Problem 12.1 The value of π is $3.1415962654.\ldots$ If C is the circumference of a circle and r is its radius, determine the value of r/C to four significant digits.

Solution:

$$C = 2\pi r \Rightarrow \frac{r}{C} = \frac{1}{2\pi} = 0.159154943.$$

To four significant digits we have $\boxed{\dfrac{r}{C} = 0.1592}$

**Problem 12.2** The base of natural logarithms is $e = 2.718281828\ldots$

(a) Express e to five significant digits.
(b) Determine the value of e^2 to five significant digits.
(c) Use the value of e you obtained in part (a) to determine the value of e^2 to five significant digits.

[Part (c) demonstrates the hazard of using rounded-off values in calculations.]

Solution: The value of e is: $e = 2.718281828$

(a) To five significant figures $\boxed{e = 2.7183}$
(b) e^2 to five significant figures is $\boxed{e^2 = 7.3891}$
(c) Using the value from part (a) we find $\boxed{e^2 = 7.3892}$ which is not correct in the fifth digit.

Problem 12.3 A machinist drills a circular hole in a panel with a nominal radius $r = 5$ mm. The actual radius of the hole is in the range $r = 5 \pm 0.01$ mm. (a) To what number of significant digits can you express the radius? (b) To what number of significant digits can you express the area of the hole?

5 mm
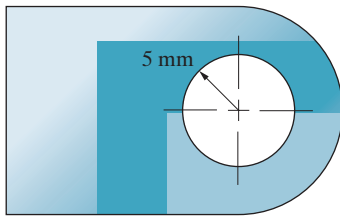
Solution:

(a) The radius is in the range $r_1 = 4.99$ mm to $r_2 = 5.01$ mm. These numbers are not equal at the level of three significant digits, but they are equal if they are rounded off to two significant digits.

$\boxed{\text{Two: } r = 5.0 \text{ mm}}$

(b) The area of the hole is in the range from $A_1 = \pi r_1^2 = 78.226$ m^2 to $A_2 = \pi r_2^2 = 78.854$ m^2. These numbers are equal only if rounded to one significant digit:

$\boxed{\text{One: } A = 80 \text{ mm}^2}$

Problem 12.4 The opening in the soccer goal is 25 ft wide and 8 ft high, so its area is 24 ft $\times$ 8 ft $= 192$ ft^2. What is its area in m^2 to three significant digits?

Solution:

$$A = 192 \text{ ft}^2 \left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right)^2 = 17.8 \text{ m}^2$$

$\boxed{A = 17.8 \text{ m}^2}$

Problem 12.5 The Burj Dubai, scheduled for completion in 2008, will be the world's tallest building with a height of 705 m. The area of its ground footprint will be 8000 m^2. Convert its height and footprint area to U.S. customary units to three significant digits.

Solution:

$$h = 705 \text{ m} \left(\frac{3.281 \text{ ft}}{1 \text{ m}} \right) = 2.31 \times 10^3 \text{ ft}$$

$$A = 8000 \text{ m}^2 \left(\frac{3.218 \text{ ft}}{1 \text{ m}} \right)^2 = 8.61 \times 10^4 \text{ ft}^2$$

$$\boxed{h = 2.31 \times 10^3 \text{ ft}, \quad A = 8.61 \times 10^4 \text{ ft}^2}$$

Problem 12.6 Suppose that you have just purchased a Ferrari F355 coupe and you want to know whether you can use your set of SAE (U.S. Customary Units) wrenches to work on it. You have wrenches with widths $w = 1/4$ in, $1/2$ in, $3/4$ in, and 1 in, and the car has nuts with dimensions $n = 5$ mm, 10 mm, 15 mm, 20 mm, and 25 mm. Defining a wrench to fit if w is no more than 2% larger than n, which of your wrenches can you use?

