times10^{-3}$ m/s^2

$F_x = ma_x \qquad m = 2$ kg

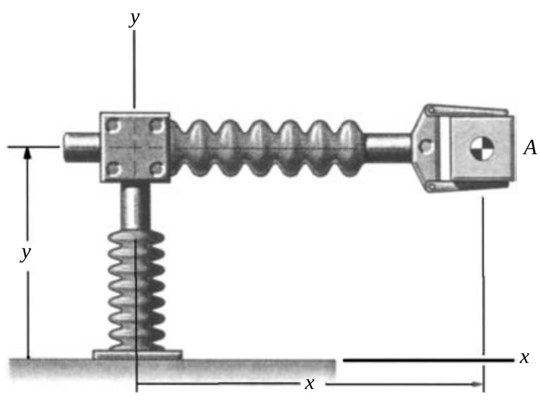
$F_y - mg = ma_y$

Solving,

$\underline{F_x = 0.096\text{ N}}$

$\underline{F_y = 19.764\text{ N}}$

Problem 14.26 The robot manipulator described in Problem 14.25 is reprogrammed so that it is stationary at $t = 0$ and the components of its acceleration are $a_x = 400 - 0.8v_x$ mm/s^2, $a_y = 200 - 0.4\ v_y$ mm/s^2 from $t = 0$ to $t = 2$ s, where v_x and v_y are the components of robot's velocity in mm/s. The y axis points upward. What are the x and y components of the total force exerted by the jaws of the manipulator on the 2-kg widget A at $t = 1$ s?



Solution:

$$a_x = \frac{dv_x}{dt} = 400 - 0.8v_x$$

$$\int_0^t dt = \int_0^{v_x} \frac{dv_x}{(400 - 0.8v_x)}$$

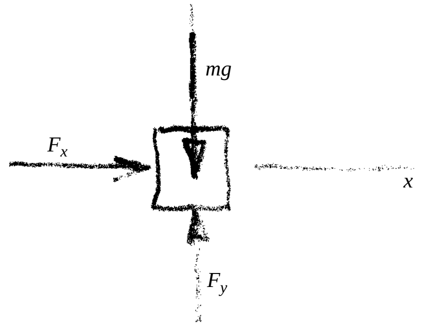
$$t = \frac{1}{(-0.8)} \ln(400 - 0.8v_x)\Big|_0^{v_x}$$

$$(-0.8t) = \ln\left(\frac{400 - 0.8v_x}{400}\right)$$

or $400 - 0.8v_x = 400e^{-0.8t}$

$$v_x = \frac{1}{(0.8)}(400)(1 - e^{-0.8t})$$

At $t = 1$ s, $\quad v_x = 275.3$ mm/s



A similar analysis for v_y yields

$v_y = 164.8$ mm/s at $t = 1$ s.

At $t = 1$ s,

$a_x = 400 - 0.8\ v_x = 179.7$ mm/s^2

$a_y = 200 - 0.4\ v_y = 134.1$ mm/s^2

$m = 2$ Kg

$g = 9.81$ m/s^2

$a_x = 0.180$ m/s^2

$a_y = 0.134$ m/s^2

$\sum F_x$: $\quad F_x = ma_x$

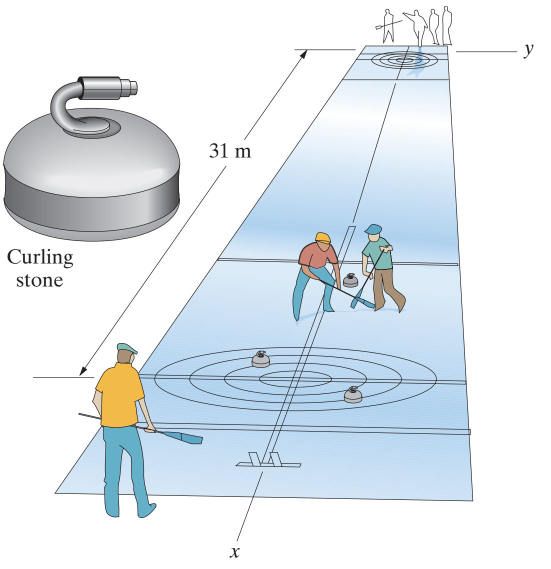
$\sum F_y$: $\quad F_y - mg = ma_y$

Solving,

$\underline{F_x = 0.359 \text{ N}}$

$\underline{F_y = 19.89 \text{ N}}$

Problem 14.27 In the sport of curling, the object is to slide a "stone" weighting 44 N into the center of a target located 31 m from the point of release. In terms of the coordinate system shown, the point of release is at $x = 0$, $y = 0$. Suppose that a shot comes to rest at $x = 31$ m, $y = 1$ m. Assume that the coefficient of kinetic friction is constant and equal to $\mu_k = 0.01$. What were the x and y components of the stone's velocity at release?



Solution: The stone travels at an angle relative to the x axis.

$$\theta = \tan^{-1}\left(\frac{1 \text{ m}}{31 \text{ m}}\right) = 1.85^\circ$$

The accelerations and distances are related as

$$a_x = v_x \frac{dv_x}{dx} = -(0.01)(9.81 \text{ m/s}^2)\cos(1.85^\circ) = -0.098 \text{ m/s}^2$$

$$\int_{v_{x0}}^{0} v_x dv_x = \int_0^{31 \text{ m}} (-0.098 \text{ m/s}^2)dx,$$

$$0 - \frac{v_{x0}^2}{2} = -(0.098 \text{ m/s}^2)(31 \text{ m}) \Rightarrow v_{x0} = 3.04 \text{ m/s}.$$

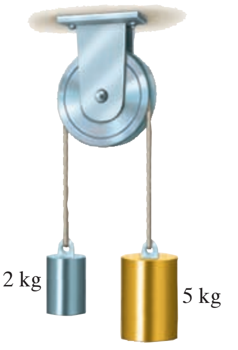
$$a_y = v_y \frac{dv_y}{dy} = -(0.01)(0.098 \text{ m/s}^2)\sin(1.85^\circ) = -0.00316 \text{ m/s}^2$$

$$\int_{v_{y0}}^{0} v_y dv_y = \int_0^{1 \text{ m}} (-0.00316 \text{ m/s}^2)dx,$$

$$0 - \frac{v_{y0}^2}{2} = -(0.00316 \text{ m/s}^2)(1 \text{ m}) \Rightarrow v_{y0} = 0.00316 \text{ m/s}^2.$$

$$\boxed{v_{x0} = 3.04 \text{ m/s}, v_{y0} = 0.00316 \text{ m/s}^2.}$$

Problem 14.28 The two masses are released from rest. How fast are they moving at $t = 0.5$ s? (See Example 14.3.)



Solution: The free-body diagrams are shown. The governing equations are

$$\Sigma F_{y\ \text{left}} : T - 2(9.81)\ \text{N} = (2\ \text{kg})a$$

$$\Sigma F_{y\ \text{right}} : T - 5(9.81)\ \text{N} = -(5\ \text{kg})a$$

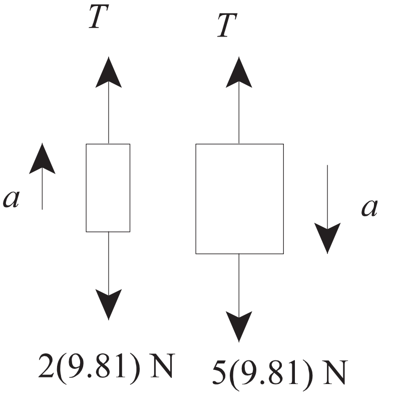
Solving, we find

$$T = 28.0\ \text{N},\ a = 4.20\ \text{m/s}^2.$$

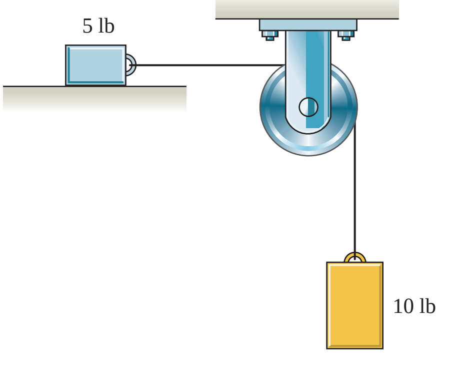
To find the velocity, we integrate the acceleration

$$v = at = (4.20\ \text{m/s})(0.5\ \text{s}) = 2.10\ \text{m/s}.$$

$$\boxed{v = 2.10\ \text{m/s}.}$$



Problem 14.29 The two weights are released from rest. The horizontal surface is smooth. (a) What is the tension in the cable after the weights are released? (b) How fast are the weights moving one second after they are released?



Solution: The free-body diagrams are shown. The governing equations are

$$\Sigma F_{xA} : T = \left(\frac{5\ \text{lb}}{32.2\ \text{ft/s}^2}\right)a$$

$$\Sigma F_{yB} : T - (10\ \text{lb}) = -\left(\frac{10\ \text{lb}}{32.2\ \text{ft/s}^2}\right)a$$

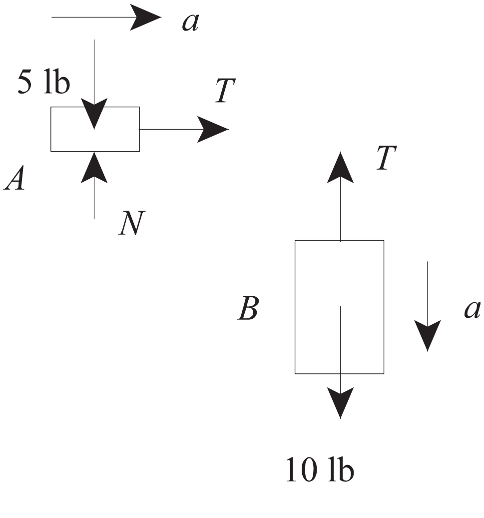
Solving, we find

$$T = 3.33\ \text{lb},\ a = 21.5\ \text{ft/s}^2.$$

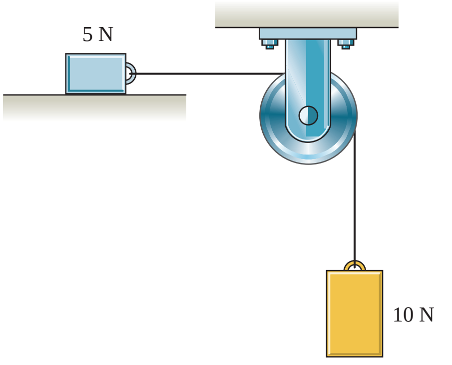
To find the velocity we integrate the acceleration

$$v = at = (21.5\ \text{ft/s}^2)(1\ \text{s}) = 21.5\ \text{ft/s}.$$

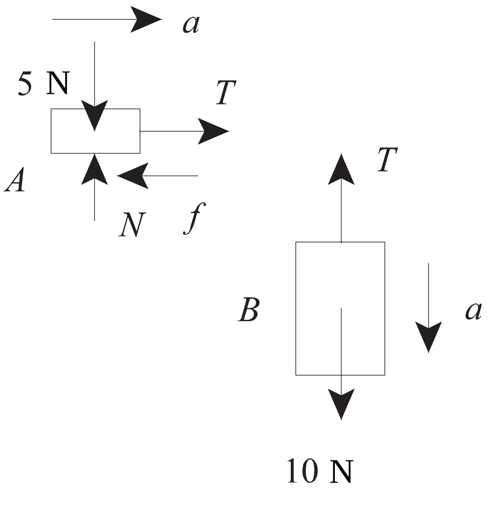
$$\boxed{(a)\ T = 3.33\ \text{lb}, \quad (b)\ v = 21.5\ \text{ft/s}.}$$



Problem 14.30 The two weights are released from rest. The coefficient of kinetic friction between the horizontal surface and the 5-N weight is $\mu_k = 0.18$. (a) What is the tension in the cable after the weights are released? (b) How fast are the weights moving one second after they are released?



Solution: The free-body diagrams are shown. The governing equations are



$$\Sigma F_{xA} : T - f = \left(\frac{5 \text{ N}}{9.81 \text{ m/s}^2}\right) a,$$

$$\Sigma F_{yA} : N - 5 \text{ N} = 0,$$

$$\Sigma F_{yB} : T - (10 \text{ N}) = -\left(\frac{10 \text{ N}}{9.81 \text{ m/s}^2}\right) a,$$

$$f = (0.18)N.$$

Solving, we find

$$T = 3.93 \text{ N}, a = 5.95 \text{ m/s}^2,$$

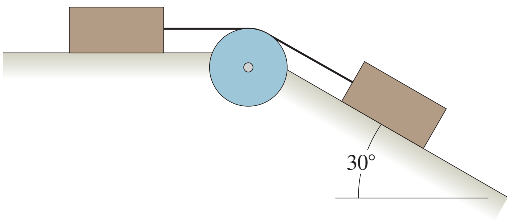
$$N = 5 \text{ N}, f = 0.9 \text{ N}.$$

To find the velocity we integrate the acceleration

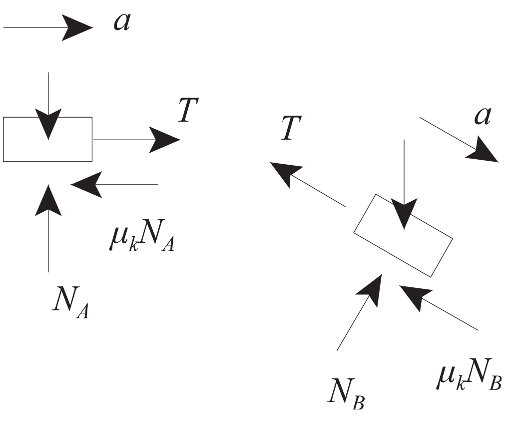
$$v = at = (5.95 \text{m/s}^2)(1 \text{ s}) = 5.95 \text{ m/s}.$$

$$\boxed{(a)\ T = 3.93 \text{ N} \quad (b)\ v = 5.95 \text{ m/s}.}$$

Problem 14.31 The mass of each box is 14 kg. One second after they are released from rest, they have moved 0.3 m from their initial positions. What is the coefficient of kinetic friction between the boxes and the surface?



Solution: We will first use the kinematic information to find the acceleration a



$$a = \text{constant},$$

$$v = at,$$

$$d = \frac{1}{2}at^2$$

$$0.3\text{ m} = \frac{1}{2}a(1\text{ s})^2$$

$$a = 0.6\text{ m/s}^2.$$

From the free-body diagrams we have four equations of motion:

$$\Sigma F_{xA} : T - \mu_k N_A = (14\text{ kg})a,$$

$$\Sigma F_{yA} : N_A - (14\text{ kg})(9.18\text{ m/s}^2) = 0$$

$$\Sigma F_{\searrow B} : (14\text{ kg})(9.81\text{ m/s}^2)\sin 30^\circ - T - \mu_k N_B = (14\text{ kg})a,$$

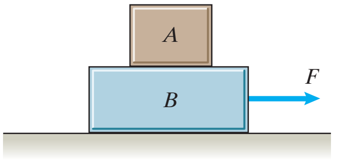
$$\Sigma F_{\nearrow B} : N_B - (14\text{ kg})(9.81\text{ m/s}^2)\cos 30^\circ = 0$$

Solving these equations, we find

$$T = 36.2\text{ N},\ N_A = 137\text{ N},\ N_B = 119\text{ N},\ \mu_k = 0.202.$$

$$\boxed{\mu_k = 0.202.}$$

Problem 14.32 The masses $m_A = 15$ kg and $m_B = 30$ kg, and the coefficients of friction between all of the surfaces are $\mu_s = 0.4$ and $\mu_k = 0.35$. The blocks are stationary when the constant force F is applied. Determine the resulting acceleration of block B if (a) $F = 200$ N; (b) $F = 400$ N.

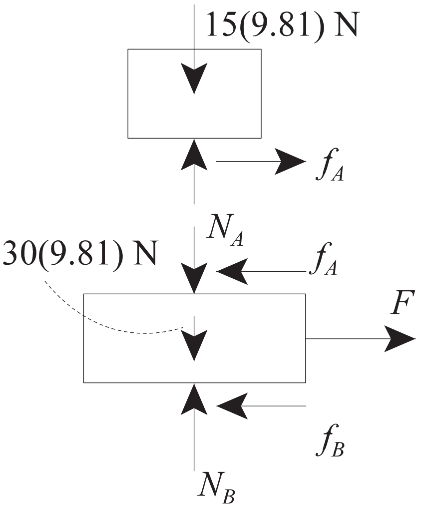


Solution: Assume that no motion occurs anywhere. Then

$$N_B = (45 \text{ kg})(9.81 \text{ m/s}^2) = 441 \text{ N}$$

$$f_{B\,\max} = \mu_s N = 0.4(441 \text{ N}) = 177 \text{ N}.$$

The blocks will slip as long as $F > 177$ N. Assume that blocks A and B move together with the common acceleration a.



$$\Sigma F_{xA} : f_A = (15 \text{ kg})a$$

$$\Sigma F_{yA} : N_A - (15 \text{ kg})(9.81 \text{ m/s}^2) = 0$$

$$\Sigma F_{xB} : F - f_A - f_B = (30 \text{ kg})a$$

$$\Sigma F_{yB} : N_B - (30 \text{ kg})(9.81 \text{ m/s}^2) - N_A = 0$$

$$\text{Slip at } B : f_B = (0.35)N_B$$

$$f_{A\,\max} = (0.4)N_A$$

(a) $F = 200$ N. Solving we find that $a = 1.01$ m/s^2, $f_A = 15.2$ N, $f_{A\,\max} = 58.9$ N. Since $f_A < f_{A\,\max}$, we know that our assumption is correct (the blocks move together).

(b) $F = 400$ N. Solving we find that $a = 5.46$ m/s^2, $f_A = 81.8$ N, $f_{A\,\max} = 58.9$ N. Since $f_A > f_{A\,\max}$, we know that our assumption is wrong (the blocks will not move together, but slip will occur at all surfaces). The equations are now

$$\Sigma F_{xA} : (0.35)N_A = (15 \text{ kg})a_A$$

$$\Sigma F_{yA} : N_A - (15 \text{ kg})(9.81 \text{ m/s}^2) = 0$$

$$\Sigma F_{xB} : F - (0.35)(N_A + N_B) = (30 \text{ kg})a_B$$

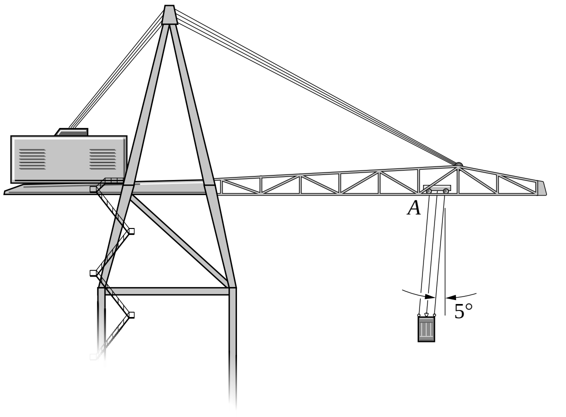
$$\Sigma F_{yB} : N_B - (30 \text{ kg})(9.81 \text{ m/s}^2) - N_A = 0$$

Solving we find that $a_A = 3.43$ m/s^2, $a_B = 6.47$ m/s^2.

$$\boxed{(a)\ a_B = 1.01 \text{ m/s}^2, \quad (b)\ a_B = 6.47 \text{ m/s}^2.}$$

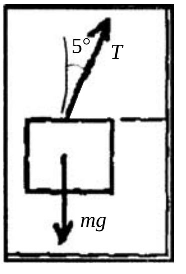
Problem 14.33 The crane's trolley at A moves to the right with constant acceleration, and the 800-kg load moves without swinging.

(a) What is the acceleration of the trolley and load?
(b) What is the sum of the tensions in the parallel cables supporting the load?



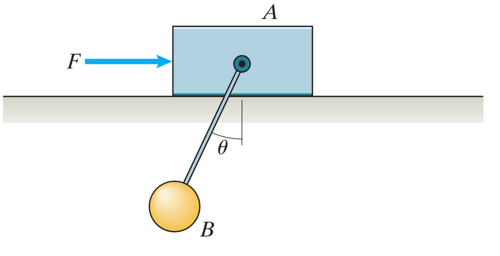
Solution: (a) From Newton's second law, $T\sin\theta = ma_x$, and $T\cos\theta - mg = 0$. Solve $a_x = \dfrac{T\sin\theta}{m}$, $T = \dfrac{mg}{\cos\theta}$, from which

$$\boxed{a = g\tan\theta = 9.81(\tan 5^\circ) = 0.858 \text{ m/s}^2}$$



(b) $$\boxed{T = \frac{800(9.81)}{\cos 5^\circ} = 7878 \text{ N}}$$

Problem 14.34 The mass of A is 30 kg and the mass of B is 5 kg. The horizontal surface is smooth. The constant force F causes the system to accelerate. The angle $\theta = 20^\circ$ is constant. Determine F.



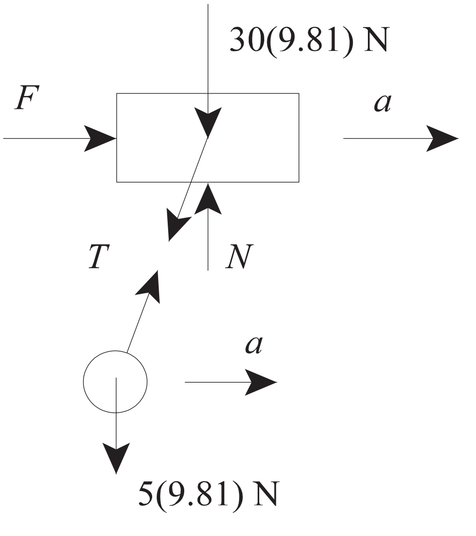
Solution: We have four unknowns (F, T, N, a) and four equations of motion:

$$\Sigma F_{xA} : F - T\sin\theta = (30 \text{ kg})a,$$

$$\Sigma F_{yA} : N - T\cos\theta - (30 \text{ kg})(9.81 \text{ m/s}^2) = 0,$$

$$\Sigma F_{xB} : T\sin\theta = (5 \text{ kg})a,$$

$$\Sigma F_{yB} : T\cos\theta - (5 \text{ kg})(9.81 \text{ m/s}^2) = 0.$$

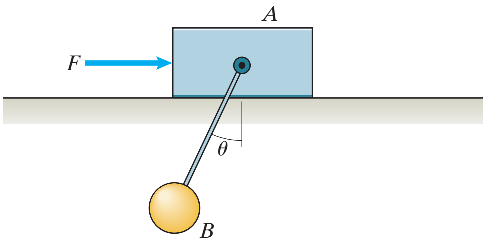


Solving, we find

$$T = 52.2 \text{ N}, a = 3.57 \text{ m/s}^2, N = 343 \text{ N},$$

$$\boxed{F = 125 \text{ N.}}$$

Problem 14.35 The mass of A is 30 kg and the mass of B is 5 kg. The coefficient of kinetic friction between A and the horizontal surface is $\mu_k = 0.2$. The constant force F causes the system to accelerate. The angle $\theta = 20^\circ$ is constant. Determine F.



Solution: We have four unknowns (F, T, N, a) and four equations of motion:

$$\Sigma F_{xA} : F - T\sin\theta - \mu_k N = (30 \text{ kg})a,$$

$$\Sigma F_{yA} : N - T\cos\theta - (30 \text{ kg})(9.81 \text{ m/s}^2) = 0,$$

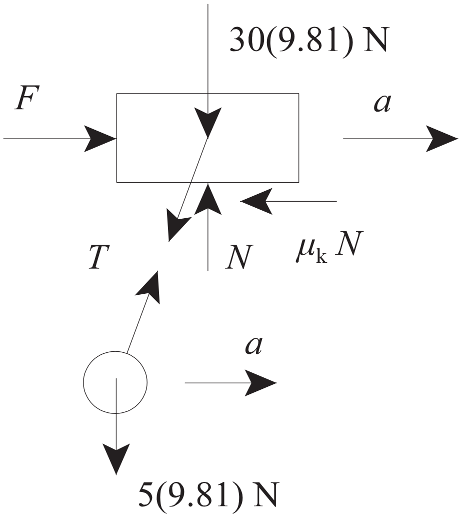
$$\Sigma F_{xB} : T\sin\theta = (5 \text{ kg})a,$$

$$\Sigma F_{yB} : T\cos\theta - (5 \text{ kg})(9.81 \text{ m/s}^2) = 0.$$

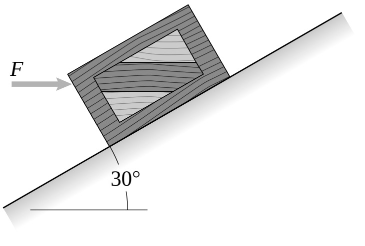
Solving, we find

$$T = 52.2 \text{ N}, a = 3.57 \text{ m/s}^2, N = 343 \text{ N},$$

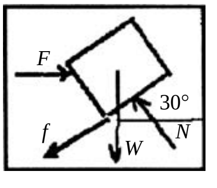
$$\boxed{F = 194 \text{ N.}}$$



Problem 14.36 The 445 N crate is initially stationary. The coefficients of friction between the crate and the inclined surface are $\mu_s = 0.2$ and $\mu_k = 0.16$. Determine how far the crate moves from its initial position in 2 s if the horizontal force $F = 400$ N.



Solution: Denote $W = 445$ N, $g = 9.81$ m/s^2, $F = 400$ N, and $\theta = 30^\circ$. Choose a coordinate system with the positive x axis parallel to the inclined surface. (See free body diagram next page.) The normal force exerted by the surface on the box is $N = F\sin\theta + W\cos\theta = 585.4$ N. The sum of the non-friction forces acting to move the box is $F_c = F\cos\theta - W\sin\theta = 124.1$ N. Slip only occurs if $|F_c| \geq |N\mu_s|$ which $124.1 > 117.1$ (N), *so slip occurs.*



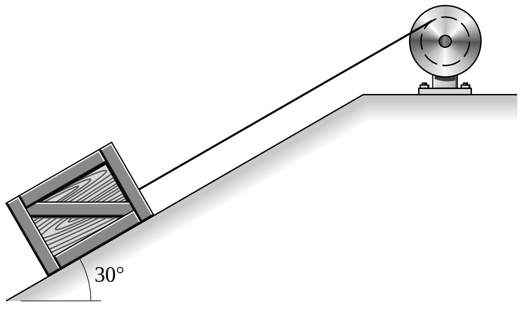
The direction of slip is determined from the sign of the sum of the non friction forces: $F_c > 0$, implies that the box slips up the surface, and $F_c < 0$ implies that the box slips down the surface (if the condition for slip is met). Since $F_c > 0$ *the box slips up the surface*. After the box slips, the sum of the forces on the box parallel to the surface is $\sum F_x = F_c - \text{sgn}(F_c)\mu_k N$, where $\text{sgn}(F_c) = \dfrac{F_c}{|F_c|}$. From Newton's second law, $\sum F_x = \left(\dfrac{W}{g}\right)a$, from which $a = \dfrac{g}{W}(F_c - \text{sgn}(F_c)\mu_k N) = 0.65$ m/s^2. The velocity is $v(t) = at$ m/s, since $v(0) = 0$. The displacement is $s = \dfrac{a}{2}t^2 ft$, since $s(0) = 0$. The position after 2 s is $\boxed{s(2) = 1.35 \text{ m}}$ up the inclined surface.

Problem 14.37 In Problem 14.36, determine how far the crate moves from its initial position in 2 s if the horizontal force $F = 133.4$ N.

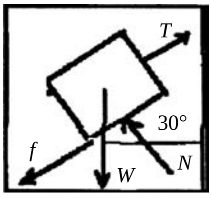
Solution: Use the definitions of terms given in the solution to Problem 14.36. For $F = 133.4$ N, $N = F\sin\theta + W\cos\theta = 452$ N, and $F_c = F\cos\theta - W\sin\theta = -106.8$ N from which, $|F_c| = 106.8 > |\mu_s N| = 90.4$, so slip occurs. Since $F_c < 0$, *the box will slip down the surface*. From the solution to Problem 14.36, after slip occurs, $a = \left(\frac{g}{W}\right)(F_c - \mathrm{sgn}(F_c)\mu_k N) = -0.761$ m/s^2. The position is $s(t) = \frac{a}{2}t^2$. At 2 seconds, $\boxed{s(2) = -1.52 \text{ m}}$ down the surface.