Solution: Convert the metric size n to inches, and compute the percentage difference between the metric sized nut and the SAE wrench. The results are:

$$5 \text{ mm} \left(\frac{1 \text{ inch}}{25.4 \text{ mm}} \right) = 0.19685.. \text{ in}, \quad \left(\frac{0.19685 - 0.25}{0.19685} \right) 100$$

$$= -27.0\%$$

$$10 \text{ mm} \left(\frac{1 \text{ inch}}{25.4 \text{ mm}} \right) = 0.3937.. \text{ in}, \quad \left(\frac{0.3937 - 0.5}{0.3937} \right) 100 = -27.0\%$$

$$15 \text{ mm} \left(\frac{1 \text{ inch}}{25.4 \text{ mm}} \right) = 0.5905.. \text{ in}, \quad \left(\frac{0.5905 - 0.5}{0.5905} \right) 100 = +15.3\%$$

$$20 \text{ mm} \left(\frac{1 \text{ inch}}{25.4 \text{ mm}} \right) = 0.7874.. \text{ in}, \quad \left(\frac{0.7874 - 0.75}{0.7874} \right) 100 = +4.7\%$$

$$25 \text{ mm} \left(\frac{1 \text{ inch}}{25.4 \text{ mm}} \right) = 0.9843.. \text{ in}, \quad \left(\frac{0.9843 - 1.0}{0.9843} \right) 100 = -1.6\%$$

A negative percentage implies that the metric nut is smaller than the SAE wrench; a positive percentage means that the nut is larger then the wrench. Thus within the definition of the 2% fit, the 1 in wrench will fit the 25 mm nut. **The other wrenches cannot be used.**
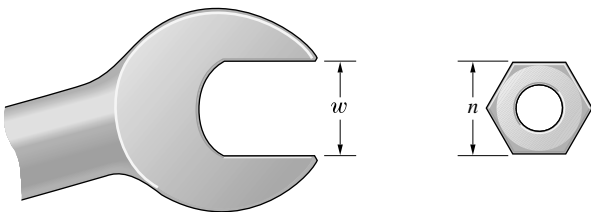
Problem 12.7 Suppose that the height of Mt. Everest is known to be between 29,032 ft and 29,034 ft. Based on this information, to how many significant digits can you express the height (a) in feet? (b) in meters?

Solution:

(a) $h_1 = 29032$ ft

   $h_2 = 29034$ ft

   The two heights are equal if rounded off to four significant digits. The fifth digit is not meaningful.

   $\boxed{\text{Four: } h = 29{,}030 \text{ ft}}$

(b) In meters we have

   $$h_1 = 29032 \text{ ft} \left(\frac{1 \text{ m}}{3.281 \text{ ft}} \right) = 8848.52 \text{ m}$$

   $$h_2 = 29034 \text{ ft} \left(\frac{1 \text{ m}}{3.281 \text{ ft}} \right) = 8849.13 \text{ m}$$

   These two heights are equal if rounded off to three significant digits. The fourth digit is not meaningful.

   $\boxed{\text{Three: } h = 8850 \text{ m}}$

Problem 12.8 The maglev (magnetic levitation) train from Shanghai to the airport at Pudong reaches a speed of 430 km/h. Determine its speed (a) in mi/h; (b) ft/s.

Solution:

(a) $v = 430 \dfrac{\text{km}}{\text{h}} \left(\dfrac{0.6214 \text{ mi}}{1 \text{ km}} \right) = 267 \text{ mi/h}$ $\boxed{v = 267 \text{ mi/h}}$

(b) $v = 430 \dfrac{\text{km}}{\text{h}} \left(\dfrac{1000 \text{ m}}{1 \text{ km}} \right) \left(\dfrac{1 \text{ ft}}{0.3048 \text{ m}} \right) \left(\dfrac{1 \text{ h}}{3600 \text{ s}} \right) = 392 \text{ ft/s}$

   $\boxed{v = 392 \text{ ft/s}}$

Problem 12.9 In the 2006 Winter Olympics, the men's 15-km cross-country skiing race was won by Andrus Veerpalu of Estonia in a time of 38 minutes, 1.3 seconds. Determine his average speed (the distance traveled divided by the time required) to three significant digits (a) in km/h; (b) in mi/h.

Solution:

(a) $v = \dfrac{15 \text{ km}}{\left(38 + \frac{1.3}{60} \right) \text{min}} \left(\dfrac{60 \text{ min}}{1 \text{ h}} \right) = 23.7 \text{ km/h}$ $\boxed{v = 23.7 \text{ km/h}}$

(b) $v = (23.7 \text{ km/h}) \left(\dfrac{1 \text{ mi}}{1.609 \text{ km}} \right) = 14.7 \text{ mi/h}$ $\boxed{v = 14.7 \text{ mi/h}}$

Problem 12.10 The Porsche's engine exerts 229 ft-lb (foot-pounds) of torque at 4600 rpm. Determine the value of the torque in N-m (Newton-meters).

Solution:

$$T = 229 \text{ ft-lb} \left(\frac{1 \text{ N}}{0.2248 \text{ lb}} \right) \left(\frac{1 \text{ m}}{3.281 \text{ ft}} \right) = 310 \text{ N-m} \quad \boxed{T = 310 \text{ N-m}}$$

Problem 12.11 The *kinetic energy* of the man in Active Example 12.1 is defined by $\frac{1}{2}mv^2$, where m is his mass and v is his velocity. The man's mass is 68 kg and he is moving at 6 m/s, so his kinetic energy is $\frac{1}{2}(68 \text{ kg})$ $(6 \text{ m/s})^2 = 1224 \text{ kg-m}^2/\text{s}^2$. What is his kinetic energy in U.S. Customary units?

Solution:

$$T = 1224 \text{ kg-m}^2/\text{s}^2 \left(\frac{1 \text{ slug}}{14.59 \text{ kg}} \right) \left(\frac{1 \text{ ft}}{0.3048 \text{ m}} \right)^2 = 903 \text{ slug-ft}^2/\text{s}$$

$$\boxed{T = 903 \text{ slug-ft}^2/\text{s}}$$

Problem 12.12 The acceleration due to gravity at sea level in SI units is $g = 9.81 \text{ m/s}^2$. By converting units, use this value to determine the acceleration due to gravity at sea level in U.S. Customary units.

Solution: Use Table 1.2. The result is:

$$g = 9.81 \left(\frac{\text{m}}{\text{s}^2} \right) \left(\frac{1 \text{ ft}}{0.3048 \text{ m}} \right) = 32.185\ldots \left(\frac{\text{ft}}{\text{s}^2} \right) = 32.2 \left(\frac{\text{ft}}{\text{s}^2} \right)$$

Problem 12.13 A *furlong per fortnight* is a facetious unit of velocity, perhaps made up by a student as a satirical comment on the bewildering variety of units engineers must deal with. A furlong is 660 ft (1/8 mile). A fortnight is 2 weeks (14 nights). If you walk to class at 2 m/s, what is your speed in furlongs per fortnight to three significant digits?

Solution:

$$v = 2 \text{ m/s} \left(\frac{1 \text{ ft}}{0.3048 \text{ m}} \right) \left(\frac{1 \text{ furlong}}{660 \text{ ft}} \right) \left(\frac{3600 \text{ s}}{\text{hr}} \right) \left(\frac{24 \text{ hr}}{1 \text{ day}} \right) \left(\frac{14 \text{ day}}{1 \text{ fortnight}} \right)$$

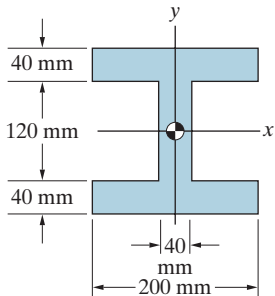$$\boxed{v = 12{,}000 \frac{\text{furlongs}}{\text{fortnight}}}$$

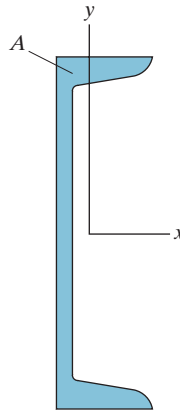**Problem 12.14** Determine the cross-sectional area of the beam (a) in m²; (b) in in².