Problem 14.38 The crate has a mass of 120 kg, and the coefficients of friction between it and the sloping dock are $\mu_s = 0.6$ and $\mu_k = 0.5$.

(a) What tension must the winch exert on the cable to start the stationary crate sliding up the dock?
(b) If the tension is maintained at the value determined in part (a), what is the magnitude of the crate's velocity when it has moved 2 m up the dock?



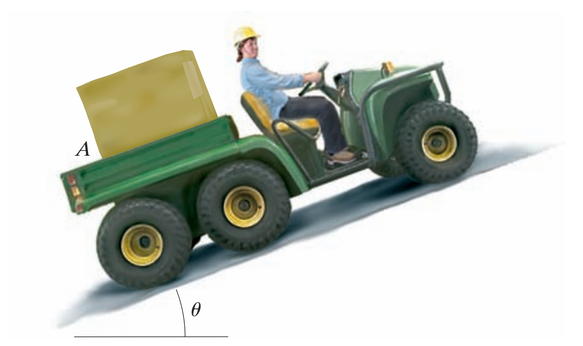
Solution: Choose a coordinate system with the x axis parallel to the surface. Denote $\theta = 30°$.



(a) The normal force exerted by the surface on the crate is $N = W\cos\theta = 120(9.81)(0.866) = 1019.5$ N. The force tending to move the crate is $F_c = T - W\sin\theta$, from which the tension required to start slip is $\boxed{T = W(\sin\theta) + \mu_s N = 1200.3 \text{ N.}}$

(b) After slip begins, the force acting to move the crate is $F = T - W\sin\theta - \mu_k N = 101.95$ N. From Newton's second law, $F = ma$, from which $a = \left(\frac{F}{m}\right) = \frac{101.95}{120} = 0.8496$ m/s^2. The velocity is $v(t) = at = 0.8496t$ m/s, since $v(0) = 0$. The position is $s(t) = \frac{a}{2}t^2$, since $s(0) = 0$. When the crate has moved 2 m up the slope, $t_{10} = \sqrt{\frac{2(2)}{a}} = 2.17$ s and the velocity is $\boxed{v = a(2.17) = 1.84 \text{ m/s.}}$

Problem 14.39 The coefficients of friction between the load A and the bed of the utility vehicle are $\mu_s = 0.4$ and $\mu_k = 0.36$. If the floor is level ($\theta = 0$), what is the largest acceleration (in m/s^2) of the vehicle for which the load will not slide on the bed?



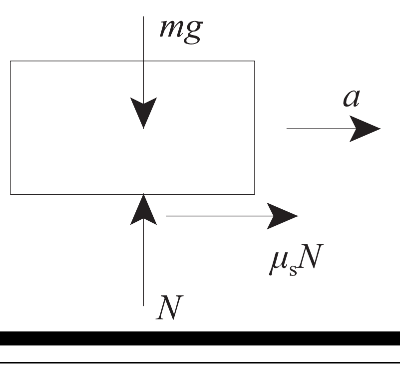
Solution: The load is on the verge of slipping. There are two unknowns (a and N). The equations are

$$\Sigma F_x : \mu_s N = ma, \quad \Sigma F_y : N - mg = 0$$

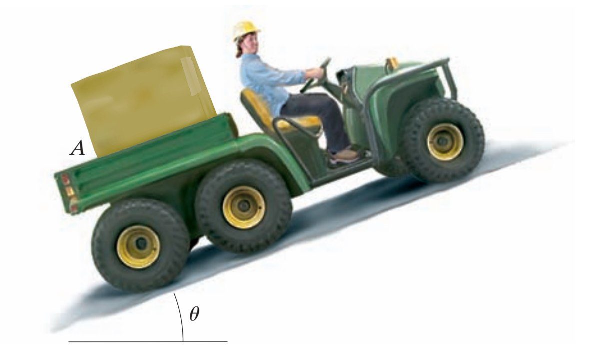
Solving we find

$$a = \mu_s g = (0.4)(9.81 \text{ m/s}^2) = 3.92 \text{ m/s}^2.$$

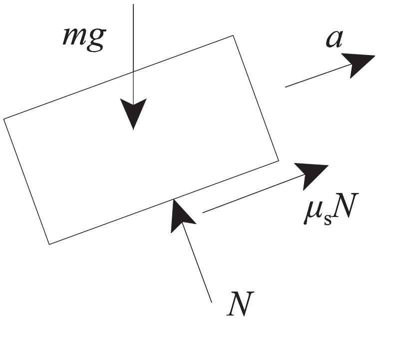
$$\boxed{a = 3.92 \text{ m/s}^2.}$$



Problem 14.40 The coefficients of friction between the load A and the bed of the utility vehicle are $\mu_s = 0.4$ and $\mu_k = 0.36$. The angle $\theta = 20^\circ$. Determine the largest forward and rearward acceleration of the vehicle for which the load will not slide on the bed.



Solution: The load is on the verge of slipping. There are two unknowns (a and N).



Forward: The equations are

$$\Sigma F_x : \mu_s N - mg\sin\theta = ma,$$

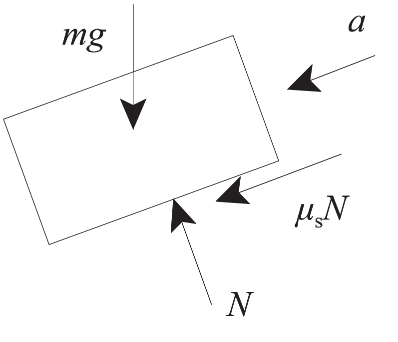
$$\Sigma F_y : N - mg\cos\theta = 0$$

Solving we find

$$a = g(\mu_s\cos\theta - \sin\theta)$$

$$= (9.81 \text{ m/s}^2)([0.4]\cos 20^\circ - \sin 20^\circ)$$

$$\boxed{a = 0.332 \text{ m/s}^2.}$$



Rearward: The equations are

$$\Sigma F_x : -\mu_s N - mg\sin\theta = -ma,$$

$$\Sigma F_y : N - mg\cos\theta = 0$$

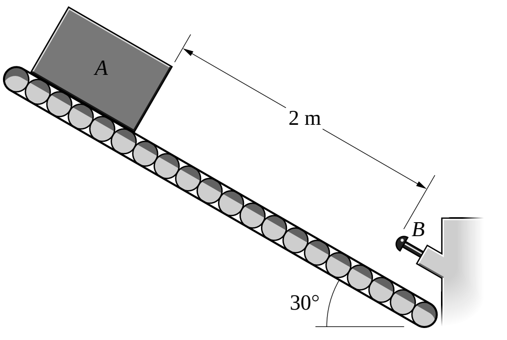
Solving we find

$$a = g(\mu_s\cos\theta + \sin\theta)$$

$$= (9.81 \text{ m/s}^2)([0.4]\cos 20^\circ + \sin 20^\circ)$$

$$\boxed{a = 7.04 \text{ m/s}^2.}$$

Problem 14.41 The package starts from rest and slides down the smooth ramp. The hydraulic device B exerts a constant 2000-N force and brings the package to rest in a distance of 100 mm from the point where it makes contact. What is the mass of the package?



Solution: Set $g = 9.81$ m/s^2

First analyze the motion before it gets to point B.

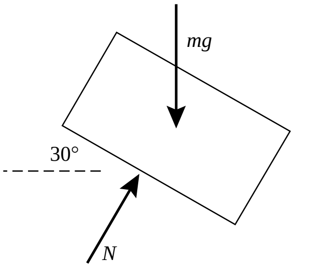
$$\sum F_{\searrow} : mg\sin 30^\circ = ma$$

$$a = g\sin 30^\circ, \quad v = (g\sin 30^\circ)t, \quad s = (g\sin 30^\circ)\frac{t^2}{2}$$

When it gets to B we have

$$s = 2\text{ m} = (g\sin 30^\circ)\frac{t^2}{2} \Rightarrow t = 0.903\text{ s}$$

$$v = (g\sin 30^\circ)(0.903\text{ s}) = 4.43\text{ m/s}$$



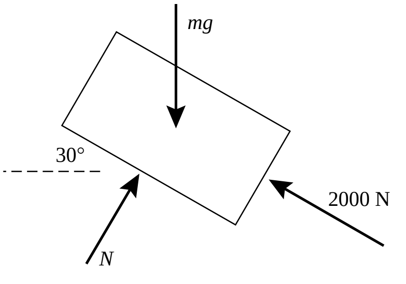
Now analyze the motion after it hits point B.

$$\sum F_{\searrow} : mg\sin 30^\circ - 2000\text{ N} = ma$$

$$a = v\frac{dv}{ds} = g\sin 30^\circ - \frac{2000\text{ N}}{m}$$

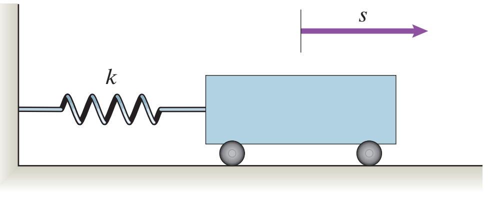
$$\int_{4.43\text{ m/s}}^{0} v\,dv = \int_0^{0.1\text{ m}} \left(g\sin 30^\circ - \frac{2000\text{ N}}{m}\right) ds$$

$$0 - \frac{(4.43\text{ m/s})^2}{2} = \left(g\sin 30^\circ - \frac{2000\text{ N}}{m}\right)(0.1\text{ m})$$



Solving the last equation we find $\boxed{m = 19.4\text{ kg}}$

Problem 14.42 The force exerted on the 10-kg mass by the linear spring is $F = -ks$, where k is the spring constant and s is the displacement of the mass relative to its position when the spring is unstretched. The value of k is 40 N/m. The mass is in the position $s = 0$ and is given an initial velocity of 4 m/s toward the right. Determine the velocity of the mass as a function of s.

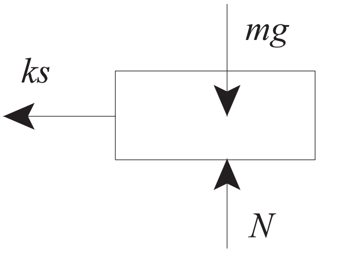


Strategy: : Use the chain rule to write the acceleration as

$$\frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = \frac{dv}{ds}v.$$

Solution: The equation of motion is $-ks = ma$

$$a = v\frac{dv}{ds} = -\frac{k}{m}s \Rightarrow \int_{v_0}^{v} v\,dv = -\int_0^s \frac{k}{m}ds \Rightarrow \frac{v^2}{2} - \frac{v_0^2}{2} = -\frac{k}{m}\frac{s^2}{2}$$



Thus

$$v = \pm\sqrt{v_0^2 - \frac{k}{m}s^2} = \pm\sqrt{(4\text{ m/s})^2 - \frac{40\text{ N/m}}{10\text{ kg}}s^2}$$

$$\boxed{v = \pm 2\sqrt{4 - s^2}\text{ m/s}}$$

Problem 14.43 The 450-kg boat is moving at 10 m/s when its engine is shut down. The magnitude of the hydrodynamic drag force (in newtons) is $40v^2$, where v is the magnitude of the velocity in m/s. When the boat's velocity has decreased to 1 m/s, what distance has it moved from its position when the engine was shut down?

Solution: The equation of motion is

$$F = -40v^2 = (450\text{ kg})a \Rightarrow a = -\frac{40}{450}v^2$$

To integrate, we write the acceleration as

$$a = v\frac{dv}{ds} = -\frac{40}{450}v^2 \Rightarrow \int_{10\text{ m/s}}^{1\text{ m/s}} \frac{dv}{v} = -\frac{40}{450}\int_0^s ds \Rightarrow \ln\left(\frac{1\text{ m/s}}{10\text{ m/s}}\right) = -\frac{40}{450}s$$

$$s = -\frac{450}{40}\ln(0.1) = 25.9\text{ m}.$$

$$\boxed{s = 25.9\text{ m.}}$$

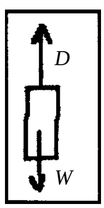
Problem 14.44 A sky diver and his parachute weigh 890 N. He is falling vertically at 30.5 m/s when his parachute opens. With the parachute open, the magnitude of the drag force (in Newton) is $0.5v^2$. (a) What is the magnitude of the sky diver's acceleration at the instant the parachute opens? (b) What is the magnitude of his velocity when he has descended 6.1 m from the point where his parachute opens?

Solution: Choose a coordinate system with s positive downward. For brevity denote $C_d = 0.5$, $W = 890$ N, $g = 9.81$ m/s^2. From Newton's second law $W - D = \left(\frac{W}{g}\right)\left(\frac{dv}{dt}\right)$, where $D = 0.5v^2$. Use the chain rule to write $v\frac{dv}{ds} = -\frac{0.5v^2 g}{W} + g = g\left(1 - \frac{C_d v^2}{W}\right)$.

(a) At the instant of full opening, the initial velocity has not decreased, and the magnitude of the acceleration is

$$\boxed{|a_{\text{init}}| = \left|g\left(1 - \frac{C_d}{W}v^2\right)\right| = 24\text{ g} = 235.3\text{ m/s}^2.}$$

(b) Separate variables and integrate: $\dfrac{v\,dv}{1 - \dfrac{C_d v^2}{W}} = g\,ds$, from which



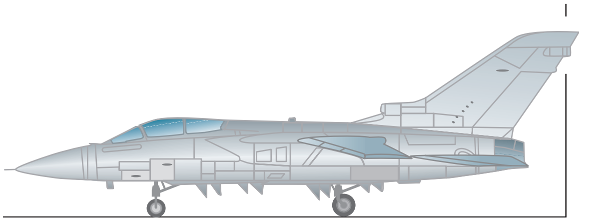
$$\ln\left(1 - \frac{C_d v^2}{W}\right) = -\left(\frac{2C_d g}{W}\right)s + C.$$ Invert and solve:

$$v^2 = \left(\frac{W}{C_d}\right)\left(1 - Ce^{-\frac{2C_d g}{W}s}\right).$$ At $s = 0$, $v(0) = 30.5$ m/s, from which $C = 1 - \dfrac{C_d 7.75^4}{W} = -1.03$, and

$$v^2 = \left(\frac{W}{C_d}\right)\left(1 + 1.03e^{-\frac{2C_d g}{W}s}\right).$$ At $s = 6.1$ m the velocity is

$$\boxed{v(s = 6.1) = \sqrt{2W\left(1 + 1.03e^{-\frac{g}{W}(6.1)}\right)} = 8.53\text{ m/s}}$$

Problem 14.45 The Panavia Tornado, with a mass of 18,000 kg, lands at a speed of 213 km/h. The decelerating force (in newtons) exerted on it by its thrust reversers and aerodynamic drag is $80{,}000 + 2.5v^2$, where v is the airplane's velocity in m/s. What is the length of the airplane's landing roll? (See Example 14.4.)



Solution:

$$v_0 = 213 \text{ km/h}\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 59.2 \text{ m/s}$$

$$\Sigma F = -(80{,}000 + 2.5v^2) = (18{,}000 \text{ kg})a$$

$$a = v\frac{dv}{ds} = -\frac{80{,}000 + 2.5v^2}{18{,}000}$$

$$\int_{v_0}^{0} \frac{v\,dv}{80{,}000 + 2.5v^2} = -\int_0^s \frac{ds}{18{,}000}$$

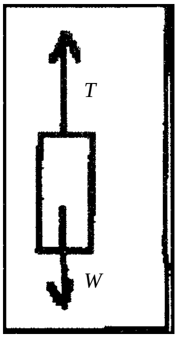
$$\frac{1}{5}\ln\left(\frac{80{,}000}{80{,}000 + 2.5v_0^2}\right) = -\frac{s}{18{,}000}$$

$$\boxed{x = 374 \text{ m.}}$$

Problem 14.46 A 890 N bungee jumper jumps from a bridge 39.6 m above a river. The bungee cord has an unstretched length of 18.3 m and has a spring constant $k = 204$ N/m. (a) How far above the river is the jumper when the cord brings him to a stop? (b) What maximum force does the cord exert on him?

Solution: Choose a coordinate system with s positive downward. Divide the fall into two parts: (1) the free fall until the bungee unstretched length is reached, (2) the fall to the full extension of the bungee cord. For Part (1): From Newton's second law $\frac{ds}{dt} = g$. Use the chain rule to write: $v\frac{dv}{ds} = g$. Separate variables and integrate:

$v^2(s) = 2$ gs, since $v(0) = 0$. At $s = 18.3$ m, $v(s = 18.3) = \sqrt{2 \text{ gs}} = 18.93$ m/s. For Part (2):
From Newton's second law



$$W - T = \frac{W}{g}\left(\frac{dv}{dt}\right), \text{ where } T = k(s - 18.3).$$

Use the chain rule to write:

$$v\frac{dv}{ds} = g - \frac{gk}{W}(s - 18.3) = g\left(1 - \frac{k}{W}(s - 18.3)\right) \; (s \geq 18.3 \text{ m}).$$

Separate variables and integrate:

$$v^2(s) = 2\,gs\left(1 - \frac{k}{2W}(s - 39.6)\right) + C. \text{ At } s = 18.3,$$

$$v^2(s = 18.3) = [2 \text{ gs}]_{s=18.3} = 39.6 \text{ g},$$

from which $C = -\frac{gk}{W}(18.3^2) = -753$. The velocity is

$$v^2(s) = -\frac{gk}{W}s^2 + 2g\left(1 + \frac{18.3\,k}{W}\right)s - \frac{gk}{W}(18.3^2).$$

When the jumper is brought to a stop, $v(s_{stop}) = 0$, from which $s^2 + 2bs + c = 0$, where $b = -\left(\frac{W}{k} + 18.3\right)$, and $c = 18.3^2$. The solution:

$$s_{\text{stop}} = -b \pm \sqrt{b^2 - c} = 36, = 9.29 \text{ m}.$$

(a) The first value represents the maximum distance on the first excursion, from which

$$\boxed{h = 39.6 - 36 = 3.6 \text{ m}}$$

is the height above the river at which he comes to a stop. (b) The maximum force exerted by the bungee cord is

$$\boxed{F = k(s - 18.3) = 204(36 - 18.3) = 3610.8 \text{ N}}$$

Problem 14.47 A helicopter weighs 91.2 kN. It takes off vertically from sea level, and its upward velocity is given as a function of its altitude h in metre by $v = 66 - 0.01\ h$ m/s

(a) How long does it take the helicopter to climb to an altitude of 1219 m?
(b) What is the sum of the vertical forces on the helicopter when its altitude is 610 m?

Solution:

(a) $v = \dfrac{dh}{dt} = 66 - 0.01h, \Rightarrow \displaystyle\int_0^t dt = \int_0^{1219} \frac{dh}{66 - 0.01h}$

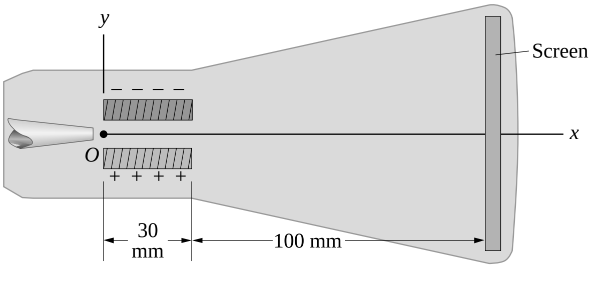
$\Rightarrow \boxed{t = 20.5\ \text{s}}$

(b) $a = v\dfrac{dv}{dh} = (66 - 0.01h)(-0.01) = -0.66 + 0.0001h$

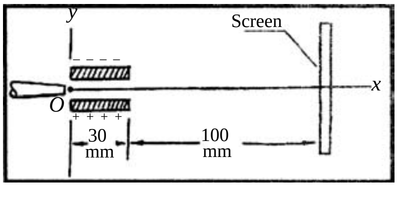
At $h = 610$ m we have

$a = -0.6\ \text{m/s}^2 \Rightarrow \boxed{F = \left(\dfrac{91200\ \text{N}}{9.81\ \text{m/s}^2}\right)(-0.6\ \text{m/s}^2) = -5578\ \text{N}}$

Problem 14.48 In a cathode-ray tube, an electron (mass $= 9.11 \times 10^{-31}$ kg) is projected at O with velocity $\mathbf{v} = (2.2 \times 10^7)\mathbf{i}$ (m/s). While the electron is between the charged plates, the electric field generated by the plates subjects it to a force $\mathbf{F} = -eE\mathbf{j}$, where the charge of the electron $e = 1.6 \times 10^{-19}$ C (coulombs) and the electric field strength $E = 15$ kN/C. External forces on the electron are negligible when it is not between the plates. Where does the electron strike the screen?



Solution: For brevity denote $L = 0.03$ m, $D = 0.1$ m. The time spent between the charged plates is $t_p = \dfrac{L}{V} = \dfrac{3 \times 10^{-2}\ \text{m}}{2.2 \times 10^7\ \text{m/s}} = 1.3636 \times 10^{-9}$ s. From Newton's second law, $\mathbf{F} = m_e\mathbf{a}_p$. The acceleration due to the charged plates is



$$\mathbf{a}_p = \frac{-eE}{m_e}\mathbf{j} = -\frac{(1.6 \times 10^{-19})(15 \times 10^3)}{9.11 \times 10^{-31}}\mathbf{j} = \mathbf{j}2.6345 \times 10^{15}\ \text{m/s}^2.$$

The velocity is $\mathbf{v}_y = -\mathbf{a}_p t$ and the displacement is $\mathbf{y} = \dfrac{\mathbf{a}_p}{2}t^2$. At the exit from the plates the displacement is $\mathbf{y}_p = -\dfrac{\mathbf{a}_p t_p^2}{2} = -\mathbf{j}2.4494 \times 10^{-3}$ (m). The velocity is $\mathbf{v}_{yp} = -\mathbf{a}_p t = -\mathbf{j}3.59246 \times 10^6$ m/s. The time spent in traversing the distance between the plates and the screen is $t_{ps} = \dfrac{D}{V} = \dfrac{10^{-1}\ \text{m}}{2.2 \times 10^7\ \text{m/s}} = 4.5455 \times 10^{-9}$ s. The vertical displacement at the screen is

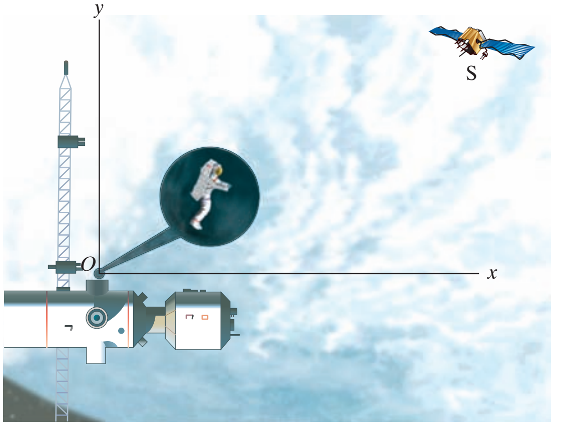
$$\boxed{\begin{aligned}\mathbf{y}_s = \mathbf{v}_{yp}t_{ps} + \mathbf{y}_p &= -\mathbf{j}(3.592456 \times 10^6)(4.5455 \times 10^{-9}) \\ &- \mathbf{j}2.4494 \times 10^{-3} = -18.8\mathbf{j}\ \text{(mm)}\end{aligned}}$$

Problem 14.49 In Problem 14.48, determine where the electron strikes the screen if the electric field strength is $E = 15\sin(\omega t)$ kN/C, where the frequency $\omega = 2 \times 10^9\ \text{s}^{-1}$.

Solution: Use the solution to Problem 14.48. Assume that the electron enters the space between the charged plates at $t = 0$, so that at that instant the electric field strength is zero. The acceleration due to the charged plates is $\mathbf{a} = -\frac{eE}{m_e}\mathbf{j} = -\frac{(1.6 \times 10^{-19}\ \text{C})(15000 \sin\omega t\ \text{N/C})}{9.11 \times 10^{-31}\ \text{kg}}\mathbf{j} = -\mathbf{j}(2.6345 \times 10^{15})$ $\sin\omega t\,(\text{m/s}^2)$. The velocity is $\mathbf{v}_y = \mathbf{j}\frac{(2.6345 \times 10^{15})}{\omega}\cos\omega t + \mathbf{C}$. Since $\mathbf{v}_y(0) = 0$ $\mathbf{C} = -\frac{2.6345 \times 10^{15}}{2 \times 10^9}\mathbf{j} = -\mathbf{j}1.3172 \times 10^6$. The displacement is $\mathbf{y} = \mathbf{j}\frac{(2.6345 \times 10^{15})}{\omega^2}\sin\omega t + \mathbf{C}t$, since $\mathbf{y}(0) = 0$. The time spent between the charged plates is (see Problem 14.48) $t_p = 1.3636 \times 10^{-9}$ s, from which $\omega t_p = 2.7273$ rad. At exit from the plates, the vertical velocity is $\mathbf{v}_{yp} = \mathbf{j}\frac{2.6345 \times 10^{15}}{2 \times 10^9}\cos(\omega t_p) + \mathbf{C} = -\mathbf{j}2.523 \times 10^6$ (m/s).

The displacement is $\mathbf{y}_p = \mathbf{j}\frac{2.6345 \times 10^{15}}{4 \times 10^{18}}\sin(\omega t_p) + \mathbf{C}t_p = -\mathbf{j}1.531 \times 10^{-3}$ (m). The time spent between the plates and the screen is $t_{ps} = 4.5455 \times 10^{-9}$ s. The vertical deflection at the screen is $\boxed{\mathbf{y}_s = \mathbf{y}_p + \mathbf{v}_{yp}t_{ps} = -13\mathbf{j}(\text{mm})}$

Problem 14.50 An astronaut wants to travel from a space station to a satellites S that needs repair. She departs the space station at O. A spring-loaded launching device gives her maneuvering unit an initial velocity of 1 m/s (relative to the space station) in the y direction. At that instant, the position of the satellite is $x = 70$ m, $y = 50$ m, $z = 0$, and it is drifting at 2 m/s (relative to the station) in the x direction. The astronaut intercepts the satellite by applying a constant thrust parallel to the x axis. The total mass of the astronaut and her maneuvering unit is 300 kg. (a) How long does it take the astronaut to reach the satellite? (b) What is the magnitude of the thrust she must apply to make the intercept? (c) What is the astronaut's velocity *relative to the satellite* when she reaches it?

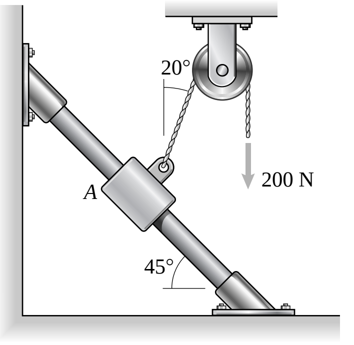


Solution: The path of the satellite relative to the space station is $x_s(t) = 2t + 70$ m, $y_s(t) = 50$ m. From Newton's second law, $T = ma_x$, $0 = ma_y$. Integrate to obtain the path of the astronaut, using the initial conditions $v_x = 0$, $v_y = 1$ m/s, $x = 0$, $y = 0$. $y_a(t) = t$, $x_a(t) = \frac{T}{2m}t^2$. (a) When the astronaut intercepts the x path of the satellite, $y_a(t_{\text{int}}) = y_s(t_{\text{int}})$, from which $\boxed{t_{\text{int}} = 50\ \text{s}}$. (b) The intercept of the y-axis path occurs when $x_a(t_{\text{int}}) = x_s(t_{\text{int}})$, from which $\frac{T}{2m}t_{\text{int}}^2 = 2t_{\text{int}} + 70$, from which

$$\boxed{T = (2m)\left(\frac{2t_{\text{int}} + 70}{t_{\text{int}}^2}\right) = 2(300)\left(\frac{170}{2500}\right) = 40.8\ \text{N}.}$$

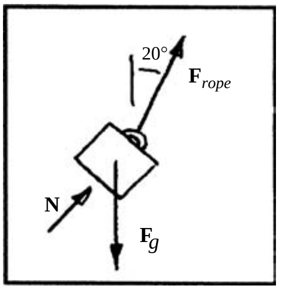
(c) The velocity of the astronaut relative to the space station is $\mathbf{v} = \mathbf{i}\left(\frac{T}{m}\right)t_{\text{int}} + \mathbf{j} = 6.8\mathbf{i} + \mathbf{j}$. The velocity of the satellite relative to the space station is $\mathbf{v}_s = 2\mathbf{i}$. The velocity of the astronaut relative to the satellite is $\boxed{\mathbf{v}_{a/s} = \mathbf{i}(6.8 - 2) + \mathbf{j} = 4.8\mathbf{i} + \mathbf{j}\ (\text{m/s})}$

Problem 14.51 What is the acceleration of the 8-kg collar A relative to the smooth bar?