Solution:

$$A = (200 \text{ mm})^2 - 2(80 \text{ mm})(120 \text{ mm}) = 20800 \text{ mm}^2$$

(a) $A = 20800 \text{ mm}^2 \left(\frac{1 \text{ m}}{1000 \text{ mm}} \right)^2 = 0.0208 \text{ m}^2 \quad \boxed{A = 0.0208 \text{ m}^2}$

(b) $A = 20800 \text{ mm}^2 \left(\frac{1 \text{ in}}{25.4 \text{ mm}} \right)^2 = 32.2 \text{ in}^2 \quad \boxed{A = 32.2 \text{ in}^2}$

Problem 12.15 The cross-sectional area of the C12×30 American Standard Channel steel beam is $A = 8.81$ in^2. What is its cross-sectional area in mm^2?

Solution:

$$A = 8.81 \text{ in}^2 \left(\frac{25.4 \text{ mm}}{1 \text{ in}} \right)^2 = 5680 \text{ mm}^2$$

Problem 12.16 A pressure transducer measures a value of 300 lb/in^2. Determine the value of the pressure in pascals. A pascal (Pa) is one newton per meter squared.

Solution: Convert the units using Table 12.2 and the definition of the Pascal unit. The result:

$$300 \left(\frac{\text{lb}}{\text{in}^2} \right) \left(\frac{4.448 \text{ N}}{1 \text{ lb}} \right) \left(\frac{12 \text{ in}}{1 \text{ ft}} \right)^2 \left(\frac{1 \text{ ft}}{0.3048 \text{ m}} \right)^2$$

$$= 2.0683 \ldots (10^6) \left(\frac{\text{N}}{\text{m}^2} \right) = 2.07(10^6) \text{ Pa}$$

Problem 12.17 A horsepower is 550 ft-lb/s. A watt is 1 N-m/s. Determine how many watts are generated by the engines of the passenger jet if they are producing 7000 horsepower.

Solution:

$$P = 7000 \text{ hp} \left(\frac{550 \text{ ft-lb/s}}{1 \text{ hp}} \right) \left(\frac{1 \text{ m}}{3.281 \text{ ft}} \right) \left(\frac{1 \text{ N}}{0.2248 \text{ lb}} \right) = 5.22 \times 10^6 \text{ W}$$

$$\boxed{P = 5.22 \times 10^6 \text{ W}}$$

Problem 12.18 Distributed loads on beams are expressed in units of force per unit length. If the value of a distributed load is 400 N/m, what is its value in lb/ft?.
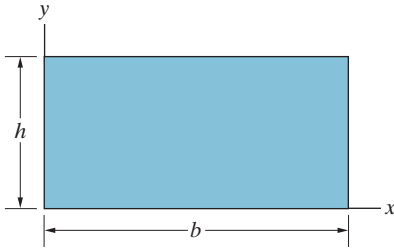
Solution:

$$w = 400 \text{ N/m} \left(\frac{0.2248 \text{ lb}}{1 \text{ N}} \right) \left(\frac{1 \text{ m}}{3.281 \text{ ft}} \right) = 27.4 \text{ lb/ft} \qquad \boxed{w = 27.4 \text{ lb/ft}}$$

Problem 12.19 The moment of inertia of the rectangular area about the x axis is given by the equation

$$I = \tfrac{1}{3}bh^3.$$

The dimensions of the area are $b = 200$ mm and $h = 100$ mm. Determine the value of I to four significant digits in terms of (a) mm^4; (b) m^4; (c) in^4.

Solution:

(a) $\quad I = \dfrac{1}{3}(200 \text{ mm})(100 \text{ mm})^3 = 66.7 \times 10^6 \text{ mm}^4$

(b) $\quad I = 66.7 \times 10^6 \text{ mm}^4 \left(\dfrac{1 \text{ m}}{1000 \text{ mm}}\right)^4 = 66.7 \times 10^{-6} \text{ m}^4$

(c) $\quad I = 66.7 \times 10^6 \text{ mm}^4 \left(\dfrac{1 \text{ in}}{25.4 \text{ mm}}\right)^4 = 160 \text{ in}^4$

Problem 12.20 In Example 12.3, instead of Einstein's equation consider the equation $L = mc$, where the mass m is in kilograms and the velocity of light c is in meters per second. (a) What are the SI units of L? (b) If the value of L in SI units is 12, what is the value in U.S. Customay base units?