Solution: For brevity, denote $\theta = 20°$, $\alpha = 45°$, $F = 200$ N, $m = 8$ kg. The force exerted by the rope on the collar is $\mathbf{F}_{rope} = 200(\mathbf{i}\sin\theta + \mathbf{j}\cos\theta) = 68.4\mathbf{i} + 187.9\mathbf{j}$ (N). The force due to gravity is $\mathbf{F}_g = -gm\mathbf{j} = -78.5\mathbf{j}$ N. The unit vector parallel to the bar, positive upward, is $\mathbf{e}_B = -\mathbf{i}\cos\alpha + \mathbf{j}\sin\alpha$. The sum of the forces acting to move the collar is $\sum F = F_c = \mathbf{e}_B \cdot \mathbf{F}_{rope} + \mathbf{e}_B \cdot \mathbf{F}_g = |\mathbf{F}_{rope}|\sin(\alpha - \theta) - gm\sin\alpha = 29.03$ N. The collar tends to slide up the bar since $F_c > 0$. From Newton's second law, the acceleration is

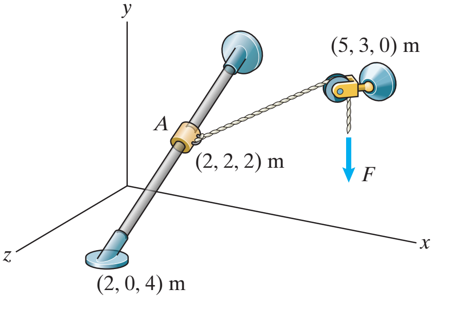
$$\boxed{a = \frac{F_c}{m} = 3.63 \text{ m/s}^2.}$$



Problem 14.52 In Problem 14.51, determine the acceleration of the 8-kg collar A relative to the bar if the coefficient of kinetic friction between the collar and the bar is $\mu_k = 0.1$.

Solution: Use the solution to Problem 14.51. $F_c = |\mathbf{F}_{rope}|\sin(\alpha - \theta) - gm\sin\alpha = 29.03$ N. The normal force is perpendicular to the bar, with the unit vector $\mathbf{e}_N = \mathbf{i}\sin\alpha + \mathbf{j}\cos\alpha$. The normal force is $N = \mathbf{e}_N \cdot \mathbf{F}_{rope} + \mathbf{e}_N \cdot \mathbf{F}_g = |\mathbf{F}_{rope}|\cos(\alpha - \theta) - gm\cos\alpha = 125.77$ N. The collar tends to slide up the bar since $F_c > 0$. The friction force opposes the motion, so that the sum of the forces on the collar is $\sum F = F_c - \mu_k N = 16.45$ N. From Newton's second law, the acceleration of the collar is $\boxed{a = \frac{16.45}{8} = 2.06 \text{ m/s}^2}$ up the bar.

Problem 14.53 The force $F = 50$ N. What is the magnitude of the acceleration of the 20-N collar A along the smooth bar at the instant shown?



Solution: The force in the rope is

$$\mathbf{F}_{rope} = (50 \text{ N})\frac{(5-2)\mathbf{i} + (3-2)\mathbf{j} + (0-2)\mathbf{k}}{\sqrt{(5-2)^2 + (3-2)^2 + (0-2)^2}}$$

The force of gravity is $\mathbf{F}_{grav} = -(20 \text{ N})\mathbf{j}$

The unit vector along the bar is

$$\mathbf{e}_{bar} = \frac{(2-2)\mathbf{i} + (2-0)\mathbf{j} + (2-4)\mathbf{k}}{\sqrt{(2-2)^2 + (2-0)^2 + (2-4)^2}}$$

The component of the total force along the bar is

$$F_{bar} = \mathbf{F} \cdot \mathbf{e}_{bar} = 14.2 \text{ N}$$

Thus $\boxed{14.2 \text{ N} = \left(\frac{20 \text{ N}}{9.81 \text{ m/s}^2}\right) a \Rightarrow a = 6.96 \text{ m/s}^2}$

Problem 14.54* In Problem 14.53, determine the magnitude of the acceleration of the 20-N collar A along the bar at the instant shown if the coefficient of static friction between the collar and the bar is $\mu_k = 0.2$.

Solution: Use the results from Problem 14.53.

The magnitude of the total force exerted on the bar is

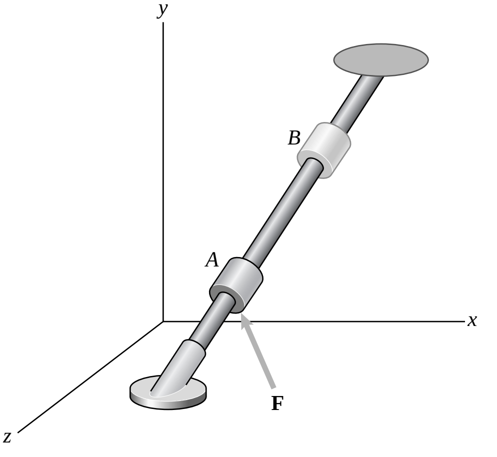
$$F = |\mathbf{F}_{rope} + \mathbf{F}_{grav}| = 48.6 \text{ N}$$

The normal force is $N = \sqrt{F^2 - F_{bar}{}^2} = 46.5$ N

The total force along the bar is now $F_{bar} - 0.2N = 4.90$ N

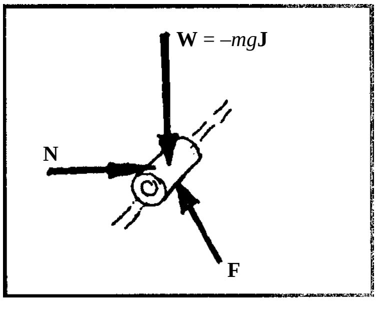
Thus $\boxed{4.90 \text{ N} = \left(\dfrac{20 \text{ N}}{9.81 \text{ m/s}^2}\right) a \Rightarrow a = 2.4 \text{ m/s}^2}$

Problem 14.55 The 6-kg collar starts from rest at position A, where the coordinates of its center of mass are (400, 200, 200) mm, and slides up the smooth bar to position B, where the coordinates of its center of mass are (500, 400, 0) mm under the action of a constant force $\mathbf{F} = -40\mathbf{i} + 70\mathbf{j} - 40\mathbf{k}$ (N). How long does it take to go from A to B?



Strategy: There are several ways to work this problem. One of the most straightforward ways is to note that the motion is along the straight line from A to B and that only the force components parallel to line AB cause acceleration. Thus, a good plan would be to find a unit vector from A toward B and to project all of the forces acting on the collar onto line AB. The resulting constant force (tangent to the path), will cause the acceleration of the collar. We then only need to find the distance from A to B to be able to analyze the motion of the collar.

Solution: The unit vector from A toward B is $\mathbf{e}_{AB} = \mathbf{e}_t = 0.333\mathbf{i} + 0.667\mathbf{j} - 0.667\mathbf{k}$ and the distance from A to B is 0.3 m. The free body diagram of the collar is shown at the right. There are three forces acting on the collar. These are the applied force $\mathbf{F}$, the weight force $\mathbf{W} = -mg\mathbf{j} = -58.86\mathbf{j}$ (N), and the force $\mathbf{N}$ which acts normal to the smooth bar. Note that $\mathbf{N}$, by its definition, will have no component tangent to the bar. Thus, we need only consider $\mathbf{F}$ and $\mathbf{W}$ when finding force components tangent to the bar. Also note that $\mathbf{N}$ is the force that the bar exerts on the collar to keep it in line with the bar. This will be important in the next problem.



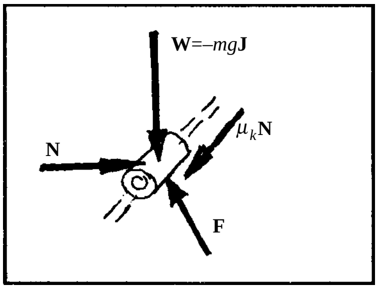
The equation of motion for the collar is $\sum \mathbf{F}_{\text{collar}} = \mathbf{F} + \mathbf{W} + \mathbf{N} = m\mathbf{a}$. In the direction tangent to the bar, the equation is $(\mathbf{F} + \mathbf{W}) \cdot \mathbf{e}_{AB} = ma_t$.

The projection of $(\mathbf{F} + \mathbf{W})$ onto line AB yields a force along AB which is $|\mathbf{F}_{AB}| = 20.76$ N. The acceleration of the 6-kg collar caused by this force is $a_t = 3.46$ m/s^2. We now only need to know how long it takes the collar to move a distance of 0.3 m, starting from rest, with this acceleration. The kinematic equations are $v_t = a_t t$, and $s_t = a_t t^2/2$. We set $s_t = 0.3$ m and solve for the time. The time required is $t = 0.416$ s

Problem 14.56* In Problem 14.55, how long does the collar take to go from A to B if the coefficient of kinetic friction between the collar and the bar is $\mu_k = 0.2$?

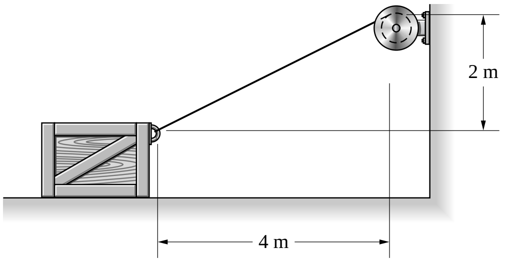
Strategy: This problem is almost the same as problem 14.55. The major difference is that now we must calculate the magnitude of the normal force, $\mathbf{N}$, and then must add a term $\mu_k|\mathbf{N}|$ to the forces tangent to the bar (in the direction from B toward A — opposing the motion). This will give us a new acceleration, which will result in a longer time for the collar to go from A to B.

Solution: We use the unit vector $\mathbf{e}_{AB}$ from Problem 14.55. The free body diagram for the collar is shown at the right. There are four forces acting on the collar. These are the applied force $\mathbf{F}$, the weight force $\mathbf{W} = -mg\mathbf{j} = -58.86\ \mathbf{j}$ (N), the force $\mathbf{N}$ which acts normal to the smooth bar, and the friction force $\mathbf{f} = -\mu_k|\mathbf{N}|\mathbf{e}_{AB}$. The normal force must be equal and opposite to the components of the forces $\mathbf{F}$ and $\mathbf{W}$ which are perpendicular (not parallel) to AB. The friction force is parallel to AB. The magnitude of $|\mathbf{F}+\mathbf{W}|$ is calculate by adding these two known forces and then finding the magnitude of the sum. The result is that $|\mathbf{F}+\mathbf{W}| = 57.66$ N. From Problem 14.55, we know that the component of $|\mathbf{F}+\mathbf{W}|$ tangent to the bar is $|\mathbf{F}_{AB}| = 20.76$ N. Hence, knowing the total force and its component tangent to the bar, we can find the magnitude of its component normal to the bar. Thus, the magnitude of the component of $|\mathbf{F}+\mathbf{W}|$ normal to the bar is 53.79 N. This is also the magnitude of the normal force $\mathbf{N}$. The equation of motion for the collar is $\sum \mathbf{F}_{\text{collar}} = \mathbf{F}+\mathbf{W}+\mathbf{N}-\mu_k|\mathbf{N}|\mathbf{e}_{AB} = m\mathbf{a}$. In the direction tangent to the bar, the equation is $(\mathbf{F}+\mathbf{W})\cdot\mathbf{e}_{AB} - \mu_k|\mathbf{N}| = ma_t$.

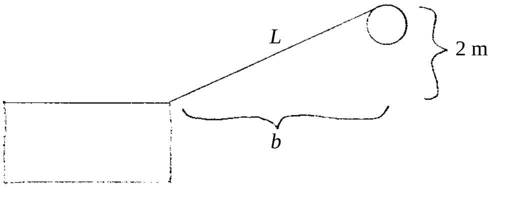


The force tangent to the bar is $F_{AB} = (\mathbf{F}+\mathbf{W})\cdot\mathbf{e}_{AB} - \mu_k|\mathbf{N}| = 10.00$ N. The acceleration of the 6-kg collar caused by this force is $a_t = 1.667\ \text{m/s}^2$. We now only need to know how long it takes the collar to move a distance of 0.3 m, starting from rest, with this acceleration. The kinematic equations are $v_t = a_t t$, and $s_t = a_t t^2/2$. We set $s_t = 0.3$ m and solve for the time. The time required is $t = 0.600$ s

Problem 14.57 The crate is drawn across the floor by a winch that retracts the cable at a constant rate of 0.2 m/s. The crate's mass is 120 kg, and the coefficient of kinetic friction between the crate and the floor is $\mu_k = 0.24$. (a) At the instant shown, what is the tension in the cable? (b) Obtain a "quasi-static" solution for the tension in the cable by ignoring the crate's acceleration. Compare this solution with your result in (a).

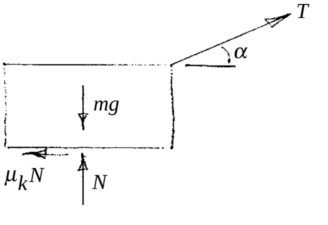


Solution:



(a) Note that $b^2 + (2)^2 = L^2$, so $b\dfrac{db}{dt} = L\dfrac{dL}{dt}$ and $b\dfrac{d^2b}{dt^2} + \left(\dfrac{db}{dt}\right)^2 = L\dfrac{d^2L}{dt^2} + \left(\dfrac{dL}{dt}\right)^2$.

Setting $b = 4$ m, $dL/dt = -0.2$ m/s and $d^2L/dt^2 = 0$, we obtain $d^2b/dt^2 = -0.0025\ \text{m/s}^2$. The crate's acceleration toward the right is $a = 0.0025\ \text{m/s}^2$.



From the free-body diagram,

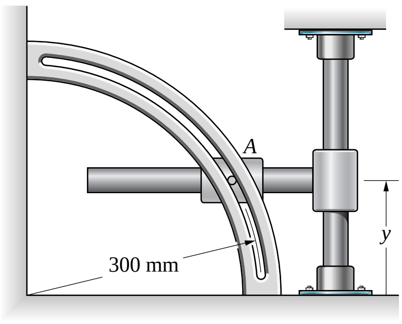
$$T\sin\alpha + N - mg = 0, \quad (1)$$

$$T\cos\alpha - \mu_k N = ma, \quad (2)$$

where $\alpha = \arctan(2/4) = 26.6^\circ$. Solving Eqs (1) and (2), we obtain $\underline{T = 282.3\ \text{N}}$.

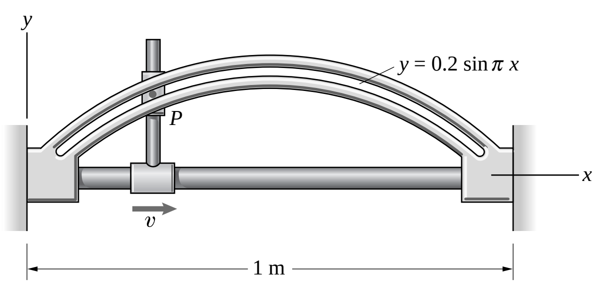
(b) Solving Eqs (1) and (2) with $a = 0$, we obtain $\underline{T = 282.0\ \text{N}}$.

Problem 14.58 If $y = 100$ mm, $\dfrac{dy}{dt} = 600$ mm/s, and $\dfrac{d^2y}{dt^2} = -200$ mm/s^2, what horizontal force is exerted on the 0.4 kg slider A by the smooth circular slot?



Solution: The horizontal displacement is $x^2 = R^2 - y^2$. Differentiate twice with respect to time: $x\dfrac{dx}{dt} = -y\dfrac{dy}{dt}$, $\left(\dfrac{dx}{dt}\right)^2 + x\dfrac{d^2x}{dt^2} = -\left(\dfrac{dy}{dt}\right)^2 - y\left(\dfrac{d^2y}{dt^2}\right)$, from which. $\dfrac{d^2x}{dt^2} = -\left(\dfrac{1}{x}\right)\left(\left(\dfrac{y}{x}\right)^2 + 1\right)\left(\dfrac{dy}{dt}\right)^2 - \left(\dfrac{y}{x}\right)\dfrac{d^2y}{dt^2}$. Substitute: $\dfrac{d^2x}{dt^2} = -1.3612$ m/s^2. From Newton's second law, $\boxed{F_h = ma_x = -1.361(0.4) = -0.544 \text{ N}}$

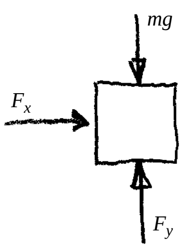
Problem 14.59 The 1-kg collar P slides on the vertical bar and has a pin that slides in the curved slot. The vertical bar moves with constant velocity $v = 2$ m/s. The y axis is vertical. What are the x and y components of the total force exerted on the collar by the vertical bar and the slotted bar when $x = 0.25$ m?



Solution:

$$v_x = \frac{dx}{dt} = 2 \text{ m/s, constant}$$

$$a_x = 0 = \frac{d^2x}{dt^2}$$



$$y = 0.2\sin(\pi x)$$

$$v_y = 0.2\pi\cos(\pi x)\frac{dx}{dt}$$

$$a_y = -0.2\pi^2\sin(\pi x)\left(\frac{dx}{dt}\right)^2 + 0.2\pi\cos(\pi x)\frac{d^2x}{dt^2}$$

when $x = 0.25$ m,

$$a_x = 0$$

$$a_y = -0.2\pi^2\sin\left(\frac{\pi}{4}\right)(2)^2 \text{ m/s}$$

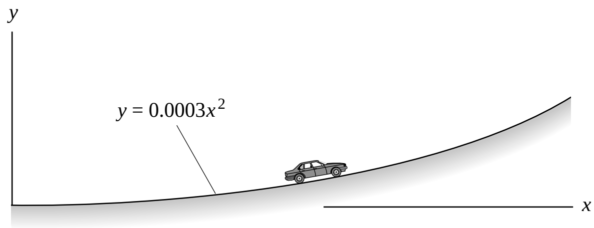
$$a_y = -5.58 \text{ m/s}^2$$

$$\sum F_x: \quad F_x = m(0)$$

$$\sum F_y: \quad F_y - mg = ma_y$$

Solving, $\underline{F_x = 0,\ F_y = 4.23 \text{ N}}$

Problem 14.60* The 1360-kg car travels along a straight road of increasing grade whose vertical profile is given by the equation shown. The magnitude of the car's velocity is a constant 100 km/h. When $x = 200$ m, what are the x and y components of the total force acting on the car (including its weight)?



Strategy: You know that the tangential component of the car's acceleration is zero. You can use this condition together with the equation for the profile of the road to determine the x and y components of the car's acceleration.

Solution:

(1) $v = \sqrt{\left(\dfrac{dx}{dt}\right)^2 + \left(\dfrac{dy}{dt}\right)^2} = 100 \text{ km/h} = 27.78 \text{ m/s, const.}$

(2) $y = 0.0003x^2$

(3) $\dfrac{dy}{dt} = 0.0006x\dfrac{dx}{dt}$

(4) $\dfrac{d^2y}{dt^2} = 0.0006\left(\dfrac{dx}{dt}\right)^2 + 0.0006x\dfrac{d^2x}{dt^2}$

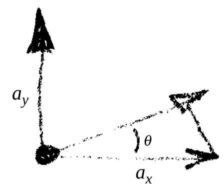
The component of acceleration parallel to the path is zero.

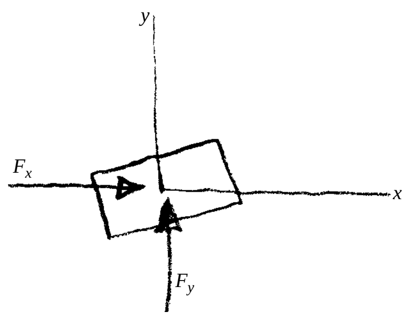
$\tan\theta = \dfrac{dy}{dx} = 0.0006\,x$

At $x = 200$ m, $\theta = 0.1194$ rad

$$\theta = 6.84^\circ$$

(5) $a_x \cos\theta + a_y \sin\theta = 0$





Solving eqns (1) through (5) simultaneously, we get

$a_x = -0.054 \text{ m/s}^2, \quad v_x = 27.6 \text{ m/s}$

$a_y = 0.450 \text{ m/s}^2, \quad v_y = 3.31 \text{ m/s}$

$m = 1360$ kg

$\sum F_x: \quad F_x = ma_x$

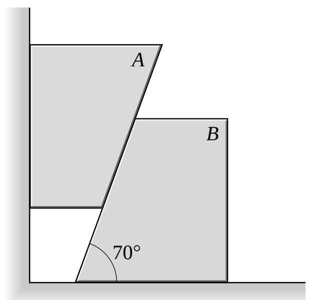
$\sum F_y: \quad F_y = ma_y$

Solving, $\underline{F_x = -73.4 \text{ N}}$

$\underline{F_y = 612 \text{ N}}$

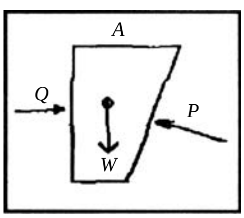
Problem 14.61* The two 445 N blocks are released from rest. Determine the magnitudes of their accelerations if friction at all contacting surfaces is negligible.

Strategy: Use the fact the components of the accelerations of the blocks perpendicular to their mutual interface must be equal.



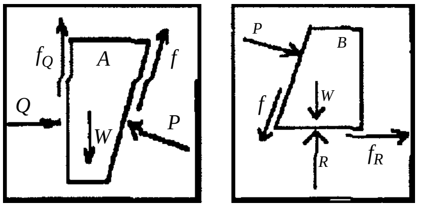
Solution: The relative motion of the blocks is constrained by the surface separating the blocks. The equation of the line separating the blocks is $y = x\tan 70^\circ$, where y is positive upward and x is positive to the right. A positive displacement of block A results in a negative displacement of B (as contact is maintained) from which $s_A = -s_B\tan 70^\circ$, and from which $\dfrac{d^2s_A}{dt^2} = -\dfrac{d^2s_B}{dt^2}\tan 70^\circ$. Thus (1) $a_A = -a_B\tan 70^\circ$.

From Newton's second law: for block A, (2) $\sum F_y = -W + P\cos 70^\circ = ma_A$, for block B, (3) $\sum F_x = P\sin 70^\circ = ma_B$, from which $a_A = -\dfrac{W}{m} + \dfrac{a_B}{\tan 70^\circ}$ m/s^2. Use (1) to obtain $a_A = -\dfrac{g}{1+\cot^2 70^\circ} = -8.66 \text{ m/s}^2$ and $a_B = -\dfrac{a_A}{\tan 70^\circ} = 3.15 \text{ m/s}^2$, where a_A is positive upward and a_B is positive to the right.



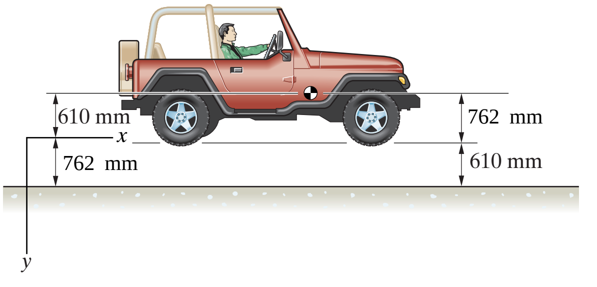
Problem 14.62* The two 445 N blocks are released from rest. The coefficient of kinetic friction between all contacting surfaces is $\mu_k = 0.1$. How long does it take block A to fall 0.305 m?

Solution: Use the results of the solution to Problem 14.61. Denote by Q the normal force at the wall, and by P the normal force at the contacting surface, and R the normal force exerted by the floor on block B. For a_A positive upward and a_B positive to the right, (1) $a_A = -a_B \tan 70^\circ$ so long as contact is maintained. From Newton's second law for block A, (2) $\sum F_x = Q - P\sin 70^\circ + f\cos 70^\circ = 0$, (3) $\sum F_y = -W + f_Q + f\cos 70^\circ + P\cos 70^\circ = ma_A$. For block B: (4) $\sum F_x = P\sin 70^\circ - f\cos 70^\circ - f_R = ma_B$, (5) $\sum F_y = -W + R - P\cos 70^\circ - f\sin 70^\circ = 0$. In addition: (6) $f = \mu_k P$, (7) $f_R = \mu_k R$, (8) $f_q = \mu_k Q$. Solve these eight equations by iteration: $a_A = -7.53$ m/s^2, $a_B = 2.74$ m/s^2. *Check*: (1) The effect of friction should reduce the downward acceleration of A in Problem 3.61, and (2) for $\mu_k = 0$, this should reduce to the solution to Problem 14.61. *check*.



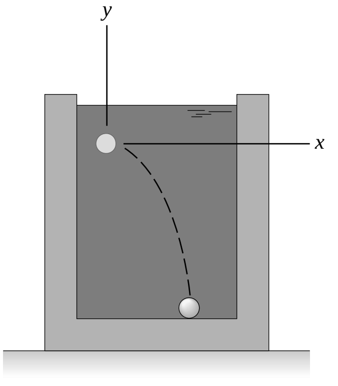
The displacement is $y = \dfrac{a_A}{2}t^2$ m, from which, for $y = -0.305$ m, $t = \sqrt{-\dfrac{2}{a_A}} = 0.284$ s

Problem 14.63 The 3000-N vehicle has left the ground after driving over a rise. At the instant shown, it is moving horizontally at 30 km/h and the bottoms of its tires are 610 mm above the (approximately) level ground. The earth-fixed coordinate system is placed with its origin 762 mm above the ground, at the height of the vehicle's center of mass when the tires first contact the ground (Assume that the vehicle remains horizontal.) When that occurs, the vehicle's center of mass initially continues moving downward and then rebounds upward due to the flexure of the suspension system. While the tires are in contact with the ground, the force exerted on them by the ground is $-2400\mathbf{i} - 18000y\mathbf{j}$ (N), where y is the vertical position of the center of mass in metre. When the vehicle rebounds what is the vertical component of the velocity, of the center of mass at the instant the wheels leave the ground? (The wheels leave the ground when the center of mass is at $y = 0$.)



Solution: This analysis follows that of Example 14.3. The equation for velocity used to determine how far down the vehicle compresses its springs also applies as the vehicle rebounds. From Example 14.3, we know that the vehicle comes to rest with $v_Y = 0$ and $y = 0.305$ m. Following the Example, the velocity on the rebound is given by $\int_0^{v_Y} v_Y\,dv_Y = \int_{0.305}^{0} 9.81(0.305 - 15y)\,dy$. Evaluation, we get $v_Y = 3.44$ m/s. ($+y$ is down). Note that the vertical velocity component on rebound is the negative of the vertical velocity of impact.

Problem 14.64* A steel sphere in a tank of oil is given an initial velocity $\mathbf{v} = 2\mathbf{i}$ (m/s) at the origin of the coordinate system shown. The radius of the sphere is 15 mm. The density of the steel is 8000 kg/m^3 and the density of the oil is 980 kg/m^3. If V is the sphere's volume, the (upward) buoyancy force on the sphere is equal to the weight of a volume V of oil. The magnitude of the hydrodynamic drag force $\mathbf{D}$ on the sphere as it falls is $|\mathbf{D}| = 1.6|\mathbf{v}|$ N, where $|\mathbf{v}|$ is the magnitude of the sphere's velocity in m/s. What are the x and y components of the sphere's velocity at $t = 0.1$ s?



Solution:

$$B = \rho_{\text{oil}} V_g$$

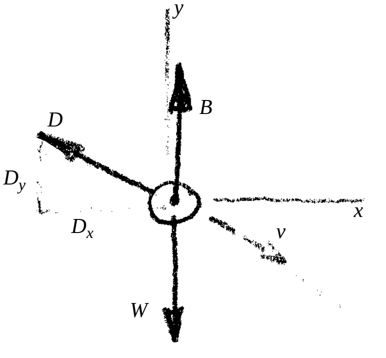
$$W = \rho_{\text{STEEL}} V_g$$

$$V = \frac{4}{3}\pi r^3$$

$$\sum F_x : \quad m_s \frac{dv_x}{dt} = -d_x = -1.6 v_x$$

$$\sum F_y : \quad m_s \frac{dv_y}{dt} = B - W - d_y$$

$$m_s \frac{dv_y}{dt} = (\rho_{\text{oil}} - \rho_{\text{STEEL}}) V_g - 1.6 v_y$$



Rewriting the equations, we get

$$\sum F_x : \quad \frac{dv_x}{dt} = -\frac{1.6}{m_s} v_x$$

$$\int_{v_{xo}}^{v_x} \frac{dv_x}{v_x} = -\frac{1.6}{m_s} \int_0^t dt$$

$$\ln(v_x)|_{v_{x0}}^{v_x} = -\frac{1.6}{m_s} t|_0^t$$

$$v_x = v_{x0} e^{-\frac{1.6}{m_s} t}$$

Substituting, we have $m_s = 0.114$ kg

$v_{x0} = 2$ m/s. At $t = 0.1$ s,

$\underline{v_x = 0.486 \text{ m/s}}$

$$\sum F_y : \quad \frac{dv_y}{dt} = \frac{(\rho_{\text{oil}} - \rho_{\text{STEEL}}) V_g}{m_s} - \frac{1.6}{m_s} v_y$$

Let $a = (\rho_{\text{oil}} - \rho_{\text{STEEL}}) V_{g/m_s} = -8.608$

$$b = -\frac{1.6}{m_s} = -14.147$$

$$\frac{dv_y}{dt} = a + b v_y$$

$$\int_0^{v_y} \frac{dy}{a + b v_y} = \int_0^t dt$$

Integrating, we get

$$\frac{1}{b} \ln(a + b v_y)|_0^{v_y} = t$$

$$\ln\left(\frac{a + b v_y}{a}\right) = bt$$

$$a + b v_y = a e^{bt}$$

$$v_y = \frac{a}{b}(e^{bt} - 1)$$

Substituting numerical values for a and b, and setting $t = 0.1$ s

$\underline{v_y = -0.461 \text{ m/s}}$

Problem 14.65* In Problem 14.64, what are the x and y coordinates of the sphere at $t = 0.1$ s?

Solution: From the solution to Problem 14.64, $\frac{dx}{dt} = v_x = v_{x0}e^{-\frac{1.6}{m_s}t} = v_{x0}e^{bt}$ where $v_{x0} = 2$ m/s and $m_s = 0.114$ kg.

Also, $\frac{dy}{dt} = v_y = \frac{a}{b}(e^{bt} - 1)$

where $a = (\rho_{\text{oil}} - \rho_{\text{STEEL}})V_{g/m} = -8.608$

$$b = -\frac{1.6}{m_s} = -14.147$$

Integrating the v_x and v_y eqns, noting that $x = 0$, $y = 0$, at $t = 0$, we get

$$x = \left(\frac{v_{xo}}{b}\right)(e^{bt} - 1)$$

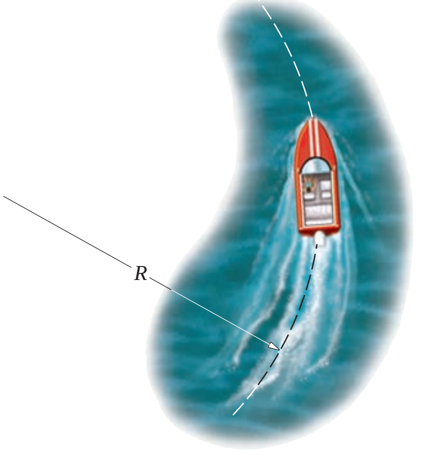
$$y = \frac{a}{b^2}(e^{bt} - 1) - \frac{a}{b}t$$

Solving at $t = 0.1$ s,

$\underline{x = 0.1070 \text{ m} = 107.0 \text{ mm}}$

$\underline{y = -0.0283 \text{ m} = -28.3 \text{ mm}}$

Problem 14.66 The boat in Active Example 14.5 weighs 1200 N with its passengers. Suppose that the boat is moving at a constant speed of 20 m/s in a circular path with radius $R = 40$ m. Determine the tangential and normal components of force acting on the boat.



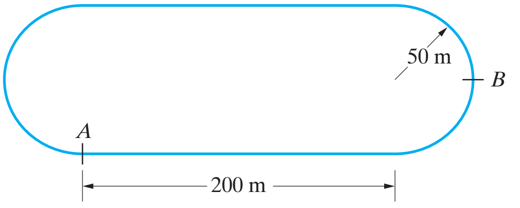
Solution: Since the speed is constant, the tangential acceleration is zero. We have

$F_t = ma_t = 0,$

$$F_n = ma_n = m\frac{v^2}{R} = \left(\frac{1200 \text{ N}}{9.81 \text{ m/s}^2}\right)\frac{(20 \text{ m/s})^2}{40 \text{ m}} = 1223 \text{ N}.$$

$\boxed{F_t = 0, F_n = 1223 \text{ N}.}$

Problem 14.67 In preliminary design studies for a sun-powered car, it is estimated that the mass of the car and driver will be 100 kg and the torque produced by the engine will result in a 60-N tangential force on the car. Suppose that the car starts from rest on the track at A and is subjected to a constant 60-N tangential force. Determine the magnitude of the car's velocity and the normal component of force on the car when it reaches B.



Solution: We first find the tangential acceleration and use that to find the velocity at B.

$$F_t = ma_t \Rightarrow 60 \text{ N} = (100 \text{ kg})\, a_t \Rightarrow a_t = 0.6 \text{ m/s}^2,$$

$$a_t = v\frac{dv}{ds} \Rightarrow \int_0^v v\,dv = \int_0^s a_t\,ds \Rightarrow \frac{v^2}{2} = a_t s,$$

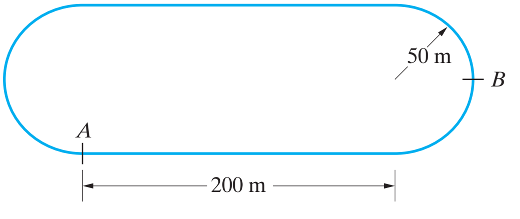
$$v_B = \sqrt{2a_t s_B} = \sqrt{2(0.6 \text{ m/s}^2)\left(200 \text{ m} + \frac{\pi}{2}[50 \text{ m}]\right)} = 18.3 \text{ m/s}.$$

The normal component of the force is

$$F_n = ma_n = m\frac{v^2}{R} = (100 \text{ kg})\frac{(18.3 \text{ m/s})^2}{50 \text{ m}} = 668 \text{ N}.$$

$$\boxed{v_B = 18.3 \text{ m/s}, \quad F_n = 668 \text{ N}.}$$

Problem 14.68 In a test of a sun-powered car, the mass of the car and driver is 100 kg. The car starts from rest on the track at A, moving toward the right. The tangential force exerted on the car (in newtons) is given as a function of time by $\Sigma F_t = 20 + 1.2t$. Determine the magnitude of the car's velocity and the normal component of force on the car at $t = 40$ s.



Solution: We first find the tangential acceleration and use that to find the velocity v and distance s as functions of time.

$$\Sigma F_t = (20 + 1.2t)N = (100 \text{ kg})\, a_t$$

$$a_t = \frac{dv}{dt} = 0.2 + 0.012t$$

$$v = 0.2t + 0.006t^2$$

$$s = 0.1t^2 + 0.002t^3$$

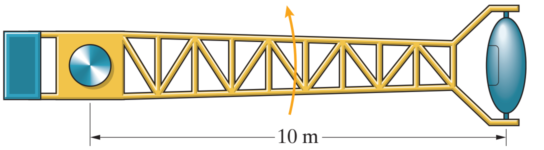
At $t = 40$ s, we have $v = 17.6$ m/s, $s = 288$ m.

For this distance the car will be on the curved part of the track. The normal component of the force is

$$F_n = ma_n = m\frac{v^2}{R} = (100 \text{ kg})\frac{(17.6 \text{ m/s})^2}{50 \text{ m}} = 620 \text{ N}.$$

$$\boxed{v_B = 17.6 \text{ m/s}, \quad F_n = 620 \text{ N}.}$$

Problem 14.69 An astronaut candidate with a mass of 72 kg is tested in a centrifuge with a radius of 10 m. The centrifuge rotates in the horizontal plane. It starts from rest at time $t = 0$ and has a constant angular acceleration of $0.2\ \text{rad/s}^2$. Determine the magnitude of the horizontal force exerted on him by the centrifuge (a) at $t = 0$; (b) at $t = 10$ s.



Solution: The accelerations are

$$a_t = r\alpha = (10\ \text{m})\ (0.2\ \text{rad/s}^2) = 2\ \text{m/s}^2$$

$$a_n = r\omega^2 = r(\alpha t)^2 = (10\ \text{m})\ (0.2\ \text{rad/s}^2)^2 t^2 = (0.4\ \text{m/s}^4)t^2$$

(a)

At $t = 0$

$$F_t = ma_t = (72\ \text{kg})(2\ \text{m/s}^2) = 144\ \text{N},\ F_n = ma_n = 0$$

$$F = \sqrt{F_t^2 + F_n^2} = 144\ \text{N}.$$

(b)

At $t = 10$ s

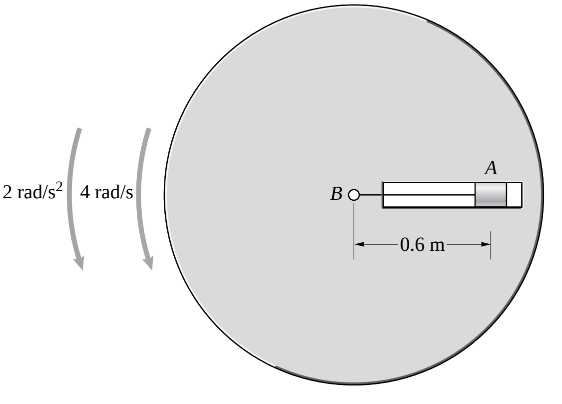
$$F_t = ma_t = (72\ \text{kg})(2\ \text{m/s}^2) = 144\ \text{N},$$

$$F_n = ma_n = (72\ \text{kg})(0.4\ \text{m/s}^4)(10\ \text{s})^2 = 2880\ \text{N}$$

$$F = \sqrt{F_t^2 + F_n^2} = 2880\ \text{N}.$$

$$\boxed{(a)\ F = 144\ \text{N},\quad (b)\ F = 2880\ \text{N}.}$$

Problem 14.70 The circular disk lies *in the horizontal plane*. At the instant shown, the disk rotates with a counterclockwise angular velocity of 4 rad/s and a counterclockwise angular acceleration of $2\ \text{rad/s}^2$. The 0.5-kg slider A is supported horizontally by the smooth slot and the string attached at B. Determine the tension in the string and the magnitude of the horizontal force exerted on the slider by the slot.



Solution:

$\omega = 4$ rad/s

$\alpha = 2\ \text{rad/s}^2$

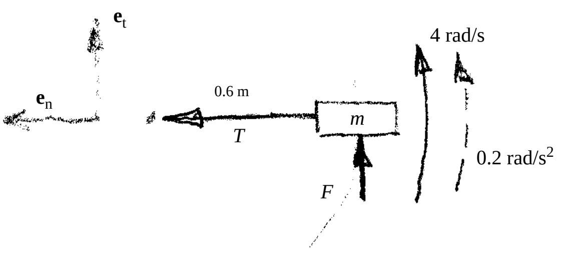
$R = 0.6$ m

$m = 0.5$ kg

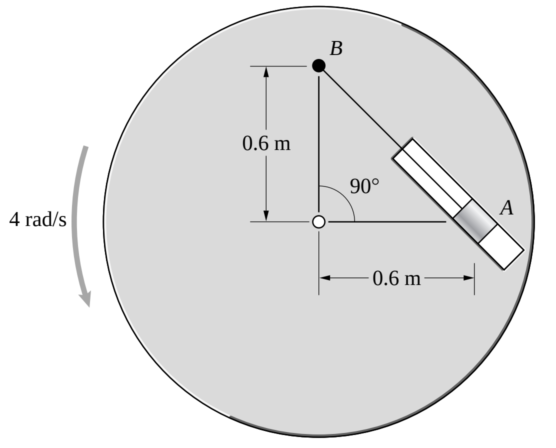
$\sum F_n : \quad T = mR\omega^2$

$\sum F_t : \quad F = mR\alpha$

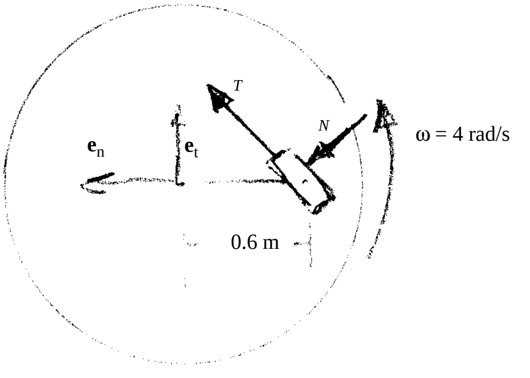
Solving, $\underline{T = 4.8\ \text{N},\ F = 0.6\ \text{N}}$



Problem 14.71 The circular disk lies *in the horizontal plane* and rotates with a constant counterclockwise angular velocity of 4 rad/s. The 0.5-kg slider A is supported horizontally by the smooth slot and the string attached at B. Determine the tension in the string and the magnitude of the horizontal force exerted on the slider by the slot.



Solution:



$$R = 0.6 \text{ m}$$

$$\omega = 4 \text{ rad/s}$$

$$m = 0.5 \text{ kg}$$

$$\alpha = 0$$

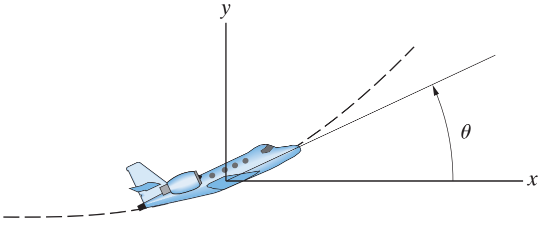
$$\sum F_n: \quad N\cos 45^\circ + T\sin 45^\circ = mR\omega^2$$

$$\sum F_t: \quad -N\sin 45^\circ + T\cos 45^\circ = mR\alpha = 0$$

Solving, $\underline{N = T = 3.39 \text{ N}}$

Problem 14.72 The 142 kN airplane is flying in the vertical plane at 128 m/s. At the instant shown the angle $\theta = 30^\circ$ and the cartesian components of the plane's acceleration are $a_x = -1.83 \text{ m/s}^2$, $a_y = 9.1 \text{ m/s}^2$.

(a) What are the tangential and normal components of the total force acting on the airplane (including its weight)?
(b) What is $d\theta/dt$ in degrees per second?



Solution:

$$\mathbf{F} = \left(\frac{142000 \text{ N}}{9.81 \text{ m/s}^2}\right)(-1.83\mathbf{i} + 9.1\mathbf{j}) \text{ m/s}^2 = (-26523\mathbf{i} + 132613\mathbf{j}) \text{ N}$$

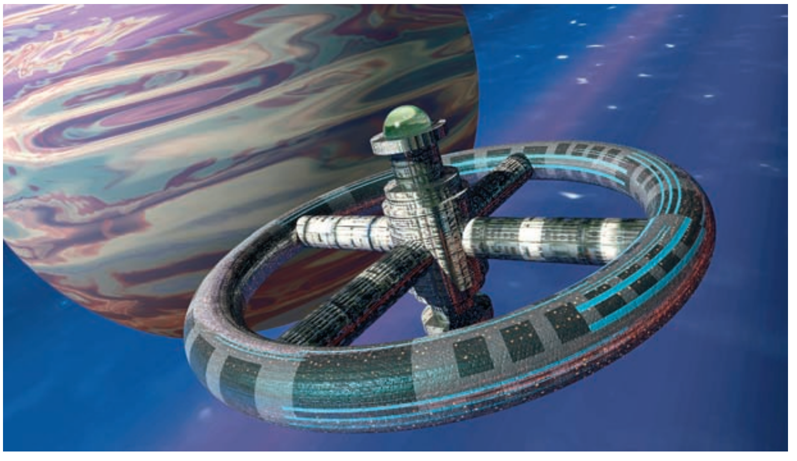
(a)
$$\boxed{\begin{aligned} F_t &= \mathbf{F}\cdot(\cos 30^\circ\mathbf{i} + \sin 30^\circ\mathbf{j}) = 43324 \text{ N} \\ F_n &= \mathbf{F}\cdot(-\sin 30^\circ\mathbf{i} + \cos 30^\circ\mathbf{j}) = 128102 \text{ N} \end{aligned}}$$

(b)
$$a_n = \frac{128102 \text{ N}}{(142000 \text{ N})/9.81 \text{ m/s}^2} = 8.83 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} \Rightarrow \rho = \frac{v^2}{a_n} = \frac{(128 \text{ m/s})^2}{(8.83 \text{ m/s}^2)} = 1855.3 \text{ m}$$

$$v = \rho\dot{\theta} \Rightarrow \boxed{\dot{\theta} = \frac{128 \text{ m/s}}{1855.3 \text{ m}} = 0.0690 \text{ rad/s}\left(\frac{180\ ^\circ}{\pi \text{ rad}}\right) = 3.95^\circ/\text{s}}$$

Problem 14.73 Consider a person with a mass of 72 kg who is in the space station described in Example 14.7. When he is in the occupied outer ring, his simulated weight in newtons is $\frac{1}{2}(72\text{ kg})(9.81\text{ m/s}^2) = 353$ N. Suppose that he climbs to a position in one of the radial tunnels that leads to the center of the station. Let r be his distance in meters from the center of the station. (a) Determine his simulated weight in his new position in terms of r. (b) What would his simulated weight be when he reaches the center of the station?



Solution: The distance to the outer ring is 100 m.

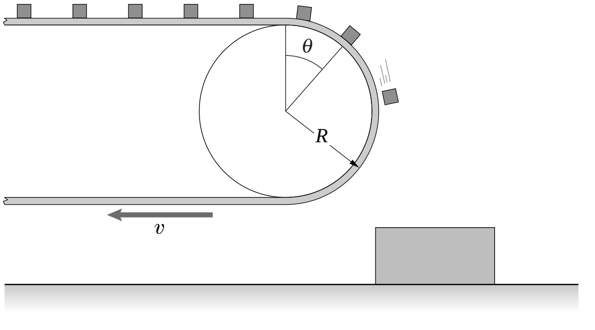
(a) At a distance r the weight would be $W = \frac{r}{100\text{ m}}(353\text{ N}) = (3.53\text{ N/m})r$

$\boxed{W = (3.53\text{ N/m})r.}$

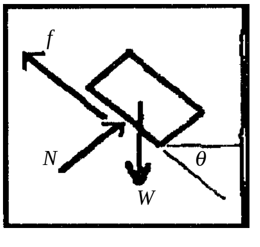
(b) At the center, $r = 0$

$\boxed{W = 0.}$

Problem 14.74 Small parts on a conveyor belt moving with constant velocity v are allowed to drop into a bin. Show that the angle θ at which the parts start sliding on the belt satisfies the equation $\cos\theta - \frac{1}{\mu_s}\sin\theta = \frac{v^2}{gR}$, where μ_s is the coefficient of static friction between the parts and the belt.

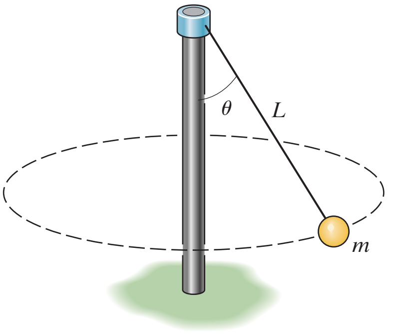


Solution: The condition for sliding is $\sum F_t = -mg\sin\theta + f = 0$, where $-mg\sin\theta$ is the component of weight acting tangentially to the belt, and $f = \mu_s$ N is the friction force tangential to the belt. From Newton's second law the force perpendicular to the belt is $N - mg\cos\theta = -m\frac{v^2}{R}$, from which the condition for slip is $-mg\sin\theta + \mu_s mg\cos\theta - \mu_s m\frac{v^2}{R} = 0$. Solve: $\cos\theta - \frac{1}{\mu_s}\sin\theta = \frac{v^2}{gR}$

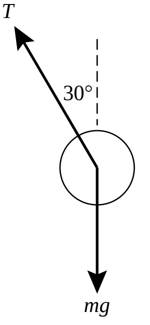


Problem 14.75 The 1-kg mass m rotates around the vertical pole in a horizontal circular path. The angle $\alpha = 30^\circ$ and the length of the string is $L = 1.22$ m. What is the magnitude of the velocity of the mass?

Strategy: Notice that the vertical acceleration of the mass is zero. Draw the free-body diagram of the mass and write Newton's second law in terms of tangential and normal components.



Solution:



$$\sum F_\uparrow : T\cos 30^\circ - mg = 0$$

$$\sum F_n : T\sin 30^\circ = m\frac{v^2}{\rho} = m\frac{v^2}{L\sin 30^\circ}$$

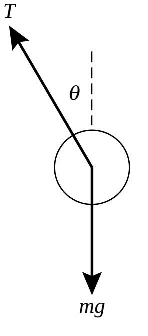
Solving we have

$$T = \frac{mg}{\cos 30^\circ},\quad v^2 = g(L\sin 30^\circ)\tan 30^\circ$$

$$\boxed{v = \sqrt{\frac{(9.81 \text{ m/s}^2)(1.22 \text{ m})\sin^2 30^\circ}{\cos 30^\circ}} = 1.86 \text{ m/s}}$$

Problem 14.76 In Problem 14.75, determine the magnitude of the velocity of the mass and the angle θ if the tension in the string is 50 N.

Solution:



$$\sum F_\uparrow : T\cos\theta - mg = 0$$

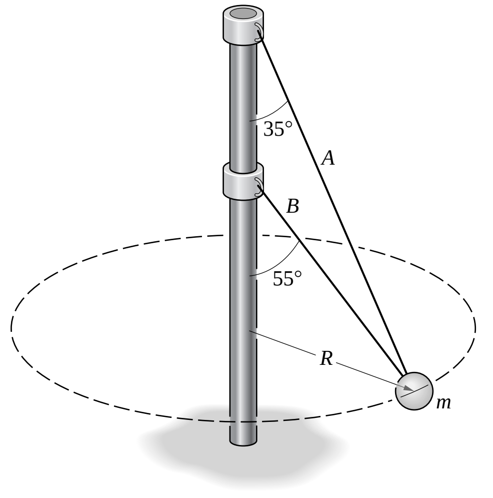
$$\sum F_n : T\sin\theta = m\frac{v^2}{L\sin\theta}$$

Solving we find $\theta = \cos^{-1}\left(\frac{mg}{T}\right)$, $v = \sqrt{\frac{(T^2 - m^2g^2)L}{Tm}}$

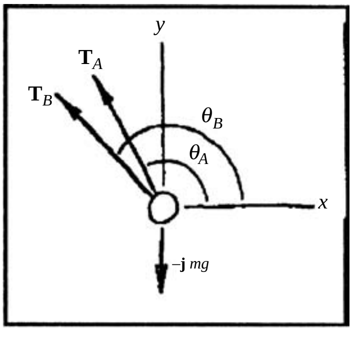
Using the problem numbers we have

$$\boxed{\begin{aligned}\theta &= \cos^{-1}\left(\frac{1 \text{ kg } 9.81 \text{ m/s}^2}{50 \text{ N}}\right) = 78.68^\circ \\ v &= \sqrt{\frac{[(50 \text{ N})^2 - (1 \text{ kg } 9.81 \text{ m/s}^2)^2]1.22 \text{ m}}{(50 \text{ N})(1 \text{ kg})}} = 7.66 \text{ m/s}\end{aligned}}$$

Problem 14.77 The 10-kg mass m rotates around the vertical pole in a horizontal circular path of radius $R = 1$ m. If the magnitude of the velocity is $v = 3$ m/s, what are the tensions in the strings A and B?



Solution: Choose a Cartesian coordinate system in the vertical plane that rotates with the mass. The weight of the mass is $\mathbf{W} = -\mathbf{j}mg = -\mathbf{j}98.1$ N. The radial acceleration is by definition directed inward:
$\mathbf{a}_n = -\mathbf{i}\left(\frac{v^2}{R}\right) = -9\mathbf{i}$ m/s^2. The angles from the horizontal are $\theta_A = 90^\circ + 35^\circ = 125^\circ$, $\theta_B = 90^\circ + 55^\circ = 145^\circ$. The unit vectors parallel to the strings, from the pole to the mass, are: $\mathbf{e}_A = +\mathbf{i}\cos\theta_A + \mathbf{j}\sin\theta_A$. $\mathbf{e}_B = +\mathbf{i}\cos\theta_B + \mathbf{j}\sin\theta_B$. From Newton's second law for the mass, $\mathbf{T} - \mathbf{W} = m\mathbf{a}_n$, from which $|\mathbf{T}_A|\mathbf{e}_A + |\mathbf{T}_B|\mathbf{e}_B - \mathbf{j}mg = -\mathbf{i}\left(m\frac{v^2}{R}\right)$. Separate components to obtain the two simultaneous equations: $|\mathbf{T}_A|\cos 125^\circ + |\mathbf{T}_B|\cos 145^\circ = -90$ N$|\mathbf{T}_A|\sin 55^\circ + |\mathbf{T}_B|\sin 35^\circ = 98.1$ N. Solve:



$\boxed{|\mathbf{T}_A| = 84 \text{ N.}}$ $\boxed{|\mathbf{T}_B| = 51 \text{ N}}$

Problem 14.78 The 10-kg mass m rotates around the vertical pole in a horizontal circular path of radius $R = 1$ m. For what range of values of the velocity v of the mass will the mass remain in the circular path described?