Solution:

(a) $\quad L = mc \Rightarrow$ $\boxed{\text{Units}(L) = \text{kg-m/s}}$

(b) $\quad L = 12 \text{ kg-m/s} \left(\dfrac{0.0685 \text{ slug}}{1 \text{ kg}}\right)\left(\dfrac{3.281 \text{ ft}}{1 \text{ m}}\right) = 2.70 \text{ slug-ft/s}$

$\boxed{L = 2.70 \text{ slug-ft/s}}$

Problem 12.21 The equation

$$\sigma = \frac{My}{I}$$

is used in the mechanics of materials to determine normal stresses in beams.

(a) When this equation is expressed in terms of SI base units, M is in newton-meters (N-m), y is in meters (m), and I is in meters to the fourth power (m^4). What are the SI units of σ?

(b) If $M = 2000$ N-m, $y = 0.1$ m, and $I = 7 \times 10^{-5}$ m^4, what is the value of σ in U.S. Customary base units?

Solution:

(a) $\quad \sigma = \dfrac{My}{I} = \dfrac{(\text{N-m})\text{m}}{\text{m}^4} = \dfrac{\text{N}}{\text{m}^2}$

(b) $\quad \sigma = \dfrac{My}{I} = \dfrac{(2000 \text{ N-m})(0.1 \text{ m})}{7 \times 10^{-5} \text{ m}^4} \left(\dfrac{1 \text{ lb}}{4.448 \text{ N}}\right)\left(\dfrac{0.3048 \text{ m}}{\text{ft}}\right)^2$

$= 59{,}700 \dfrac{\text{lb}}{\text{ft}^2}$

Problem 12.22 The acceleration due to gravity on the surface of the moon is 1.62 m/s^2. (a) What would be the mass of the C-clamp in Active Example 12.4 be on the surface of the moon? (b) What would the weight of the C-clamp in newtons be on the surface of the moon?
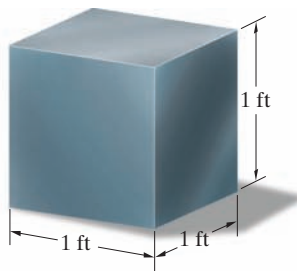
Solution:

(a) The mass does not depend on location. The mass in kg is

$$0.0272 \text{ slug} \left(\dfrac{14.59 \text{ kg}}{1 \text{ slug}}\right) = 0.397 \text{ kg} \quad \boxed{\text{mass} = 0.397 \text{ kg}}$$

(b) The weight on the surface of the moon is

$$W = mg = (0.397 \text{ kg})(1.62 \text{ m/s}^2) = 0.643 \text{ N} \quad \boxed{W = 0.643 \text{ N}}$$

Problem 12.23 The 1 ft × 1 ft × 1 ft cube of iron weighs 490 lb at sea level. Determine the weight in newtons of a 1 m × 1 m × 1 m cube of the same material at sea level.

Solution: The weight density is $\gamma = \dfrac{490 \text{ lb}}{1 \text{ ft}^3}$

The weight of the 1 m³ cube is:

$$W = \gamma V = \left(\frac{490 \text{ lb}}{1 \text{ ft}^3}\right)(1 \text{ m})^3 \left(\frac{1 \text{ ft}}{0.3048 \text{ m}}\right)^3 \left(\frac{1 \text{ N}}{0.2248 \text{ lb}}\right) = 77.0 \text{ kN}$$

Problem 12.24 The area of the Pacific Ocean is 64,186,000 square miles and its average depth is 12,925 ft. Assume that the weight per unit volume of ocean water is 64 lb/ft³. Determine the mass of the Pacific Ocean (a) in slugs; (b) in kilograms.