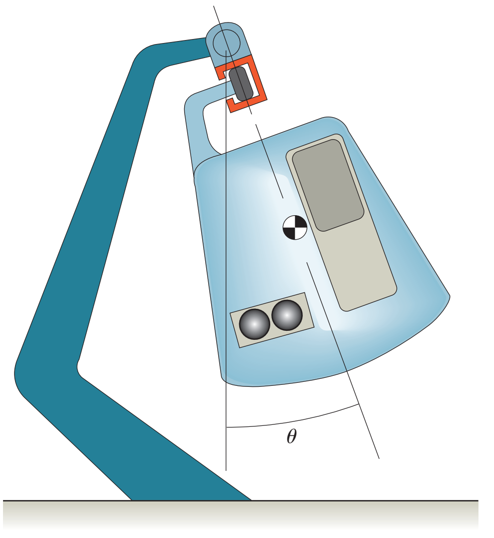
Solution: The minimum value of v will occur when the string B is impending zero, and the maximum will occur when string A is impending zero. From the solution to Problem 14.77,

$$|\mathbf{T}_A|\cos 125^\circ + |\mathbf{T}_B|\cos 145^\circ = -m\left(\frac{v^2}{R}\right),$$

$$|\mathbf{T}_A|\sin 125^\circ + |\mathbf{T}_B|\sin 145^\circ = \text{mg}.$$

These equations are to be solved for the velocity when one of the string tensions is set equal to zero. For $|\mathbf{T}_A| = 0$, $v = 3.743$ m/s. For $|\mathbf{T}_B| = 0$, $v = 2.621$ m/s. The range: $\boxed{2.62 \le v \le 3.74 \text{ m/s}}$

Problem 14.79 Suppose you are designing a monorail transportation system that will travel at 50 m/s, and you decide that the angle θ that the cars swing out from the vertical when they go through a turn must not be larger than 20°. If the turns in the track consist of circular arcs of constant radius R, what is the minimum allowable value of R? (See Active Example 14.6)



Solution: The equations of motion are

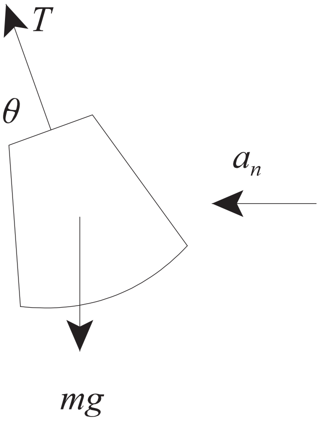
$$\Sigma F_y : T\cos\theta - mg = 0$$

$$\Sigma F_n : T\sin\theta = ma_n = m\frac{v^2}{R}$$

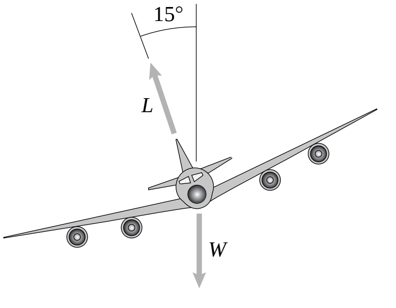
Solving we have

$$R = \frac{v^2}{g\tan\theta} = \frac{(50\text{ m/s})^2}{(9.81\text{ m/s}^2)\tan 20^\circ} = 700\text{ m}$$

$$\boxed{R = 700\text{ m.}}$$



Problem 14.80 An airplane of weight $W = 890$ kN makes a turn at constant altitude and at constant velocity $v = 183$ m/s. The bank angle is 15°. (a) Determine the lift force L. (b) What is the radius of curvature of the plane's path?



Solution: The weight is $\mathbf{W} = -\mathbf{j}W = -\mathbf{j}(890\times 10^3)$ N. The normal acceleration is $\mathbf{a}_n = \mathbf{i}\left(\frac{v^2}{\rho}\right)$. The lift is $\mathbf{L} = |\mathbf{L}|(\mathbf{i}\cos 105^\circ + \mathbf{j}\sin 105^\circ) = |\mathbf{L}|(-0.2588\mathbf{i} + 0.9659\mathbf{j})$.

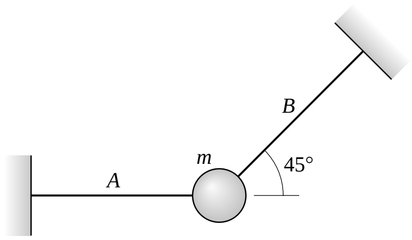
(a) From Newton's second law, $\sum \mathbf{F} = \mathbf{L} + \mathbf{W}_n = m\mathbf{a}_n$, from which, substituting values and separating the $\mathbf{j}$ components: $|\mathbf{L}|(0.9659) = 890\times 10^3$, $\boxed{|\mathbf{L}| = \frac{890\times 10^3}{0.9659} = 921420\text{ N}}$

(b) The radius of curvature is obtained from Newton's law: $|\mathbf{L}|(-0.2588) = -m\left(\frac{v^2}{\rho}\right)$, from which

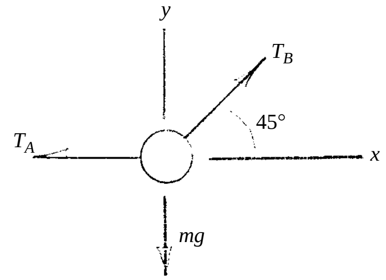
$$\boxed{\rho = \left(\frac{W}{g}\right)\left(\frac{v^2}{|\mathbf{L}|(0.2588)}\right) = 12729.6\text{ m.}}$$

Problem 14.81 The suspended 2-kg mass m is stationary.

(a) What are the tensions in the strings A and B?
(b) If string A is cut, what is the tension in string B immediately afterward?



Solution:



(a) $\sum F_x = T_B \cos 45^\circ - T_A = 0,$

$\sum F_y = T_B \sin 45^\circ - mg = 0.$

Solving yields $T_A = 19.6$ N, $T_B = 27.7$ N.

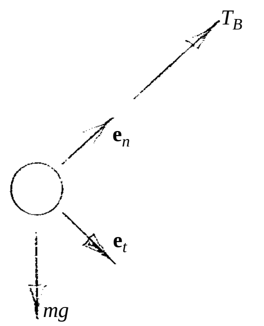
(b) Use Normal and tangential components.

$$\sum F_{\rm n} = ma_{\rm n} :$$

$$T_B - mg \cos 45^\circ = m\frac{v^2}{\rho}.$$

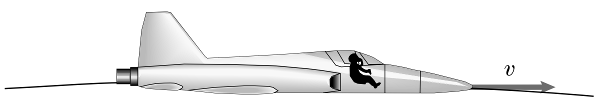
But $v = 0$ at the instant of release, so

$T_B = mg \cos 45^\circ = 13.9$ N.

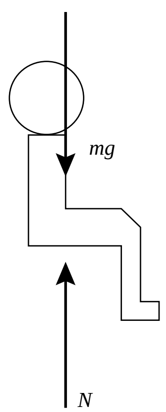


Problem 14.82 The airplane flies with constant velocity v along a circular path in the vertical plane. The radius of the airplane's circular path is 2000 m. The mass of the pilot is 72 kg.

(a) The pilot will experience "weightlessness" at the top of the circular path if the airplane exerts no net force on him at that point. Draw a free-body diagram of the pilot and use Newton's second law to determine the velocity v necessary to achieve this condition.
(b) Suppose that you don't want the force exerted on the pilot by the airplane to exceed four times his weight. If he performs this maneuver at $v =$ 200 m/s, what is the minimum acceptable radius of the circular path?



Solution: The FBD assumes that the seat pushes up on the pilot. If the seat (or shoulder straps) pushes down, we will us a negative sign for N.



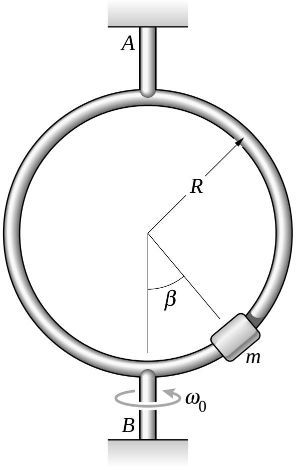
Dynamics: $\sum F_{\uparrow} : N - mg = -m\dfrac{v^2}{\rho}$

(a) $\boxed{N = 0 \Rightarrow v = \sqrt{g\rho} = \sqrt{(9.81 \text{ m/s}^2)(2000 \text{ m})} = 140 \text{ m/s}}$

(b) The force will push down on the pilot

$$N = -4mg \Rightarrow -5mg = -m\frac{v^2}{\rho} \Rightarrow \rho = \frac{v^2}{5g} \quad \boxed{\rho = 815 \text{ m}}$$

Problem 14.83 The smooth circular bar rotates with constant angular velocity ω_0 about the vertical axis AB. The radius $R = 0.5$ m. The mass m remains stationary relative to the circular bar at $\beta = 40^\circ$. Determine ω_0.



Solution:

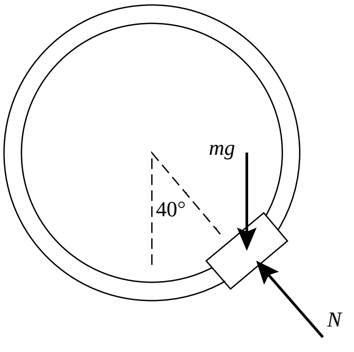
$$\sum F_\uparrow : N\cos 40^\circ - mg = 0$$

$$\sum F_\leftarrow : N\sin 40^\circ = m\frac{v^2}{\rho} = m\frac{(R\sin 40^\circ\omega_0)^2}{R\sin 40^\circ}$$

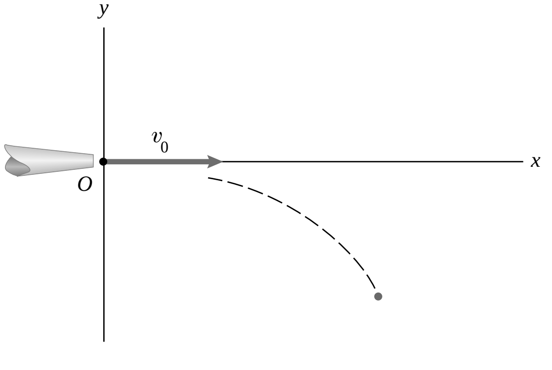
Solving we find

$$N = \frac{mg}{\cos 40^\circ}, \quad \omega_0 = \sqrt{\frac{g}{R\cos 40^\circ}}$$

$$\Rightarrow \boxed{\omega_0 = \sqrt{\frac{9.81\ \text{m/s}^2}{0.5\ \text{m}\cos 40^\circ}} = 5.06\ \text{rad/s}}$$

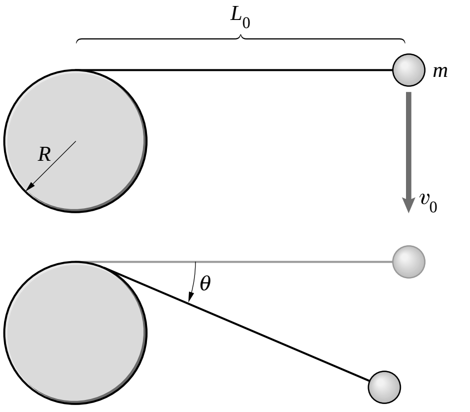


Problem 14.84 The force exerted on a charged particle by a magnetic field is $\mathbf{F} = q\mathbf{v}\times\mathbf{B}$, where q and $\mathbf{v}$ are the charge and velocity of the particle, and $\mathbf{B}$ is the magnetic field vector. A particle of mass m and positive charge q is projected at O with velocity $\mathbf{v} = v_0\mathbf{i}$ into a uniform magnetic field $\mathbf{B} = B_0\mathbf{k}$. Using normal and tangential components, show that (a) the magnitude of the particle's velocity is constant and (b) the particle's path is a circle of radius $m\dfrac{v_0}{qB_0}$.



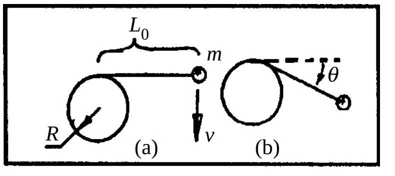
Solution: (a) The force $\mathbf{F} = q(\mathbf{v}\times\mathbf{B})$ is everywhere normal to the velocity and the magnetic field vector on the particle path. Therefore the tangential component of the force is zero, hence from Newton's second law the tangential component of the acceleration $\dfrac{dv}{dt} = 0$, from which $\boxed{v(t) = C = v_0}$, and the velocity is a constant. Since there is no component of force in the z-direction, and no initial z-component of the velocity, the motion remains in the *x-y* plane. The unit vector $\mathbf{k}$ is positive out of the paper in the figure; by application of the right hand rule the cross product $\mathbf{v}\times\mathbf{B}$ is directed along a unit vector toward the instantaneous center of the path at every instant, from which $\mathbf{F} = -|\mathbf{F}|\mathbf{e}_n$, where $\mathbf{e}_n$ is a unit vector normal to the path. The normal component of the acceleration is $\mathbf{a}_n = -(v_0^2/\rho)\mathbf{e}_n$, where ρ is the radius of curvature of the path. From Newton's second law, $\mathbf{F} = m\mathbf{a}_n$, from which $-|\mathbf{F}| = -m(v_0^2/\rho)$. The magnitude of the cross product can be written as $|\mathbf{v}\times\mathbf{B}| = v_0B_0\sin\theta = v_0B_0$, since $\theta = 90^\circ$ is the angle between $\mathbf{v}$ and $\mathbf{B}$. From which: $qv_0B_0 = m\dfrac{v_0^2}{\rho}$, from which the radius of curvature is $\boxed{\rho = \dfrac{mv_0}{qB_0}}$. Since the term on the right is a constant, the radius of curvature is a constant, and the path is a circle with radius $\dfrac{mv_0}{qB_0}$.

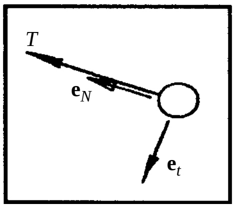
Problem 14.85 The mass m is attached to a string that is wrapped around a fixed post of radius R. At $t = 0$, the object is given a velocity v_0 as shown. Neglect external forces on m other that the force exerted by the string. Determine the tension in the string as a function of the angle θ.



Strategy: The velocity vector of the mass is perpendicular to the string. Express Newton's second law in terms of normal and tangential components.

Solution: Make a hypothetical cut in the string and denote the tension in the part connected to the mass by T. The acceleration normal to the path is $\frac{v^2}{\rho}$. The instantaneous radius of the path is $\rho = L_0 - R\theta$, from which by Newton's second law, $\sum F_n = T = m\frac{v_0^2}{L_0 - R\theta}$, from which $\boxed{T = m\frac{v_0^2}{L_0 - R\theta}}$



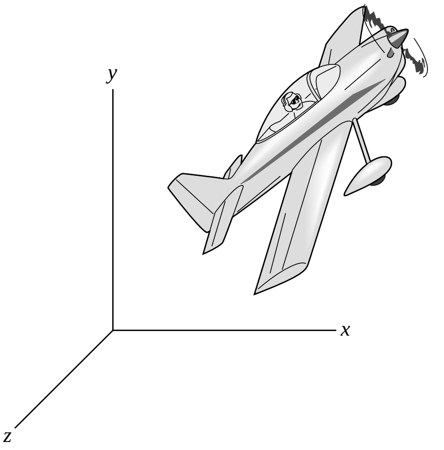


Problem 14.86 The mass m is attached to a string that is wrapped around the fixed post of radius R. At $t = 0$, the mass is given a velocity v_0 as shown. Neglect external forces on m other than the force exerted by the string. Determine the angle θ as a function of time.

Solution: Use the solution to Problem 14.85. The angular velocity is $\frac{d\theta}{dt} = \frac{v_0}{\rho}$. From Problem 14.85, $\rho = L_0 - R\theta$, from which $\frac{d\theta}{dt} = \frac{v_0}{(L_0 - R\theta)}$. Separate variables: $(L_0 - R\theta)d\theta = v_0 dt$. Integrate: $L_0\theta - \frac{R}{2}\theta^2 = v_0 t$, since $\theta(0) = 0$. In canonical form $\theta^2 + 2b\theta + c = 0$, where $b = -\frac{L}{R}$, and $c = \frac{2v_0 t}{R}$. The solution: $\theta = -b \pm \sqrt{b^2 - c} = \frac{L_0}{R} \pm \sqrt{\left(\frac{L_0}{R}\right)^2 - \frac{2v_0 t}{R}}$. The angle increases with time, so the negative sign applies. Reduce: $\boxed{\theta = \frac{L_0}{R}\left(1 - \sqrt{1 - \frac{2Rv_0 t}{L_0^2}}\right)}$

Check: When $R\theta = L_0$, the string has been fully wrapped around the post. Substitute to obtain: $\sqrt{1 - \frac{2Rv_0 t}{L_0^2}} = 0$, from which $\frac{2Rv_0 t}{L_0^2} = 1$, which is the value for impending failure as t increases because of the imaginary square root. Thus the solution behaves as expected. *check*.

Problem 14.87 The sum of the forces in newtons exerted on the 360-kg sport plane (including its weight) during an interval of time is $(-1000 + 280t)\mathbf{i} + (480 - 430t)\mathbf{j} + (720 + 200t)\mathbf{k}$, where t is the time in seconds. At $t = 0$, the velocity of the plane's center of gravity is $20\mathbf{i} + 35\mathbf{j} - 20\mathbf{k}$ (m/s). If you resolve the sum of the forces on the plane into components tangent and normal to the plane's path at $t = 2$ s, what are their values of $\sum F_t$ and $\sum F_n$?



Solution: This problem has several steps. First, we must write Newton's second law and find the acceleration of the aircraft. We then integrate the components of the acceleration (separately) to find the velocity components as functions of time. Then we evaluate the velocity of the aircraft and the force acting on the aircraft at $t = 2s$. Next, we find a unit vector along the velocity vector of the aircraft and project the total force acting on the aircraft onto this direction. Finally, we find the magnitude of the total force acting on the aircraft and the force component normal to the direction of flight. We have $a_X = (1/m)(-1000 + 280t)$, $a_Y = (1/m)(480 - 430t)$, and $a_Z = (1/m)(720 + 200t)$. Integrating and inserting the known initial velocities, we obtain the relations $v_X = v_{X0} + (1/m)(-1000t + 280t^2/2)$ (m/s) $= 20 + (1/m)(-1000t + 140t^2)$ (m/s), $v_Y = 35 + (1/m)(480t - 215t^2)$ (m/s), and $v_Z = -20 + (1/m)(720t + 100t^2)$ (m/s). The velocity at $t = 2s$ is $\mathbf{v} = 16\mathbf{i} + 35.3\mathbf{j} - 14.9\mathbf{k}$ (m/s) and the unit vector parallel to $\mathbf{v}$ is $\mathbf{e_v} = 0.386\mathbf{i} + 0.850\mathbf{j} - 0.359\mathbf{k}$. The total force acting on the aircraft at $t = 2s$ is $\mathbf{F} = -440\mathbf{i} - 380\mathbf{j} + 1120\mathbf{k}$ N. The component of $\mathbf{F}$ tangent to the direction of flight is $\sum F_t = \mathbf{F} \cdot \mathbf{e_v} = -894.5$ N. The magnitude of the total force acting on the aircraft is $|\mathbf{F}| = 1261.9$ N. The component of $\mathbf{F}$ normal to the direction of flight is given by $\sum F_n = \sqrt{|\mathbf{F}|^2 - (\sum F_t)^2} = 890.1$ N.

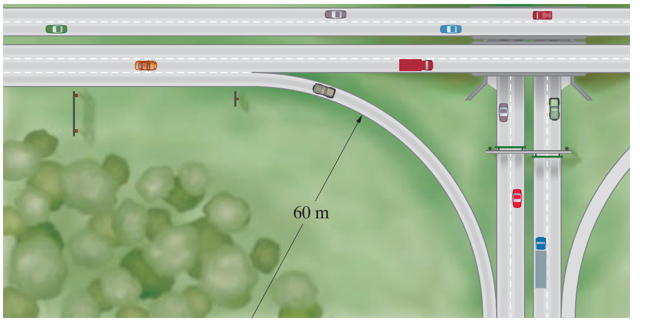
Problem 14.88 In Problem 14.87, what is the instantaneous radius of curvature of the plane's path at $t = 2$ s? The vector components of the sum of the forces in the directions tangenial and normal to the path lie in the osculating plane. Determine the components of a unit vector perpendicular to the osculating plane at $t = 2$ s.

Strategy: From the solution to problem 14.87, we know the total force vector and acceleration vector acting on the plane. We also know the direction of the velocity vector. From the velocity and the magnitude of the normal acceleration, we can determine the radius of curvature of the path. The cross product of the velocity vector and the total force vector will give a vector perpendicular to the plane containing the velocity vector and the total force vector. This vector is perpendicular to the plane of the osculating path. We need then only find a unit vector in the direction of this vector.

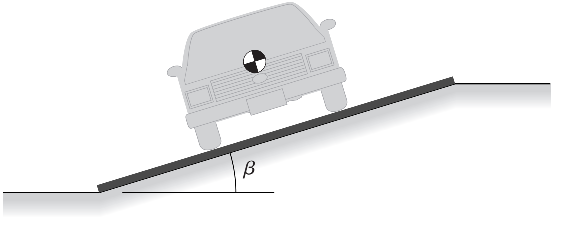
Solution: From Problem 14.87, we know at $t = 2$ s, that $a_n = \sum F_n/m = 2.47$ m/s^2. We can find the magnitude of the velocity $|\mathbf{v}| = 41.5$ m/s at this time. The radius of curvature of the path can then be found from $\rho = |\mathbf{v}|^2/a_n = 696.5$ m.

The cross product yields the desired unit vector, i.e., $\mathbf{e} = (\mathbf{F} \times \mathbf{v})/|\mathbf{F} \times \mathbf{v}| = -0.916\mathbf{i} + 0.308\mathbf{j} - 0.256\mathbf{k}$

Problem 14.89 The freeway off-ramp is circular with 60-m radius (Fig. a). The off-ramp has a slope $\beta = 15^\circ$ (Fig. b). If the coefficient of static friction between the tires of a car and the road is $\mu_s = 0.4$, what is the maximum speed at which it can enter the ramp without losing traction? (See Example 14.18.)

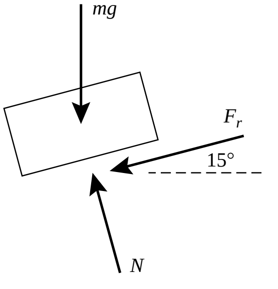


(a)



(b)

Solution:



$\rho = 60$ m, $\quad g = 9.81$ m/s^2

$$\sum F_{\uparrow} : N \cos 15^\circ - F_r \sin 15^\circ - mg = 0$$

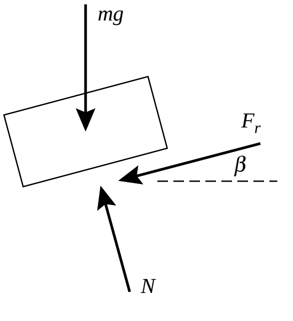
$$\sum F_{\leftarrow} : N \sin 15^\circ + F_r \cos 15^\circ = m\frac{v^2}{\rho}$$

$F_r = 0.4N$

Solving we have $\boxed{v = 21.0 \text{ m/s}}$

Problem 14.90* The freeway off-ramp is circular with 60-m radius (Fig. a). The off-ramp has a slope β (Fig. b). If the coefficient of static friction between the tires of a car and the road is $\mu_s = 0.4$ what minimum slope β is needed so that the car could (in theory) enter the off-ramp at any speed without losing traction? (See Example 14.8.)

Solution:



$$\sum F_{\uparrow} : N \cos \beta - F_r \sin \beta - mg = 0$$

$$\sum F_{\leftarrow} : N \sin \beta + F_r \cos \beta = m\frac{v^2}{\rho}$$

$F = \mu N$

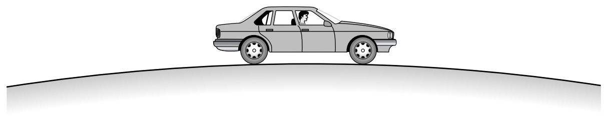
Solving we have $F_r = \dfrac{\mu m g}{\cos\beta - \mu \sin\beta}$.

If we set the denominator equal to zero, then we will always have enough friction to prevent sliding.

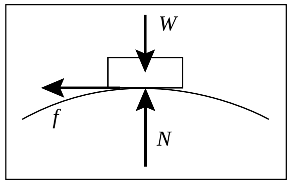
Thus $\boxed{\beta = \tan^{-1}\left(\frac{1}{\mu}\right) = \tan^{-1}\left(\frac{1}{0.4}\right) = 68.2^\circ}$

We would also need to check the low-speed case, where the car might slip down the ramp.

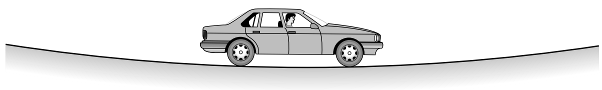
Problem 14.91 A car traveling at 30 m/s is at the top of a hill. The coefficient of kinetic friction between the tires and the road is $\mu_k = 0.8$. The instantaneous radius of curvature of the car's path is 200 m. If the driver applies the brakes and the car's wheels lock, what is the resulting deceleration of the car tangent to its path?



Solution: From Newton's second law; $N - W = -m\dfrac{v^2}{R}$ from which $N = m\left(g - \dfrac{v^2}{R}\right)$. The acceleration tangent to the path is $\dfrac{dv}{dt}$, from which $\dfrac{dv}{dt} = -\dfrac{\mu_k N}{m}$, and $\boxed{\dfrac{dv}{dt} = -\mu_k\left(g - \dfrac{v^2}{R}\right) = 4.25 \text{ m/s}^2}$



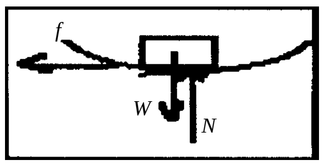
Problem 14.92 A car traveling at 30 m/s is at the bottom of a depression. The coefficient of kinetic friction between the tires and the road is $\mu_\text{k} = 0.8$. The instantaneous radius of curvature of the car's path is 200 m. If the driver applies the brakes and the car's wheel lock, what is the resulting deceleration of the car in the direction tangential to its path? Compare your answer to that of Problem 14.91.



Solution: Use the solution to Problem 14.91: $\dfrac{dv}{dt} = -\dfrac{\mu_k N}{m}$. From Newton's second law, $N - W = m\left(\dfrac{v^2}{R}\right)$, from which

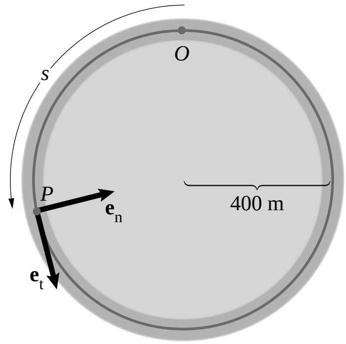
$$N = m\left(g + \left(\frac{v^2}{R}\right)\right),$$

and $\boxed{\dfrac{dv}{dt} = -\mu_k\left(g + \dfrac{v^2}{R}\right) = -11.45 \text{ m/s}^2}$



Problem 14.93 The combined mass of the motorcycle and rider is 160 kg. The motorcycle starts from rest at $t = 0$ and moves along a circular track with 400-m radius. The tangential component of acceleration as a function of time is $a_t = 2 + 0.2t\ \text{m/s}^2$. The coefficient of static friction between the tires and the track is $\mu_s = 0.8$. How long after it starts does the motorcycle reach the limit of adhesion, which means its tires are on the verge of slipping? How fast is the motorcycle moving when that occurs?

Strategy: Draw a free-body diagram showing the tangential and normal components of force acting on the motorcycle.



Solution:

$m = 160$ kg

$R = 400$ m

Along track motion:

$a_t = 2 + 0.2t\ \text{m/s}^2$

$V_t = V = 2t + 0.1t^2\ \text{m/s}$

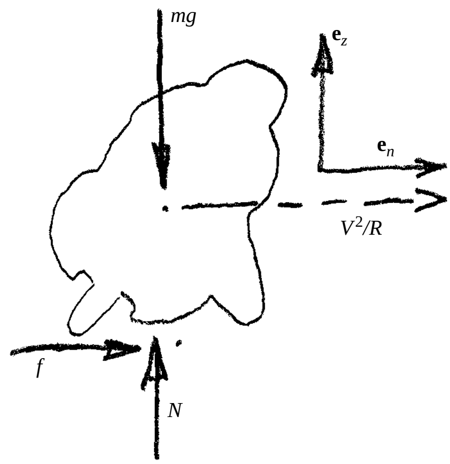
$s = t^2 + 0.1t^3/3\ \text{m}$

Forces at impending slip

$|\mathbf{F} + \mathbf{f}| = \mu_k N$ at impending slip

$|\mathbf{F} + \mathbf{f}| = \sqrt{F^2 + f^2}$ since $\mathbf{f} \perp \mathbf{F}$

Force eqns.



$\sum F_t : F = ma_t$ $\qquad R = 400$ m

$\sum F_n : f = mv^2/R$ $\qquad m = 160$ kg

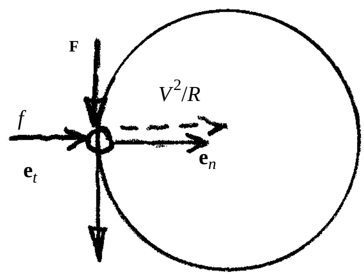
$\sum F_z : N - mg = 0$ $\qquad g = 9.81\ \text{m/s}^2$

$\mu_s = 0.8$

$\sqrt{F^2 + f^2} = \mu_s N$

$a_t = 2 + 0.2t$

$v = 2t + 0.1t^2$



Six eqns, six unknowns (F, f, v, a_t, N, t)

Solving, we have

$\underline{t = 14.4\ \text{s}}$ $\qquad F = 781$ N

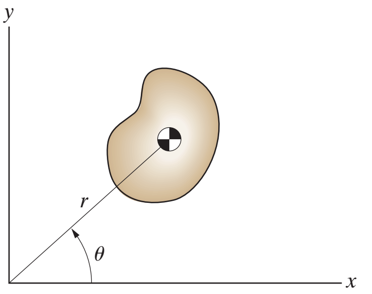
$\underline{v = 49.6\ \text{m/s}}$ $\qquad f = 983$ N

$N = 1570$ N

$a_t = 4.88\ \text{m/s}^2$

(at $t = 14.4$ s)

Problem 14.94 The center of mass of the 12-kg object moves in the $x-y$ plane. Its polar coordinates are given as functions of time by $r = 12 - 0.4t^2$ m, $\theta = 0.02t^3$ rad. Determine the polar components of the total force acting on the object at $t = 2$ s.



Solution:

$r = 12 - 0.4t^2, \quad \theta = 0.02t^3$

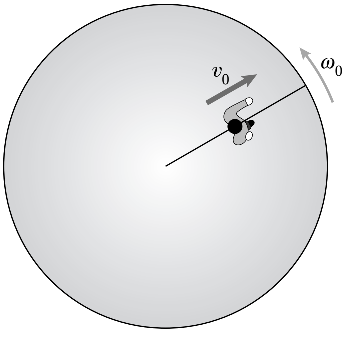
$\dot{r} = -0.8t, \quad \dot{\theta} = 0.06t^2$

$\ddot{r} = -0.8, \quad \ddot{\theta} = 0.12t$

Set t = 2 s

$$F_r = m(\ddot{r} - r\dot{\theta}^2) = (12 \text{ kg})(-0.8 - [10.4][0.24]^2) \text{ m/s}^2$$
$$= -16.8 \text{ N}$$
$$F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = (12 \text{ kg})([10.4][0.24] + 2[-1.6][0.24]) \text{ m/s}^2$$
$$= 20.7 \text{ N}$$

Problem 14.95 A 445 N person walks on a large disk that rotates with constant angular velocity $\omega_0 = 0.3$ rad/s. He walks at a constant speed $v_0 = 1.52$ m/s along a straight radial line painted on the disk. Determine the polar components of the horizontal force exerted on him when he is 1.83 m from the center of the disk. (How are these forces exerted on him?)



Solution:

$r = 1.83$ m, $\dot{r} = 1.52$ m/s, $\ddot{r} = 0$, $\dot{\theta} = 0.3$ rad/s, $\ddot{\theta} = 0$

$$F_r = m(\ddot{r} - r\dot{\theta}^2) = \left(\frac{445 \text{ N}}{9.81 \text{ m/s}^2}\right)(0 - [1.83 \text{ m}][0.3 \text{ rad/s}]^2)$$
$$= -7.47 \text{ N}$$
$$F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = \left(\frac{445 \text{ N}}{9.81 \text{ m/s}^2}\right)(0 + 2[1.52 \text{ m/s}][0.3 \text{ rad/s}])$$
$$= 41.5 \text{ N}$$

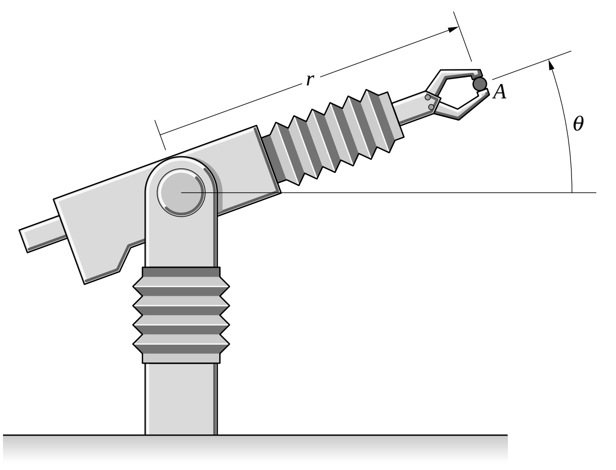
The forces are exerted as friction between the disk and the man's feet.

Problem 14.96 The robot is programmed so that the 0.4-kg part A describes the path

$$r = 1 - 0.5\cos(2\pi t) \text{ m},$$

$$\theta = 0.5 - 0.2\sin(2\pi t) \text{ rad}.$$

Determine the radial and transverse components of the force exerted on A by the robot's jaws at $t = 2$ s.



Solution: The radial component of the acceleration is

$$a_r = \frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2.$$

The derivatives:

$$\frac{dr}{dt} = \frac{d}{dt}(1 - 0.5\cos 2\pi t) = \pi \sin 2\pi t,$$

$$\frac{d^2r}{dt^2} = \frac{d}{dt}(\pi \sin 2\pi t) = 2\pi^2 \cos 2\pi t;$$

$$\frac{d\theta}{dt} = \frac{d}{dt}(0.5 - 0.2\sin 2\pi t) = -0.4\pi \cos 2\pi t.$$

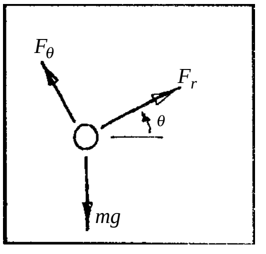
$$\frac{d^2\theta}{dt^2} = \frac{d}{dt}(-0.4\pi \cos 2\pi t) = 0.8\pi^2 \sin 2\pi t.$$

From which

$$[a_r]_{t=2} = 2\pi^2 \cos 4\pi - (1 - 0.5\cos 4\pi)(-0.4\pi \cos 4\pi)^2,$$

$$= 2\pi^2 - 0.08\pi^2 = 18.95 \text{ m/s}^2;$$

$\theta(t = 2) = 0.5$ rad.



From Newton's second law, $F_r - mg\sin\theta = ma_r$, and $F_\theta - mg\cos\theta = ma_\theta$, from which

$$\boxed{F_r = 0.4a_r + 0.4g\sin\theta = 9.46 \text{ N}.}$$

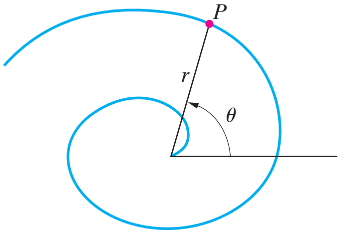
The transverse component of the acceleration is

$$a_\theta = r\left(\frac{d^2\theta}{dt^2}\right) + 2\left(\frac{dr}{dt}\right)\left(\frac{d\theta}{dt}\right),$$

from which $[a_\theta]_{t=2} = (1 - 0.5\cos 4\pi)(0.8\pi^2 \sin 4\pi) + 2(\pi \sin 4\pi)(-0.4\pi \sin 4\pi) = 0$, and

$$\boxed{F_\theta = 3.44 \text{ N}}$$

Problem 14.97 A 50-N object P moves along the spiral path $r = (0.1)\theta$ m, where θ is in radians. Its angular position is given as a function of time by $\theta = 2t$ rad, and $r = 0$ at $t = 0$. Determine the polar components of the total force acting on the object at $t = 4$ s.



Solution:

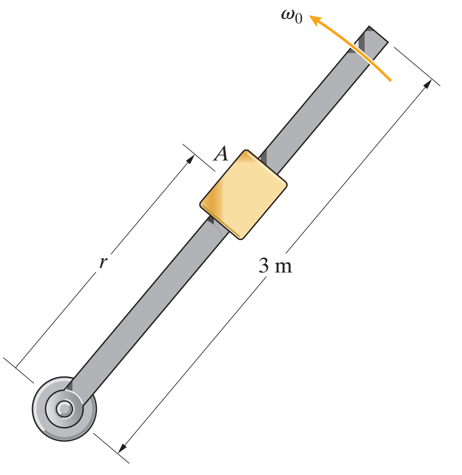
$\theta = 2t, \dot{\theta} = 2, \ddot{\theta} = 0, r = 0.1\theta = 0.2t, \dot{r} = 0.2, \ddot{r} = 0$

At $t = 4$ s, $\theta = 8, \dot{\theta} = 2, \ddot{\theta} = 0, r = 0.8, \dot{r} = 0.2, \ddot{r} = 0$

Thus

$$\boxed{\begin{aligned} F_r &= m(\ddot{r} - r\dot{\theta}^2) = \left(\frac{50 \text{ N}}{9.81 \text{ m/s}^2}\right)(0 - [0.8 \text{ m}][2 \text{ rad/s}]^2) \\ &= -16.3 \text{ N} \\ F_\theta &= m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = \left(\frac{50 \text{ N}}{9.81 \text{ m/s}^2}\right)(0 + 2[0.2 \text{ m/s}][2 \text{ rad/s}]) \\ &= 4.08 \text{ N} \end{aligned}}$$

Problem 14.98 The smooth bar rotates *in the horizontal plane* with constant angular velocity $\omega_0 = 60$ rpm. If the radial velocity of the 1-kg collar A is $v_r = 10$ m/s when its radial position is $r = 1$ m, what is its radial velocity when $r = 2$ m? (See Active Example 14.9).



Solution: Notice that no radial force acts on the collar, so the radial acceleration is zero. Write the term

$$\frac{d^2r}{dt^2} = \frac{dv_r}{dt} = \frac{dv_r}{dr}\frac{dr}{dt} = v_r\frac{dv_r}{dr}$$

The angular velocity is

$$\omega = 60 \text{ rpm}\left(\frac{2\pi \text{ rad}}{\text{rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 6.28 \text{ rad/s}.$$

Then

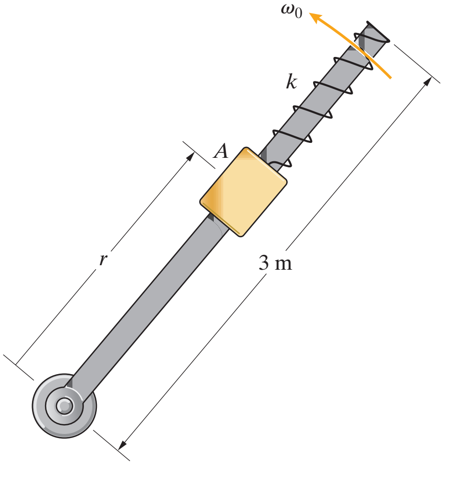
$$a_r = \frac{d^2r}{dt^2} - r\omega^2 = 0 \Rightarrow \frac{d^2r}{dt^2} = r\omega^2$$

$$\frac{d^2r}{dt^2} = v_r\frac{dv_r}{dr} = r\omega^2 \Rightarrow \int_{10\text{m/s}}^{v_r} v_r\,dv_r = \int_{1\text{m}}^{2\text{m}} \omega^2 r\,dr$$

$$\frac{v_r^2}{2} - \frac{(10 \text{ m/s})^2}{2} = (6.28 \text{ rad/s})^2\left(\frac{[2 \text{ m}]^2}{2} - \frac{[1 \text{ m}]^2}{2}\right)$$

$$\boxed{v_r = 14.8 \text{ m/s.}}$$

Problem 14.99 The smooth bar rotates *in the horizontal plane* with constant angular velocity $\omega_0 = 60$ rpm. The spring constant is $k = 20$ N/m and the unstretched length of the spring is 3 m. If the radial velocity of the 1-kg collar A is $v_r = 10$ m/s when its radial position is $r = 1$ m, what is its radial velocity when $r = 2$ m? (See Active Example 14.9.)



Solution: Notice that the only radial force comes from the spring. Write the term

$$\frac{d^2r}{dt^2} = \frac{dv_r}{dt} = \frac{dv_r}{dr}\frac{dr}{dt} = v_r\frac{dv_r}{dr}$$

The angular velocity is

$$\omega = 60 \text{ rpm}\left(\frac{2\pi \text{ rad}}{\text{rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 6.28 \text{ rad/s}.$$

The equation of motion in the radial direction is

$$\Sigma F_{rm} : -kr = ma_r \Rightarrow a_r = -\frac{k}{m}r$$

Then

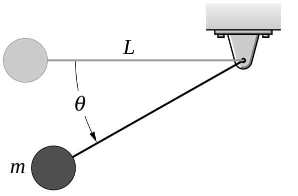
$$a_r = \frac{d^2r}{dt^2} - r\omega^2 = -\frac{k}{m}r \Rightarrow \frac{d^2r}{dt^2} = r\left(\omega^2 - \frac{k}{m}\right)$$

$$\frac{d^2r}{dt^2} = v_r\frac{dv_r}{dr} = r\left(\omega^2 - \frac{k}{m}\right) \Rightarrow \int_{10\text{ m/s}}^{v_r} v_r\,dv_r = \int_{1\text{ m}}^{2\text{ m}}\left(\omega^2 - \frac{k}{m}\right) r\,dr$$

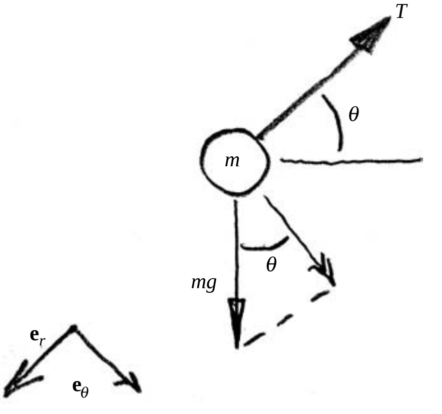
$$\frac{v_r^2}{2} - \frac{(10 \text{ m/s})^2}{2} = \left[(6.28 \text{ rad/s})^2 - \frac{20 \text{ N/m}}{1 \text{ kg}}\right]\left(\frac{[2 \text{ m}]^2}{2} - \frac{[1 \text{ m}]^2}{2}\right)$$

$$\boxed{v_r = 12.6 \text{ m/s.}}$$

Problem 14.100 The 2-kg mass m is released from rest with the string horizontal. The length of the string is $L = 0.6$ m. By using Newton's second law in terms of polar coordinates, determine the magnitude of the velocity of the mass and the tension in the string when $\theta = 45^\circ$.



Solution:



$$L = 0.6 \text{ m}$$

$$m = 2 \text{ kg}$$

$$F_r = ma_r$$

$$-T + mg\sin\theta = m\left(\frac{d^2L}{dt^2} - Lw^2\right)$$

$$\sum F_\theta = ma_\theta$$

$$mg\cos\theta = m\left(2\frac{dL}{dt}w + L\alpha\right)$$

However $\dfrac{dL}{dt} = \dfrac{d^2L}{dt^2} = 0$

$$L\alpha = L\frac{dw}{d\theta}w = g\cos\theta$$

$$\int_0^w w\,dw = \frac{g}{L}\int_0^{\pi/4}\cos\theta\,d\theta$$

$$\frac{w^2}{2} = \frac{g}{L}\sin\theta\Big|_0^{\pi/4} = \frac{g}{L}\sin\frac{\pi}{4}$$

$$w = 4.81 \text{ rad/s}$$

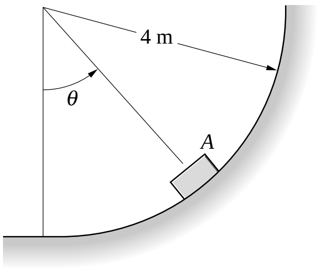
$$\underline{|v| = Lw = 2.89 \text{ m/s}}$$

$$-T + mg\sin\theta = -mLw^2$$

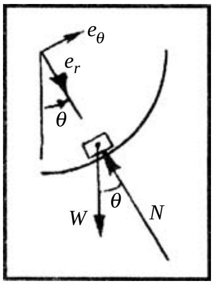
$$T = m(g\sin\theta + Lw^2)$$

$$\underline{T = 41.6 \text{ N}}$$

Problem 14.101 The 1-N block A is given an initial velocity $v_0 = 14$ m/s to the right when it is in the position $\theta = 0$, causing it to slide up the smooth circular surface. By using Newton's second law in terms of polar coordinates, determine the magnitude of the velocity of the block when $\theta = 60^\circ$.



Solution: For this problem, Newton's second law in polar coordinates states



$\sum F_r = mg\cos\theta - N = m(d^2r/dt^2 - r\omega^2)$ and

$\sum F_\theta = -mg\sin\theta = m(r\alpha + 2\omega(dr/dt))$.

In this problem, r is constant. Thus $(dr/dt) = (d^2r/dt^2) = 0$, and the equations reduce to $N = mr\omega^2 + mg\cos\theta$ and $r\alpha = -g\sin\theta$. The first equation gives us a way to evaluate the normal force while the second can be integrated to give $\omega(\theta)$. We rewrite the second equation as

$$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta}\frac{d\theta}{dt} = \omega\frac{d\omega}{d\theta} = -\left(\frac{g}{r}\right)\sin\theta$$

and then integrate $\int_{\omega_0}^{\omega_{60}} \omega\, d\omega = -\left(\frac{g}{r}\right)\int_0^{60^\circ} \sin\theta\, d\theta$. Carrying out the integration, we get

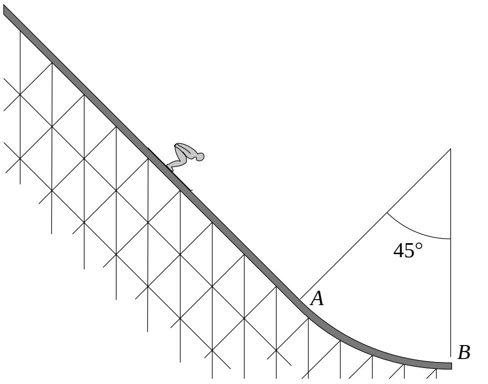
$$\frac{\omega_{60}^2}{2} - \frac{\omega_0^2}{2} = -\left(\frac{g}{r}\right)(-\cos\theta)\Big|_0^{60^\circ} = -\left(\frac{g}{r}\right)(1 - \cos 60^\circ).$$

Noting that $\omega_0 = v_0/R = 3.5$ rad/s, we can solve for $\omega_{60} = 2.05$ rad/s and $v_{60} = R\omega_{60} = 8.20$ m/s.

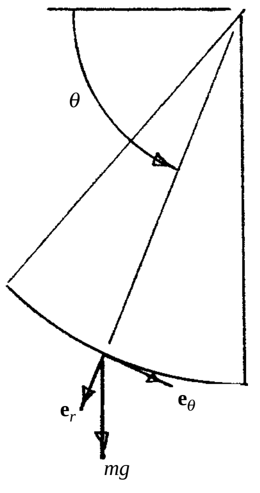
Problem 14.102 The 1-N block is given an initial velocity $v_0 = 14$ m/s to the right when it is in the position $\theta = 0$, causing it to slide up the smooth circular surface. Determine the normal force exerted on the block by the surface when $\theta = 60^\circ$.

Solution: From the solution to Problem 14.101, we have $N_{60} = mr\omega_{60}^2 + mg\cos 60^\circ$ or $N = 2.21$ N.

Problem 14.103 The skier passes point A going 17 m/s. From A to B, the radius of his circular path is 6 m. By using Newton's second law in terms of polar coordinates, determine the magnitude of the skier's velocity as he leaves the jump at B. Neglect tangential forces other than the tangential component of his weight.



Solution: In terms of the angle θ shown, the transverse component of his weight is $mg\cos\theta$. Therefore



$$\sum F_\theta = ma_\theta :$$

$$mg\cos\theta = m\left(r\frac{d^2\theta}{dt^2} + 2\cancelto{0}{\frac{dr}{dt}}\frac{d\theta}{dt}\right). \quad \textbf{(1)}$$

Note that

$$\frac{d^2\theta}{dt^2} = \frac{dw}{dt} = \frac{dw}{d\theta}\frac{d\theta}{dt} = \frac{dw}{d\theta}w,$$

So (1) becomes

$$g\cos\theta = r\frac{dw}{d\theta}w.$$

Separating variables,

$$w\,dw = \frac{g}{r}\cos\theta\, d\theta. \quad \textbf{(2)}$$

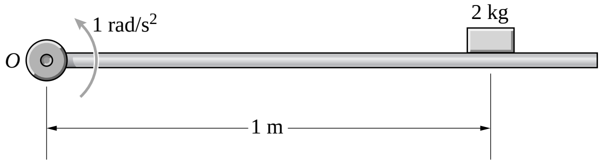
At A, $\theta = 45^\circ$ and $w = v_A/r = 17/6 = 2.83$ rad/s. Integrating (2),

$$\int_{2.83}^{w_B} w\,dw = \frac{g}{r}\int_{45^\circ}^{90^\circ}\cos\theta\, d\theta,$$

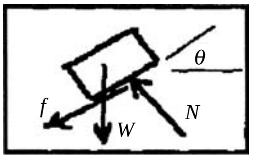
we obtain $w_B = 3.00$ rad/s. His velocity at B is $rw_B = 18.0$ m/s.

$\underline{v_B = 18.0 \text{ m/s.}}$

Problem 14.104* A 2-kg mass rests on a flat horizontal bar. The bar begins rotating *in the vertical plane* about O with a constant angular acceleration of 1 rad/s^2. The mass is observed to slip relative to the bar when the bar is 30° above the horizontal. What is the static coefficient of friction between the mass and the bar? Does the mass slip toward or away from O?

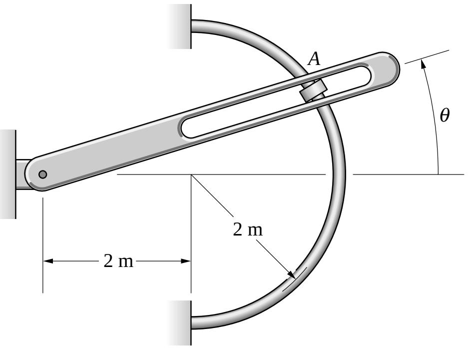


Solution: From Newton's second law for the radial component $-mg\sin\theta \pm \mu_s N = -mR\omega^2$, and for the normal component: $N - mg\cos\theta = mR\alpha$. Solve, and note that $\alpha = \dfrac{d\omega}{dt} = \omega\dfrac{d\omega}{d\theta} = 1 = const$, $\omega^2 = 2\theta$, since $\omega(0) = 0$, to obtain $-g\sin\theta \pm \mu_s(g\cos\theta + R\alpha) = -2R\theta$. For $\alpha = 1$, $R = 1$, this reduces to $\pm\mu_s(1 + g\cos\theta) = -2\theta + g\sin\theta$. Define the quantity $F_R = 2\theta - g\sin\theta$. If $F_R > 0$, the block will tend to slide away from O, the friction force will oppose the motion, and the negative sign is to be chosen. If $F_R < 0$, the block will tend to slide toward O, the friction force will oppose the motion, and the positive sign is to be chosen. The equilibrium condition is derived from the equations of motion: $\text{sgn}(F_R)\mu_s(1 + g\cos\theta) = (2\theta - g\sin\theta)$, from which $\boxed{\mu_s = \text{sgn}(F_R)\dfrac{2\theta - g\sin\theta}{1 + g\cos\theta} = 0.406}$.

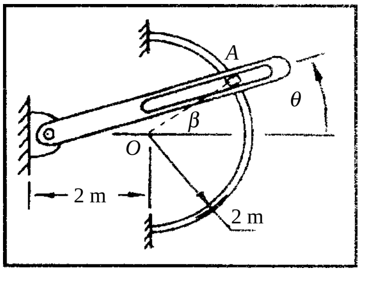


Since $F_r = -3.86 < 0$, the block will slide toward O.

Problem 14.105* The 0.25 N slider A is pushed along the circular bar by the slotted bar. The circular bar lies *in the horizontal plane*. The angular position of the slotted bar is $\theta = 10t^2$ rad. Determine the polar components of the total external force exerted on the slider at $t = 0.2$ s.



Solution: The interior angle β is between the radius from O to the slider A and the horizontal, as shown in the figure. The interior angle formed by the radius from C to the slider A and the line from O to the slider is $\beta - \theta$. The angle β is found by applying the law of sines: $\dfrac{2}{\sin\theta} = \dfrac{2}{\sin(\beta - \theta)}$ from which $\sin\theta = \sin(\beta - \theta)$ which is satisfied by $\beta = 2\theta$. The radial distance of the slider from the hinge point is also found from the sine law: $\dfrac{r}{\sin(180 - \beta)} = \dfrac{2}{\sin\theta}$, $r = \dfrac{2\sin 2\theta}{\sin\theta}$, from which $r = 4\cos\theta$. The radial component of the acceleration is $a_r = \dfrac{d^2r}{dt^2} - r\left(\dfrac{d\theta}{dt}\right)^2$. The derivatives: $\dfrac{d\theta}{dt} = \dfrac{d}{dt}(10t^2) = 20t$. $\dfrac{d^2\theta}{dt^2} = 20$. $\dfrac{dr}{dt} = -4\sin\theta\left(\dfrac{d\theta}{dt}\right) = -(80\sin\theta)t$. $\dfrac{d^2r}{dt^2} = -80\sin\theta - (1600\cos\theta)t^2$. Substitute:



$$[a_r]_{t=0.2} = [a_r = -80\sin(10t^2) - (1600\cos(10t^2))t^2$$

$$- (4\cos(10t^2))(20t)^2]_{t=0.2}$$

$$= -149.0 \text{ m/s}^2.$$

From Newton's second law, the radial component of the external force is

$$\boxed{F_r = \left(\frac{W}{g}\right)a_r = -1.158 \text{ N.}}$$

The transverse acceleration is $a_\theta = r\left(\dfrac{d^2\theta}{dt^2}\right) + 2\left(\dfrac{dr}{dt}\right)\left(\dfrac{d\theta}{dt}\right)$. Substitute:

$$[a_\theta]_{t=0.2} = [a_\theta = (4\cos(10t^2))(20)$$

$$+ 2(-80\sin(10t^2))(t)(20)(t)]_{t=0.2} = 23.84 \text{ m/s}^2.$$

The transverse component of the external force is

$$\boxed{F_\theta = \left(\frac{W}{g}\right)a_\theta = 0.185 \text{ N}}$$

Problem 14.106* The 0.25 N slider A is pushed along the circular bar by the slotted bar. The circular bar lies *in the vertical plane*. The angular position of the slotted bar is $\theta = 10t^2$ rad. Determine the polar components of the total force exerted on the slider by the circular and slotted bars at $t = 0.25$ s.

Solution: Assume that the orientation in the vertical plane is such that the $\theta = 0$ line is horizontal. Use the solution to Problem 14.105. For positive values of θ the radial component of acceleration due to gravity acts toward the origin, which by definition is the same direction as the radial acceleration. The transverse component of the acceleration due to gravity acts in the same direction as the transverse acceleration. From which the components of the acceleration due to gravity in the radial and transverse directions are $g_r = g\sin\theta$ and $g_\theta = g\cos\theta$. These are to be added to the radial and transverse components of acceleration due to the motion. From Problem 14.105, $\theta = 10t^2$ rad

$$[a_r]_{t=0.25} = [-80\sin\theta - (1600\cos\theta)t^2$$

$$- (4\cos\theta)(20t)^2]_{t=0.25} = -209 \text{ m/s}^2.$$

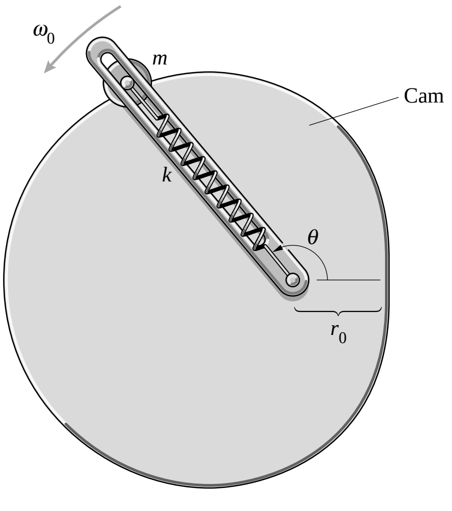
From Newton's second law for the radial component $F_r - mg\sin\theta = \left(\frac{W}{g}\right)a_r$, from which $F_r = -1.478$ N The transverse component of the acceleration is (from Problem 14.105)

$$[a_\theta]_{t=0.25} = [(4\cos\theta)(20)$$

$$+ 2(-80\sin\theta)(t)(20)(t)]_{t=0.25} = -52.14 \text{ m/s}^2.$$

From Newton's second law for transverse component $F_\theta - mg\cos\theta = \left(\frac{W}{g}\right)a_\theta$, from which $\boxed{F_\theta = -0.2025 \text{ N}}$

Problem 14.107* The slotted bar rotates *in the horizontal plane* with constant angular velocity ω_0. The mass m has a pin that fits into the slot of the bar. A spring holds the pin against the surface of the fixed cam. The surface of the cam is described by $r = r_0(2 - \cos\theta)$. Determine the polar components of the total external force exerted on the pin as functions of θ.



Solution: The angular velocity is constant, from which $\theta = \int \omega_0\,dt + C = \omega_0 t + C$. Assume that $\theta(t = 0) = 0$, from which $C = 0$. The radial acceleration is $a_r = \frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2$. The derivatives: $\frac{d\theta}{dt} = \frac{d}{dt}(\omega_0 t) = \omega_0$, $\frac{d^2\theta}{dt^2} = 0$. $\frac{dr}{dt} = \frac{d}{dt}(r_0(2-\cos\theta)) = r_0\sin\theta\left(\frac{d\theta}{dt}\right) = \omega_0 r_0\sin\theta$, $\frac{d^2r}{dt^2} = \frac{d}{dt}(\omega_0 r_0\sin\theta) = \omega_0^2 r_0\cos\theta$. Substitute: $a_r = \omega_0^2 r_0\cos\theta - r_0(2-\cos\theta)(\omega_0^2) = 2r_0\omega_0^2(\cos\theta - 1)$. From Newton's second law the radial component of the external force is

$$\boxed{F_r = ma_r = 2mr_0\omega_0^2(\cos\theta - 1).}$$

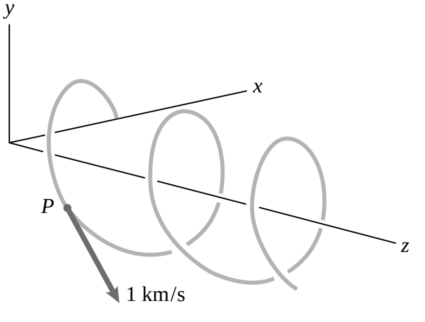
The transverse component of the acceleration is $a_\theta = r\frac{d^2\theta}{dt^2} + 2\left(\frac{dr}{dt}\right)\left(\frac{d\theta}{dt}\right)$. Substitute: $a_\theta = 2r_0\omega_0^2\sin\theta$. From Newton's second law, the transverse component of the external force is

$$\boxed{F_\theta = 2mr_0\omega_0^2\sin\theta}$$

Problem 14.108* In Problem 14.107, suppose that the unstretched length of the spring is r_0. Determine the smallest value of the spring constant k for which the pin will remain on the surface of the cam.

Solution: The spring force holding the pin on the surface of the cam is $F_r = k(r - r_0) = k(r_0(2 - \cos\theta) - r_0) = kr_0(1 - \cos\theta)$. This force acts toward the origin, which by definition is the same direction as the radial acceleration, from which Newton's second law for the pin is $\sum F = kr_0(1 - \cos\theta) = -ma_r$. From the solution to Problem 14.107, $kr_0(1 - \cos\theta) = -2mr\omega_0^2(\cos\theta - 1)$. Reduce and solve: $k = 2m\omega_0^2$. Since $\cos\theta \le 1$, $kr_0(1 - \cos\theta) \ge 0$, and $2mr_0\omega_0^2(\cos\theta - 1) \le 0$. If $k > 2m\omega_0^2$, Define $F_{\text{eq}} = kr_0(1 - \cos\theta) + 2mr\omega_0^2(\cos\theta - 1)$. If $F_{\text{eq}} > 0$ the spring force dominates over the range of θ, so that the pin remains on the cam surface. If $k < 2m\omega_0^2$, $F_{\text{eq}} < 0$ and the radial acceleration dominates over the range of θ, so that the pin will leave the cam surface at some value of θ. Thus $\boxed{k = 2m\omega_0^2}$ is the minimum value of the spring constant required to keep the pin in contact with the cam surface.

Problem 14.109 A charged particle P in a magnetic field moves along the spiral path described by $r = 1$ m, $\theta = 2z$ rad, where z is in meters. The particle moves along the path in the direction shown with a constant speed $|\mathbf{v}| = 1$ km/s. The mass of the particle is 1.67×10^{-27} kg. Determine the sum of the forces on the particle in terms of cylindrical coordinates.



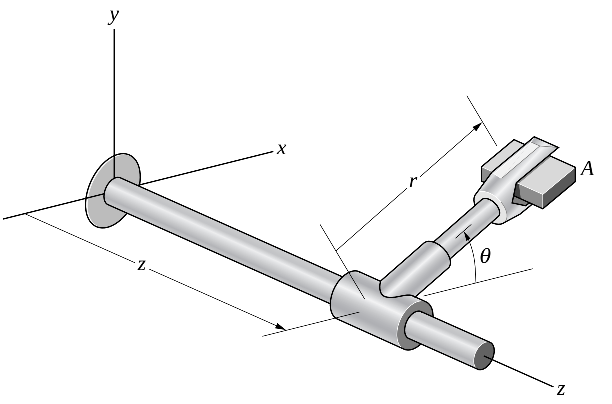
Solution: The force components in cylindrical coordinates are given by

$$\sum F_r = ma_r = m\left(\frac{d^2r}{dt^2} r\omega^2\right),$$

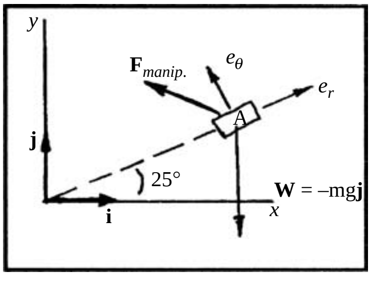
$$\sum F_\theta = ma_\theta = m\left(r\alpha + 2\left(\frac{dr}{dt}\right)\omega\right),$$

and $\sum F_z = ma_z = m\dfrac{d^2z}{dt^2}$. From the given information, $\dfrac{dr}{dt} = \dfrac{d^2r}{dt^2} = 0$. We also have that $\theta = 2z$. Taking derivatives of this, we see that $\dfrac{d\theta}{dt} = \omega = 2\dfrac{dz}{dt} = 2v_z$. Taking another derivative, we get $\alpha = 2a_z$. There is no radial velocity component so the constant magnitude of the velocity $|\mathbf{v}|^2 = v_\theta^2 + v_z^2 = r^2\omega^2 + v_z^2 = (1000 \text{ m/s})^2$. Taking the derivative of this expression with respect to time, we get $r^2\left(2\omega\dfrac{d\omega}{dt}\right) + 2v_z\dfrac{dv_z}{dt} = 0$. Noting that $\dfrac{dv_z}{dt} = \dfrac{d^2z}{dt^2}$ and that $\alpha = \dfrac{d\omega}{dt}$, we can eliminate $\dfrac{dv_z}{dt}$ from the equation. We get $2r^2\omega\alpha + 2\left(\dfrac{\omega}{2}\right)\left(\dfrac{\alpha}{2}\right)$, giving $(2r^2 + 1/2)\omega\alpha = 0$. Since $\omega \ne 0$, $\alpha = 0$, and $a_z = 0$. Substituting these into the equations of motion, we get $\omega^2 = 4(1000)^2 5$ (rad/s)^2, and $\sum F_r = -mr\omega^2 = -1.34 \times 10^{-21}$ m/s^2, $\sum F_\theta = 0$ and $\sum F_z = 0$

Problem 14.110 At the instant shown, the cylindrical coordinates of the 4-kg part A held by the robotic manipulator are $r = 0.6$ m, $\theta = 25^\circ$, and $z = 0.8$ m. (The coordinate system is fixed with respect to the earth, and the y axis points upward). A's radial position is increasing at $\dfrac{dr}{dt} = 0.2$ m/s, and $\dfrac{d^2r}{dt^2} = -0.4$ m/s^2. The angle θ is increasing at $\dfrac{d\theta}{dt} = 1.2$ rad/s and $\dfrac{d^2\theta}{dt^2} = 2.8$ rad/s^2. The base of the manipulator arm is accelerating in the z direction at $\dfrac{d^2z}{dt^2} = 2.5$ m/s^2. Determine the force vector exerted on A by the manipulator in cylindrical coordinates.



Solution: The total force acting on part A in cylindrical coordinates is given by $\sum F_r = ma_r = m\left(\dfrac{d^2r}{dt^2} - r\omega^2\right)$, $\sum F_\theta = ma_\theta = m\left(r\alpha + 2\left(\dfrac{dr}{dt}\right)\omega\right)$, and $\sum F_z = ma_z = m\dfrac{d^2z}{dt^2}$. We are given the values of every term in the right hand side of these equations. (Recall the definitions of ω and α. Substituting in the known values, we get $\sum F_r = -5.06$ N, $\sum F_\theta = 8.64$ N, and $\sum F_z = 10.0$ N. These are the total forces acting on Part A, including the weight.



To find the forces exerted on the part by the manipulator, we need to draw a free body diagram of the part and resolve the weight into components along the various axes. We get $\sum \mathbf{F} = \sum \mathbf{F}_{\text{manip}} + \mathbf{W} = m\mathbf{a}$ where the components of $\sum \mathbf{F}$ have already been determined above. In cylindrical coordinates, the weight is given as $W = -mg\sin\theta\mathbf{e}_{\text{r}} - mg\cos\theta\mathbf{e}_\theta$. From the previous equation, $\sum \mathbf{F} = \sum \mathbf{F}_{\text{manip}} - mg\sin\theta\mathbf{e}_{\text{r}} - mg\cos\theta\mathbf{e}_\theta$. Substituting in terms of the components, we get $(\sum F_{\text{manip}})_z = 11.5$ (*newtons*), $(\sum F_{\text{manip}})_\theta = 44.2$ (*newtons*) and $(\sum F_{\text{manip}})_z = 10.0$ (*newtons*).

Problem 14.111 Suppose that the robotic manipulator in Problem 14.110 is used in a space station to investigate zero-g manufacturing techniques. During an interval of time, the manipulator is programmed so that the cylindrical coordinates of the 4-kg part A are $\theta = 0.15t^2$ rad, $r = 0.5(1 + \sin\theta)$ m, and $z = 0.8(1 + \theta)$ m Determine the force vector exerted on A by the manipulator at $t = 2$ s in terms of cylindrical coordinates.

Solution:

$$\theta = 0.15t^2 \text{ rad},$$

$$\frac{d\theta}{dt} = 0.3t \text{ rad/s},$$

$$\frac{d^2\theta}{dt^2} = 0.3 \text{ rad/s}^2.$$

$$r = 0.5(1 + \sin\theta) \text{ m},$$

$$\frac{dr}{dt} = 0.5\frac{d\theta}{dt}\cos\theta \text{ m/s},$$

$$\frac{d^2r}{dt^2} = 0.5\frac{d^2\theta}{dt^2}\cos\theta - 0.5\left(\frac{d\theta}{dt}\right)^2\sin\theta \text{ m/s}^2.$$

$$z = 0.8(1 + \theta) \text{ m},$$

$$\frac{dz}{dt} = 0.8\frac{d\theta}{dt} \text{ m/s},$$

$$\frac{d^2z}{dt^2} = 0.8\frac{d^2\theta}{dt^2} \text{ m/s}^2.$$

Evaluating these expressions at $t = 2$ s, the acceleration is

$$\mathbf{a} = \left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right]\mathbf{e}_r + \left(r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}\right)\mathbf{e}_\theta + \frac{d^2z}{dt^2}\mathbf{e}_z$$

$$= -0.259\mathbf{e}_r + 0.532\mathbf{e}_\theta + 0.24\mathbf{e}_z \text{ (m/s}^2).$$

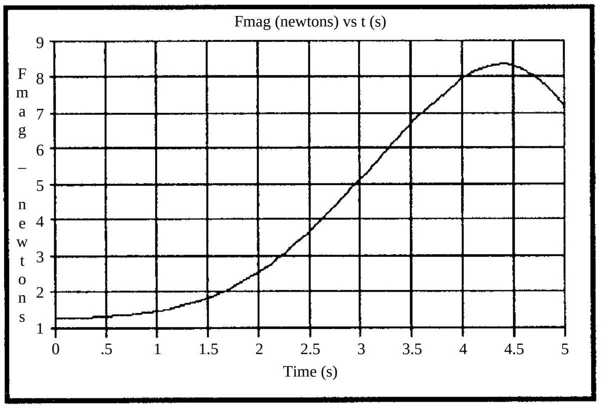
Therefore

$$\sum \mathbf{F} = m\mathbf{a}$$

$$= -1.04\mathbf{e}_r + 2.13\mathbf{e}_\theta + 0.96\mathbf{e}_z \text{ (N)}.$$

Problem 14.112* In Problem 14.111, draw a graph of the magnitude of the force exerted on part A by the manipulator as a function of time from $t = 0$ to $t = 5$ s and use it to estimate the maximum force during that interval of time.

Solution: Use a numerical solver to work problem 14.111 for a series of values of time during the required interval and plot the magnitude of the resulting force as a function of time. From the graph, the maximum force magnitude is approximately 8.4 N and it occurs at a time of about 4.4 seconds.



Problem 14.113 The International Space Station is in a circular orbit 225 miles above the earth's surface.

(a) What is the magnitude of the velocity of the space station?
(b) How long does it take to complete one revolution?

Solution: The radius of the orbit is

$$r_0 = R_E + 225 \text{ mi}$$

$$= 3960 + 225 \text{ mi}$$

$$= 2.21 \times 10^7 \text{ ft.}$$

(a) From Eq (14.24), the velocity is

$$v_0 = \sqrt{\frac{gR_E^2}{r_0}}$$

$$= \sqrt{\frac{(32.2)[(3960)(5280)]^2}{2.21 \times 10^7}}$$

$$= \underline{25200 \text{ ft/s}}(17200 \text{ mi/h}).$$

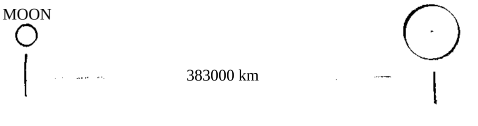
(b) Let T be the time required. Then $v_0 T = 2\pi r_0$,

$$\text{so } T = \frac{2\pi r_0}{v_0} = \underline{5500 \text{ s}} \text{ (1.53 h).}$$

Problem 14.114 The moon is approximately 383,000 km from the earth. Assume that the moon's orbit around the earth is circular with velocity given by Eq. (14.24).

(a) What is the magnitude of the moon's velocity?
(b) How long does it take to complete one revolution around the earth?

Solution:



$$R_E = 6370 \text{ km} = 6.37 \times 10^6 \text{ m}$$

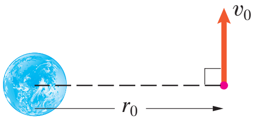
$$r_0 = 383000 \text{ km} = 3.83 \times 10^8 \text{ m}$$

$$\underline{v_0 = \sqrt{\frac{gR_E^2}{r_0}} = 1020 \text{ m/s}}$$

$$\text{Period} = 2\pi r_0 / v_0$$

$$\underline{\text{Period} = 2.36 \times 10^6 \text{ s} = 27.3 \text{ days}}$$

Problem 14.115 Suppose that you place a satellite into an elliptic earth orbit with an initial radius $r_0 = 6700$ km and an initial velocity v_0 such that the maximum radius of the orbit is 13,400 km. (a) Determine v_0. (b) What is the magnitude of the satellite's velocity when it is at its maximum radius? (See Active Example 14.10).



Solution: We have

$$\varepsilon = \frac{r_0 v_0^2}{g R_E} - 1,\ r_{max} = r_0\left(\frac{1+\varepsilon}{1-\varepsilon}\right),\ r_0 v_0 = r_{max} v_{max\,radius}.$$

Solving we find

$$\varepsilon = \frac{r_{max} - r_0}{r_{max} + r_0} = \frac{13{,}400 \text{ km} - 6700 \text{ km}}{13{,}400 \text{ km} + 6700 \text{ km}} = 0.333,$$

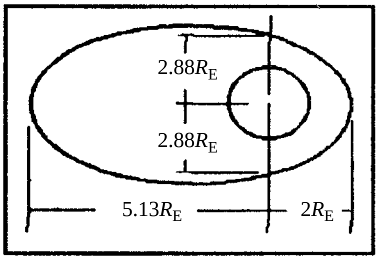
$$v_0 = \sqrt{g(1+\varepsilon)\frac{R_E{}^2}{r_0}} = \sqrt{(9.81 \text{ m/s}^2)(1.333)\frac{(6370 \text{ km})^2}{6700 \text{ km}}} = 8900 \text{ m/s},$$

$$v_{max\ \ radius} = \frac{r_0}{r_{max}} v_0 = \frac{6700 \text{ km}}{13400 \text{ km}}(8900 \text{ m/s}) = 4450 \text{ m/s}$$

$$\boxed{(a)\ v_0 = 8900 \text{ m/s},\quad (b) v_{max\ \ radius} = 4450 \text{ m/s}.}$$

Problem 14.116 A satellite is given an initial velocity $v_0 = 6700$ m/s at a distance $r_0 = 2R_E$ from the center of the earth as shown in Fig. 14.18a. Draw a graph of the resulting orbit.

Solution: The graph is shown.



Problem 14.117 The time required for a satellite in a circular earth orbit to complete one revolution increases as the radius of the orbit increases. If you choose the radius properly, the satellite will complete one revolution in 24 hours. If a satellite is placed in such an orbit directly above the equator and moving from west to east, it will remain above the same point on the earth as the earth rotates beneath it. This type of orbit, conceived by Arthur C. Clarke, is called *geosynchronous*, and is used for communication and television broadcast satellites. Determine the radius of a geosynchronous orbit in km.

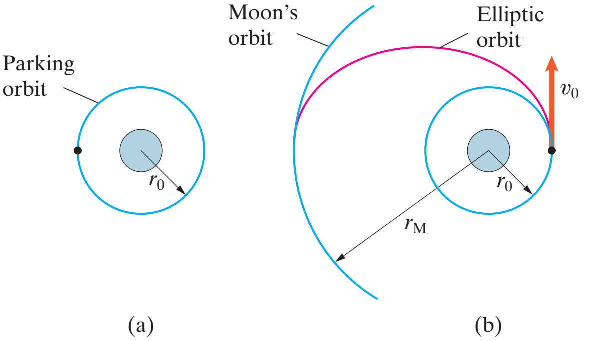
Solution: We have

$$v = \frac{2\pi r}{T}, \quad \frac{v^2}{r} = g\frac{R_E{}^2}{r^2}$$

$$r = \left(\frac{gR_E^2T^2}{4\pi^2}\right)^{1/3} = \left(\frac{[9.81\text{ m/s}^2][6370\times10^3\text{ m}]^2[24(60)(60)\text{ s}]^2}{4\pi^2}\right)^{1/3}$$

$$\boxed{r = 42.2\times10^6\text{ m} = 42{,}200\text{ km.}}$$

Problem 14.118* You can send a spacecraft from the earth to the moon in the following way. First, launch the spacecraft into a circular "parking" orbit of radius r_0 around the earth (Fig. a). Then increase its velocity in the direction tangent to the circular orbit to a value v_0 such that it will follow an elliptic orbit whose maximum radius is equal to the radius r_M of the moon's orbit around the earth (Fig. b). The radius $r_M = 382942$ km. Let $r_0 = 6693$km. What velocity v_0 is necessary to send a spacecraft to the moon? (This description is simplified in that it disregards the effect of the moon's gravity.)



(a) (b)

Solution: Note that

$$R_E = 6370\text{ km}, \quad r_0 = 6693\text{ km}, \quad r_M = 382942\text{ km}$$

First find the eccentricity:

$$r_{max} = r_M = r_0\left(\frac{1+\varepsilon}{1-\varepsilon}\right) \Rightarrow \varepsilon = \frac{r_M - r_0}{r_M + r_0}$$

Then use eq. 14.23

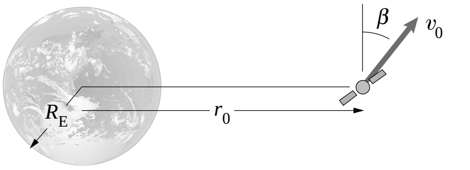
$$\varepsilon = \frac{r_0 v_0{}^2}{gR_E{}^2} - 1 \Rightarrow v_0 = R_E\sqrt{\frac{(1+\varepsilon)g}{r_0}} = R_E\sqrt{\frac{2gr_M}{r_0(r_0+r_M)}}$$

Putting in the numbers we have $\boxed{v_0 = 10820\text{ m/s}}$

Problem 14.119* At $t = 0$, an earth satellite is a distance r_0 from the center of the earth and has an initial velocity v_0 in the direction shown. Show that the polar equation for the resulting orbit is

$$\frac{r}{r_0} = \frac{(\varepsilon + 1)\cos^2\beta}{[(\varepsilon + 1)\cos^2\beta - 1]\cos\theta - (\varepsilon + 1)\sin\beta\cos\beta\sin\theta + 1},$$

where $\varepsilon = \left(\dfrac{r_0 v_0^2}{g R_E^2}\right) - 1.$



Solution: We need to modify the solution in Section 14.5 to account for this new initial condition. At $\theta = 0$ (see Fig. 14.17)

$$v_r = \frac{dr}{dt} = v_0 \sin\beta$$

and

$$v_\theta = r\frac{d\theta}{dt} = v_0 \cos\beta.$$

Therefore Eq (14.15) becomes

$$r^2\frac{d\theta}{dt} = r v_\theta = r_0 v_0 \cos\beta \quad (1)$$

Following the same steps that led to Eq. (14.21) in terms of $u = 1/r$ yields

$$u = A\sin\theta + B\cos\theta + \frac{g R_E^2}{r_0^2 v_0^2 \cos^2\beta}. \quad (2)$$

At $\theta = 0$,

$$u = \frac{1}{r_0}. \quad (3)$$

Also, note that

$$v_r = \frac{dr}{dt} = \frac{d}{dt}\left(\frac{1}{u}\right) = -\frac{1}{u^2}\frac{du}{dt} = -\frac{1}{u^2}\frac{du}{d\theta}\frac{d\theta}{dt}$$

$$= -r_0 v_0 \cos\beta\frac{du}{d\theta},$$

where we used (1). Therefore, at $\theta = 0$

$$-r_0 v_0 \cos\beta\frac{du}{d\theta} = v_0 \sin\beta. \quad (4)$$

From (2),

$$\frac{du}{d\theta} = A\cos\theta - B\sin\theta. \quad (5)$$

From conditions (3) and (4) and Eq. (5),

$$B = \frac{1}{r_0} - \frac{g R_E^2}{r_0^2 v_0^2 \cos^2\beta}$$

and

$$A = -\frac{\sin\beta}{r_0\cos\beta}.$$

Substituting these expressions for A and B into Eq (2) yields the desired result.

Problem 14.120 The Acura NSX can brake from 120 km/h to a stop in a distance of 112 m. (a) If you assume that the vehicle's deceleration is constant, what are its deceleration and the magnitude of the horizontal force its tires exert on the road? (b) If the car's tires are at the limit of adhesion (i.e., slip is impending), and the normal force exerted on the car by the road equals the car's weight, what is the coefficient of friction μ_s? (This analysis neglects the effects of horizontal and vertical aerodynamic forces).

Solution:

(a) 120 km/h = 33.3 m/s.

$$a = \frac{dv}{dt} = \frac{dv}{ds}v.$$

Integrating,

$$\int_{33.3}^{0} v\,dv = \int_{0}^{112} a\,ds,$$

we obtain $a = -4.95$ m/s^2. The magnitude of the friction force is

$$f = m|a| = \left(\frac{13000}{9.81}\right)(4.95)$$

$$= 6560 \text{ N}.$$

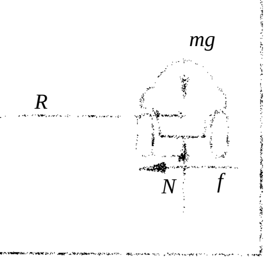
(b) The Normal force is the car's weight, so

$$\mu_s = \frac{f}{N} = \frac{6560}{13000}$$

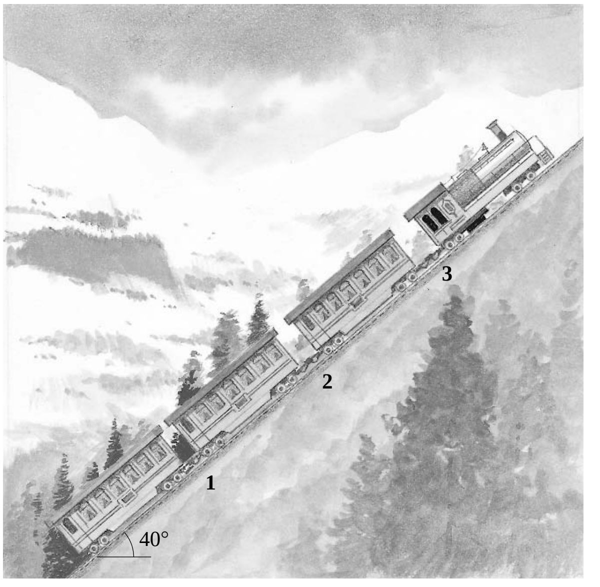
$$= 0.505.$$

Problem 14.121 Using the coefficient of friction obtained in Problem 14.120, determine the highest speed at which the NSX could drive on a flat, circular track of 600-m radius without skidding.

Solution: The free body diagram is at the right. The normal force is equal to the weight and the friction force has the same magnitude as in Problem 14.120 since $f = \mu_s N$. The equation of motion in the radial direction (from the center of curvature of the track to the car) is $\sum F_r = mv^2/R = f = \mu_s N = \mu_s mg$. Thus, we have that $mv^2/R = \mu_s mg$ or $v^2 = \mu_s Rg$. Inserting the numbers, we obtain $v = 36.7$ m/s $= 132$ km/h.



Problem 14.122 A cog engine hauls three cars of sightseers to a mountain top in Bavaria. The mass of each car, including its passengers, is 10,000 kg and the friction forces exerted by the wheels of the cars are negligible. Determine the forces in couplings 1, 2, and 3 if: (a) the engine is moving at constant velocity; (b) the engine is accelerating up the mountain at 1.2 m/s^2.



Solution: (a) The force in coupling 1 is

$$\boxed{F_1 = 10{,}000\ g \sin 40^\circ = 631\ \text{kN.}}$$

The force on coupling 2 is

$$\boxed{F_2 = 20{,}000\ g \sin(40) = 126.1\ \text{kN.}}$$

The force on coupling 3 is

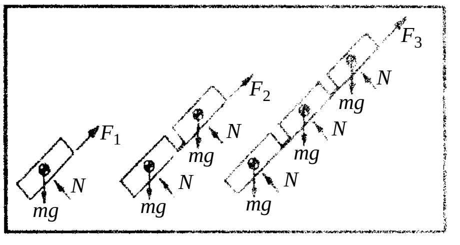
$$\boxed{F_3 = 30{,}000\ g \sin 40^\circ = 189.2\ \text{kN.}}$$

(b) From Newton's second law, $F_{1a} - mg \sin 40^\circ = ma$. Under constant acceleration up the mountain, the force on coupling 1 is

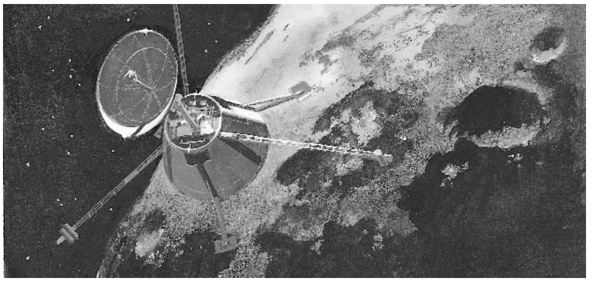
$$\boxed{F_{1a} = 10{,}000a + 10{,}000\ g \sin 40^\circ = 75.1\ \text{kN.}}$$

The force on coupling 2 is $\boxed{F_{2a} = 2F_{1a} = 150.1\ \text{kN.}}$

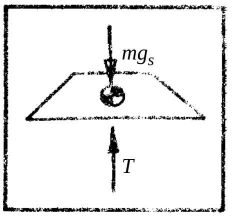
The force on coupling 3 is $\boxed{F_{2a} = 3\ F_{1a} = 225.2\ \text{kN}}$



Problem 14.123 In a future mission, a spacecraft approaches the surface of an asteroid passing near the earth. Just before it touches down, the spacecraft is moving downward at a constant velocity relative to the surface of the asteroid and its downward thrust is 0.01 N. The computer decreases the downward thrust to 0.005 N, and an onboard laser interferometer determines that the acceleration of the spacecraft relative to the surface becomes 5×10^{-6} m/s^2 downward. What is the gravitational acceleration of the asteroid near its surface?

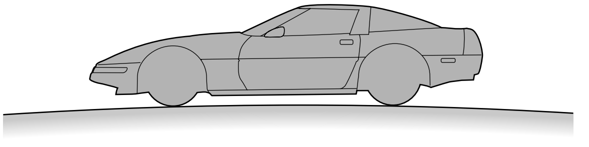


Solution: Assume that the mass of the spacecraft is negligible compared to mass of the asteroid. With constant downward velocity, the thrust balances the gravitational force: $0.01 - mg_s = 0$, where m is the mass of the space craft. With the change in thrust, this becomes $0.005 - mg_s = m(-5 \times 10^{-6})$ N/kg^2. Multiply the first equation by 0.005, the second by 0.01, and subtract: The result:

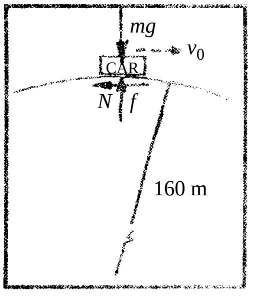


$$\boxed{g_s = \left(\frac{0.01(5 \times 10^{-6})}{(0.01 - 0.005)}\right) = 1 \times 10^{-5} \text{ N/kg}^2}$$

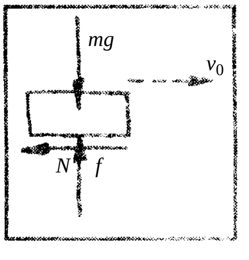
Problem 14.124 A car with a mass of 1470 kg, including its driver, is driven at 130 km/h over a slight rise in the road. At the top of the rise, the driver applies the brakes. The coefficient of static friction between the tires and the road is $\mu_s = 0.9$ and the radius of curvature of the rise is 160 m. Determine the car's deceleration at the instant the brakes are applied, and compare it with the deceleration on a level road.



Solution: First, note that 130 km/h = 36.11 m/s. We have a situation in which the car going over the rise reduces the normal force exerted on the car by the road and also reduces the braking force. The free body diagram of the car braking over the rise is shown at the right along with the free body diagram of the car braking on a level surface. For the car going over the rise, the equations of motion are $\sum F_t = -f = ma_t$, where f is the friction force. The normal equation is $\sum F_n = N - mg = mv^2/R$. The relation between friction and normal force is given as $f = \mu_s N$. Solving, we get $a_t = -1.49$ m/s^2.

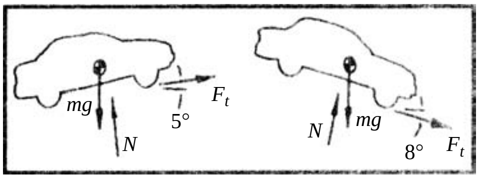


For the car braking on a level surface, the equations are $N - mg = 0$, $f = \mu_s N$, and $-f = ma_x$. Evaluating, we get $a_x = 8.83$ m/s^2. Note that the accelerations are VERY different. We conclude that at 130 km/h, a rise in the road with a radius of 160 m is not "slight". The car does not become airborne, but if the radius of curvature were smaller, the car would leave the road.



Problem 14.125 The car drives at constant velocity up the straight segment of road on the left. If the car's tires continue to exert the same tangential force on the road after the car has gone over the crest of the hill and is on the straight segment of road on the right, what will be the car's acceleration?

Solution: The tangential force on the left is, from Newton's second law, $F_t - mg\sin(5^\circ) = ma = 0$. On the right, from Newton's second law: $F_t + mg\sin(8^\circ) = ma$ from which the acceleration is



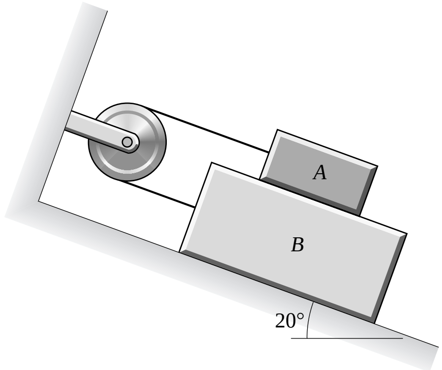
$$\boxed{a = g(\sin 5^\circ + \sin 8^\circ) = 0.2264 \text{ g}}$$

Problem 14.126 The aircraft carrier *Nimitz* weighs 91,000 tons. (A ton is 8896 N) Suppose that it is traveling at its top speed of approximately 15.4 m/s when its engines are shut down. If the water exerts a drag force of magnitude $88960v$ N, where v is the carrier's velocity in metre per second, what distance does the carrier move before coming to rest?

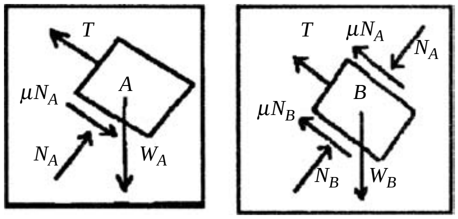
Solution: The force on the carrier is $F = -Kv$, where $K = 88960$. The acceleration is $a = \dfrac{F}{m} = -\dfrac{gK}{W}v$. Use the chain rule to write $v\dfrac{dv}{dx} = -\dfrac{gK}{W}v$. Separate variables and integrate: $dv = -\dfrac{gK}{W}dx$, $v(x) = -\dfrac{gK}{W}x + C$. The initial velocity: $v(0) = 15.4$ m/s, from which $C = v(0) = 50.63$, and $v(x) = v(0) - \dfrac{gK}{W}x$, from which, at rest,

$$\boxed{x = \frac{Wv(0)}{gK} = 4365 \text{ m} = 4.36 \text{ km}}$$

Problem 14.127 If $m_A = 10$ kg, $m_B = 40$ kg, and the coefficient of kinetic friction between all surfaces is $\mu_k = 0.11$, what is the acceleration of B down the inclined surface?



Solution: Choose a coordinate system with the origin at the wall and the x axis parallel to the plane surface. Denote $\theta = 20^\circ$. Assume that slip has begun. From Newton's second law for block A:



(1) $\sum F_x = -T + m_A g \sin\theta + \mu_k N_A = m_A a_A$,

(2) $\sum F_y = N_A - m_A g \cos\theta = 0$. From Newton's second law for block B:

(3) $\sum F_x = -T - \mu_k N_B - \mu_k N_A + m_B g \sin\theta = m_B a_B$,

(4) $\sum F_y = N_B - N_A - m_B g \cos\theta = 0$. Since the pulley is one-to-one, the sum of the displacements is $x_B + x_A = 0$. Differentiate twice:

(5) $a_B + a_A = 0$. Solving these five equations in five unknowns, $T = 49.63$ N, $N_A = 92.2$ N, $N_B = 460.9$ N, $a_A = -0.593$ m/s^2,

$\boxed{a_B = 0.593 \text{ m/s}^2}$

Problem 14.128 In Problem 14.127, if A weighs 89 N, B weighs 444.8 N, and the coefficient of kinetic friction between all surfaces is $\mu_k = 0.15$, what is the tension in the cord as B slides down the inclined surface?

Solution: From the solution to Problem 14.127,

(1) $\sum F_x = -T + m_A g \sin\theta + \mu_k N_A = m_A a_A$,

(2) $\sum F_y = N_A - m_A g \cos\theta = 0$. For block B:

(3) $\sum F_x = -T - \mu_k N_B - \mu_k N_A + m_B g \sin\theta = m_B a_B$,

(4) $\sum F_y = N_B - N_A - m_B g \cos\theta = 0$.

(5) $a_B + a_A = 0$ Solve by iteration:

$\boxed{T = 46.5 \text{ N}}$,

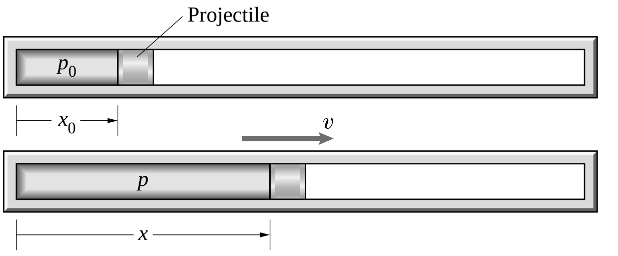
$N_A = 83.6$ N, $N_B - 501.7$ N, $a_A = -0.39$ m/s^2,

$a_B = 0.39$ m/s^2

Problem 14.129 A gas gun is used to accelerate projectiles to high velocities for research on material properties. The projectile is held in place while gas is pumped into the tube to a high pressure p_0 on the left and the tube is evacuated on the right. The projectile is then released and is accelerated by the expanding gas. Assume that the pressure p of the gas is related to the volume V it occupies by $pV^\gamma =$ constant, where γ is a constant. If friction can be neglected, show that the velocity of the projectile at the position x is

$$v = \sqrt{\frac{2p_0 A x_0^\gamma}{m(\gamma - 1)}\left(\frac{1}{x_0^{\gamma-1}} - \frac{1}{x^{\gamma-1}}\right)},$$

where m is the mass of the projectile and A is the cross-sectional area of the tube.



Solution: The force acting on the projectile is $F = pA$ where p is the instantaneous pressure and A is the area. From $pV^\gamma = K$, where $K = p_0 V_0^\gamma$ is a constant, and the volume $V = Ax$, it follows that $p = \frac{K}{(Ax)^\gamma}$, and the force is $F = KA^{1-\gamma}x^{-\gamma}$. The acceleration is

$$a = \frac{F}{m} = \frac{K}{m}A^{1-\gamma}x^{-\gamma}.$$

The equation to be integrated:

$v\frac{dv}{dx} = \frac{K}{m}A^{1-\gamma}x^{-\gamma}$, where the chain rule $\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$ has been used. Separate variables and integrate:

$$v^2 = 2\left(\frac{K}{m}\right)A^{1-\gamma}\int x^{-\gamma}\,dx + C = 2\left(\frac{K}{m}\right)A^{1-\gamma}\left(\frac{x^{1-\gamma}}{1-\gamma}\right) + C.$$

When $x = x_0$, $v_0 = 0$, therefore

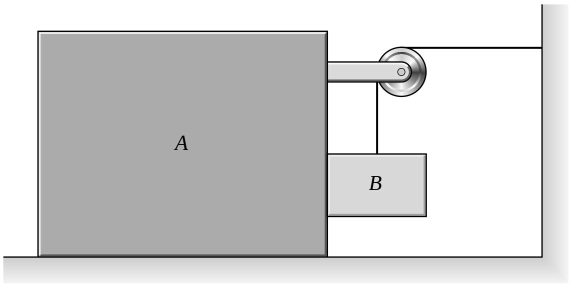
$$v^2 = 2\left(\frac{K}{m}\right)\left(\frac{A^{1-\gamma}}{1-\gamma}\right)(x^{1-\gamma} - x_0^{1-\gamma}).$$

Substitute $K = p_0 V_0^\gamma = p_0 A^\gamma x_0^\gamma$ and reduce:

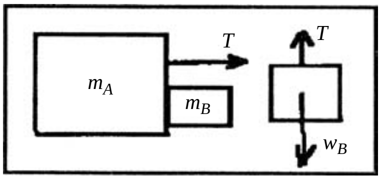
$v^2 = \frac{2p_0 A x_0^\gamma}{m(1-\gamma)}(x^{1-\gamma} - x_0^{1-\gamma})$. Rearranging:

$$\boxed{v = \sqrt{\frac{2p_0 A x_0^\gamma}{m(\gamma - 1)}\left(\frac{1}{x_0^{\gamma-1}} - \frac{1}{x^{\gamma-1}}\right)}}$$

Problem 14.130 The weights of the blocks are $W_A = 120$ N, and $W_B = 20$ N and the surfaces are smooth. Determine the acceleration of block A and the tension in the cord.



Solution: Denote the tension in the cord near the wall by T_A. From Newton's second law for the two blocks:



$$\sum F_x = T_A = \left(\frac{W_A}{g} + \frac{W_B}{g}\right) a_A.$$

For block B: $\sum F_y = T_A - W_B = \dfrac{W_B}{g} a_B$. Since the pulley is one-to-one, as the displacement of B increases *downward* (negatively) the displacement of A increases *to the right* (positively), from which $x_A = -x_B$. Differentiate twice to obtain $a_A = -a_B$. Equate the expressions to obtain:
$a\left(\dfrac{W_A}{g} + \dfrac{W_B}{g}\right) = W_B + \dfrac{W_B}{g} a$, from which

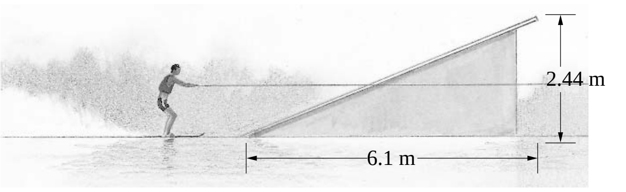
$$\boxed{a = g\left(\frac{W_B}{W_A + 2W_B}\right) = g\left(\frac{20}{160}\right) = \frac{9.81}{8} = 1.23 \text{ m/s}^2}$$

Problem 14.131 The 100-Mg space shuttle is in orbit when its engines are turned on, exerting a thrust force $\mathbf{T} = 10\mathbf{i} - 20\mathbf{j} + 10\mathbf{k}$ (kN) for 2 s. Neglect the resulting change in mass of the shuttle. At the end of the 2-s burn, fuel is still sloshing back and forth in the shuttle's tanks. What is the change in the velocity of the center of mass of the shuttle (including the fuel it contains) due to the 2-s burn?

Solution: At the completion of the burn, there are no external forces on the shuttle (it is in free fall) and the fuel sloshing is caused by internal forces that cancel, and the center of mass is unaffected. The change in velocity is

$$\boxed{\begin{aligned} \Delta\mathbf{v} &= \int_0^2 \frac{\mathbf{T}}{m}\,dt = \frac{2(10^4)}{10^5}\mathbf{i} - \frac{2(2\times 10^4)}{10^5}\mathbf{j} + \frac{2(10^4)}{10^5}\mathbf{k} \\ &= 0.2\mathbf{i} - 0.4\mathbf{j} + 0.2\mathbf{k} \text{ (m/s)} \end{aligned}}$$

Problem 14.132 The water skier contacts the ramp with a velocity of 40.2 km/h parallel to the surface of the ramp. Neglecting friction and assuming that the tow rope exerts no force on him once he touches the ramp, estimate the horizontal length of the skier's jump from the end of the ramp.



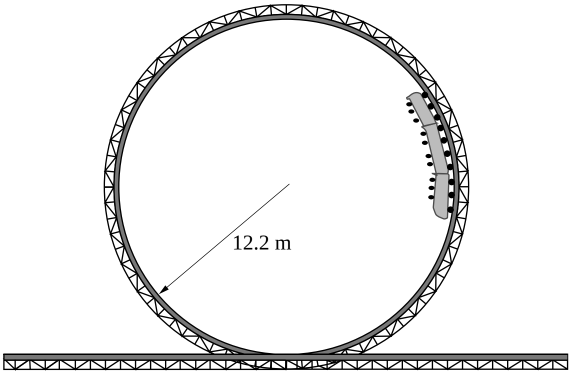
Solution: Break the path into two parts: (1) The path from the base to the top of the ramp, and (2) from the top of the ramp until impact with the water. Let u be the velocity parallel to the surface of the ramp, and let z be the distance along the surface of the ramp. From the chain rule, $u\dfrac{du}{dz} = -g\sin\theta$, where $\theta = \tan^{-1}\left(\dfrac{2.44}{6.1}\right) = 21.8^\circ$. Separate variables and integrate: $u^2 = -(2g\sin\theta)z + C$. At the base of the ramp

$$u(0) = 40.2 \text{ km/h} = 11.17 \text{ m/s}$$

from which $C = (11.17^2) = 124.8$ and $u = \sqrt{C - (2g\sin\theta)z}$. At $z = \sqrt{2.44^2 + 6.1^2} = 6.56$ m $u = 8.78$ m/s. (2) In the second part of the path the skier is in free fall. The equations to be integrated are $\dfrac{dv_y}{dt} = -g$, $\dfrac{dy}{dt} = v_y$, with $v(0) = u\sin\theta = 8.78(0.3714) = 3.26$ m/s, $y(0) = 2.44$ m. $\dfrac{dv_x}{dt} = 0$, $\dfrac{dx}{dt} = v_x$, with $v_x(0) = u\cos\theta = 8.14$ m/s, $x(0) = 0$. Integrating: $v_y(t) = -gt + 3.26$ m/s. $y(t) = -\dfrac{g}{2}t^2 + 3.26t + 2.44$ m $v_x(t) = 8.14$ m/s, $x(t) = 8.14t$. When $y(t_{\text{impact}}) = 0$, the skier has hit the water. The impact time is $t_{\text{impact}}^2 + 2bt_{\text{impact}} + c = 0$ where $b = -\dfrac{3.26}{g}$, $c = -\dfrac{4.85}{g}$. The solution $t_{\text{impact}} = -b \pm \sqrt{b^2 - c} = 1.11$ s, $= -0.45$ s. The negative values has no meaning here. The horizontal distance is

$$\boxed{x(t_{\text{impact}}) = 8.14t_{\text{impact}} = 9.05 \text{ m}}$$

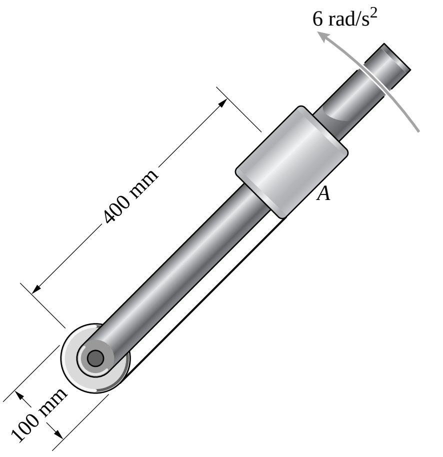
Problem 14.133 Suppose you are designing a roller-coaster track that will take the cars through a vertical loop of 12.2 m radius. If you decide that, for safety, the downward force exerted on a passenger by his seat at the top of the loop should be at least one-half the passenger's weight, what is the minimum safe velocity of the cars at the top of the loop?



Solution: Denote the normal force exerted on the passenger by the seat by N. From Newton's second law, at the top of the loop $-N - mg = -m\left(\dfrac{v^2}{R}\right)$, from which $-\dfrac{N}{m} = g - \dfrac{v^2}{R} = -\dfrac{g}{2}$. From which:

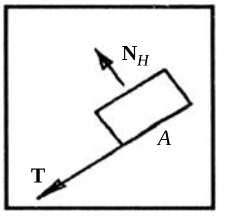
$$\boxed{v = \sqrt{\frac{3Rg}{2}} = 13.4 \text{ m/s}}$$

Problem 14.134 As the smooth bar rotates *in the horizontal plane*, the string winds up on the fixed cylinder and draws the 1-kg collar A inward. The bar starts from rest at $t = 0$ in the position shown and rotates with constant acceleration. What is the tension in the string at $t = 1$ s?



Solution: The angular velocity of the spool relative to the bar is $\alpha = 6$ rad/s^2. The acceleration of the collar relative to the bar is $\dfrac{d^2r}{dt^2} = -R\alpha = -0.05(6) = -0.3$ m/s^2. The take up velocity of the spool is

$$v_s = \int R\alpha\, dt = -0.05(6)t = -0.3t \text{ m/s.}$$



The velocity of the collar relative to the bar is
$\dfrac{dr}{dt} = -0.3t$ m/s.
The velocity of the collar relative to the bar is $dr/dt = -0.3t$ m/s. The position of the collar relative to the bar is $r = -0.15t^2 + 0.4$ m. The angular acceleration of the collar is $\dfrac{d^2\theta}{dt^2} = 6$ rad/s^2. The angular velocity of the collar is $\dfrac{d\theta}{dt} = 6t$ rad/s. The radial acceleration is $a_r = \dfrac{d^2r}{dt^2} - r\left(\dfrac{d\theta}{dt}\right)^2 = -0.3 - (-0.15t^2 + 0.4)(6t)^2$. At $t = 1$ s the radial acceleration is $a_r = -9.3$ m/s^2, and the tension in the string is

$\boxed{|T| = |ma_r| = 9.3 \text{ N}}$

Problem 14.135 In Problem 14.134, suppose that the coefficient of kinetic friction between the collar and the bar is $\mu_k = 0.2$. What is the tension in the string at $t = 1$ s?

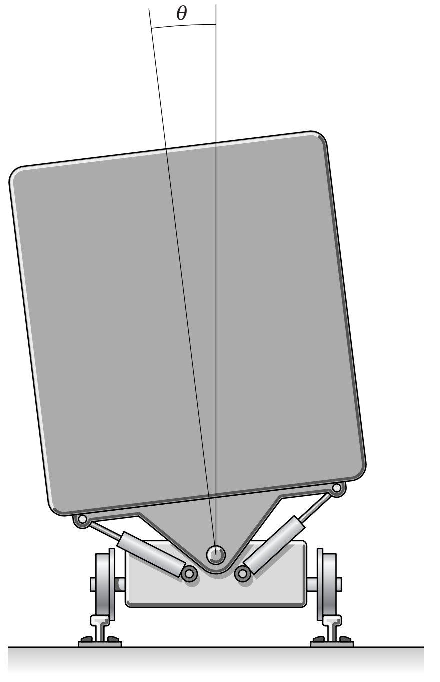
Solution: Use the results of the solution to Problem 14.134 At $t = 1$ s, the horizontal normal force is

$$N_H = |ma_\theta| = m\left|\left(r\frac{d^2\theta}{dt^2} + 2\left(\frac{dr}{dt}\right)\left(\frac{d\theta}{dt}\right)\right)\right| = 2.1 \text{ N,}$$

from which the total normal force is $N = \sqrt{N_H^2 + (mg)^2}$ From Newton's second law: $\left(-T + \mu_k\sqrt{N_H^2 + (mg^2)}\right)\mathbf{e}_r + N_H\mathbf{e}_\theta = ma_r\mathbf{e}_r + ma_\theta\mathbf{e}_\theta$, from which $-T + \mu_k\sqrt{N_H^2 + (mg)^2} = ma_r$. From the solution to Problem 14.152, $a_r = -9.3$ m/s^2. Solve: The tension is

$\boxed{T = 11.306 \text{ N}}$

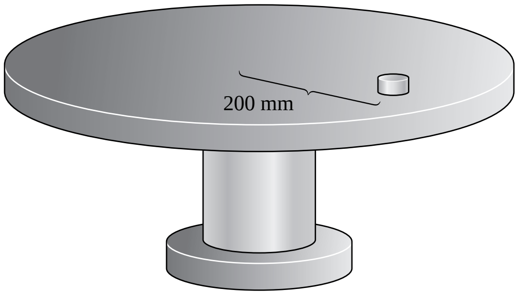
Problem 14.136 If you want to design the cars of a train to tilt as the train goes around curves in order to achieve maximum passenger comfort, what is the relationship between the desired tilt and θ, the velocity v of the train, and the instantaneous radius of curvature, ρ, of the track?



Solution: For comfort, the passenger should feel the total effects of acceleration down toward his feet, that is, apparent side (radial) accelerations should not be felt. This condition is achieved when the tilt θ is such that $mg\sin\theta - m(v^2/\rho)\cos\theta = 0$, from which

$$\boxed{\tan\theta = \frac{v^2}{\rho g}.}$$

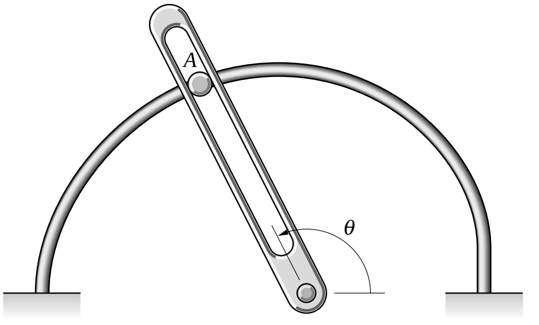
Problem 14.137 To determine the coefficient of static friction between two materials, an engineer at the U.S. National Institute of Standards and Technology places a small sample of one material on a horizontal disk whose surface is made of the other material and then rotates the disk from rest with a constant angular acceleration of 0.4 rad/s^2. If she determines that the small sample slips on the disk after 9.903 s, what is the coefficient of friction?



Solution: The angular velocity after $t = 9.903$ s is $\omega = 0.4t = 3.9612$ rad/s. The radial acceleration is $a_n = 0.2\omega^2 = 3.138$ m/s^2. The tangential acceleration is $a_t = (0.2)0.4 = 0.08$ m/s^2. At the instant before slip occurs, Newton's second law for the small sample is $\sum F = \mu_s N = \mu_s mg = m\sqrt{a_n^2 + a_t^2}$, from which

$$\boxed{\mu_s = \frac{\sqrt{a_n^2 + a_t^2}}{g} = 0.320}$$

Problem 14.138* The 1-kg slider A is pushed along the curved bar by the slotted bar. The curved bar lies *in the horizontal plane*, and its profile is described by $r = 2\left(\frac{\theta}{2\pi} + 1\right)$ m, where θ is in radians. The angular position of the slotted bar is $\theta = 2t$ rad. Determine the radial and transverse components of the total external force exerted on the slider when $\theta = 120^0$.



Solution: The radial position is $r = 2\left(\frac{t}{\pi} + 1\right)$. The radial velocity: $\frac{dr}{dt} = \frac{2}{\pi}$.

The radial acceleration is zero. The angular velocity: $\frac{d\theta}{dt} = 2$. The angular acceleration is zero. At $\theta = 120^0 = 2.09$ rad. From Newton's second law, the radial force is $F_r = ma_r$, from which

$$\boxed{\mathbf{F}_r = -\left[r\left(\frac{d\theta}{dt}\right)^2\right]\mathbf{e}_r = -10.67\mathbf{e}_r \text{ N}}$$

The transverse force is $F_\theta = ma_\theta$, from which

$$\boxed{\mathbf{F}_\theta = 2\left[\left(\frac{dr}{dt}\right)\left(\frac{d\theta}{dt}\right)\right]\mathbf{e}_\theta = 2.55\mathbf{e}_\theta \text{ N}}$$

Problem 14.139* In Problem 14.138, suppose that the curved bar lies *in the vertical plane*. Determine the radial and transverse components of the total force exerted on A by the curved and slotted bars at $t = 0.5$ s.

Solution: Assume that the curved bar is vertical such that the line $\theta = 0$ is horizontal. The weight has the components: $\mathbf{W} = (W\sin\theta)\mathbf{e}_r + (W\cos\theta)\mathbf{e}_\theta$. From Newton's second law: $F_r - W\sin\theta = ma_r$, and $F_\theta - W\cos\theta = ma_\theta$., from which $\mathbf{F}_r - g\sin 2t\mathbf{e}_r = -r(d\theta/dt)^2\mathbf{e}_r$, from which

$$F_r = \left(-2\left(\frac{t}{\pi} + 1\right)(2^2) + g\sin 2t\right),$$

at $t = 0.5$ s, $\boxed{F_r = -1.02 \text{ N}}$. The transverse component $F_\theta = 2\left(\frac{2}{\pi}\right)(2) + g\cos 2t = \left(\frac{8}{\pi} + g\cos 2t\right)$. At $t = 0.51$ s,

$$\boxed{F_\theta = 7.85 \text{ N}}$$