Solution: The volume of the ocean is

$$V = (64,186,000 \text{ mi}^2)(12,925 \text{ ft}) \left(\frac{5,280 \text{ ft}}{1 \text{ mi}}\right)^2 = 2.312 \times 10^{19} \text{ ft}^3$$

(a) $\quad m = \rho V = \left(\dfrac{64 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right)(2.312 \times 10^{19}\, ft^3) = 4.60 \times 10^{19} \text{ slugs}$

(b) $\quad m = (4.60 \times 10^{19} \text{ slugs}) \left(\dfrac{14.59 \text{ kg}}{1 \text{ slug}}\right) = 6.71 \times 10^{20} \text{ kg}$

Problem 12.25 The acceleration due to gravity at sea level is $g = 9.81 \text{ m/s}^2$. The radius of the earth is 6370 km. The universal gravitational constant is $G = 6.67 \times 10^{-11} \text{ N-m}^2/\text{kg}^2$. Use this information to determine the mass of the earth.

Solution: Use Eq. (12.3) $a = \dfrac{Gm_E}{R^2}$. Solve for the mass,

$$m_E = \frac{gR^2}{G} = \frac{(9.81 \text{ m/s}^2)(6370 \text{ km})^2 \left(10^3 \dfrac{\text{m}}{\text{km}}\right)^2}{6.67(10^{-11}) \left(\dfrac{\text{N-m}^2}{\text{kg}^2}\right)}$$

$$= 5.9679\ldots (10^{24}) \text{ kg} = 5.97(10^{24}) \text{ kg}$$

Problem 12.26 A person weighs 800 N sea level. The radius of the earth is 6372 km. What force is exerted on the person by the gravitational attraction of the earth if he is in a space station in orbit 322 km above the surface of the earth?

Solution: Use Eq. (12.5).

$$W = mg \left(\frac{R_E}{r}\right)^2 = \left(\frac{W_E}{g}\right) g \left(\frac{R_E}{R_E + H}\right)^2 = W_E \left(\frac{6372}{6372 + 322}\right)^2$$

$$= (800)(0.9519) = 262 \text{ N}$$

Problem 12.27 The acceleration due to gravity on the surface of the moon is 1.62 m/s². The moon's radius is $R_M = 1738$ km.

(a) What is the weight in newtons on the surface of the moon of an object that has a mass of 10 kg?
(b) Using the approach described in Example 12.5, determine the force exerted on the object by the gravity of the moon if the object is located 1738 km above the moon's surface.

Solution:

(a) $\quad W = mg_M = (10 \text{ kg})(1.26 \text{ m/s}^2) = 12.6 \text{ N} \quad \boxed{W = 12.6 \text{ N}}$

(b) Adapting equation 1.4 we have $a_M = g_M \left(\dfrac{R_M}{r}\right)^2$. The force is then

$$F = ma_M = (10 \text{ kg})(1.62\text{m/s}^2) \left(\frac{1738 \text{ km}}{1738 \text{ km} + 1738 \text{ km}}\right)^2 = 4.05 \text{ N}$$

$$\boxed{F = 4.05 \text{ N}}$$

Problem 12.28 If an object is near the surface of the earth, the variation of its weight with distance from the center of the earth can often be neglected. The acceleration due to gravity at sea level is $g = 9.81$ m/s^2. The radius of the earth is 6370 km. The weight of an object at sea level is mg, where m is its mass. At what height above the earth does the weight of the object decrease to 0.99 mg?

Solution: Use a variation of Eq. (12.5).

$$W = mg \left(\frac{R_E}{R_E + h} \right)^2 = 0.99 \text{ mg}$$

Solve for the radial height,

$$h = R_E \left(\frac{1}{\sqrt{0.99}} - 1 \right) = (6370)(1.0050378 - 1.0)$$

$$= 32.09 \ldots \text{km} = 32,100 \text{ m} = 32.1 \text{ km}$$

Problem 12.29 The planet Neptune has an equatorial diameter of 49,532 km and its mass is 1.0247×10^{26} kg. If the planet is modeled as a homogeneous sphere, what is the acceleration due to gravity at its surface? (The universal gravitational constant is $G = 6.67 \times 10^{-11}$ h-m^2/kg^2.)

Solution:
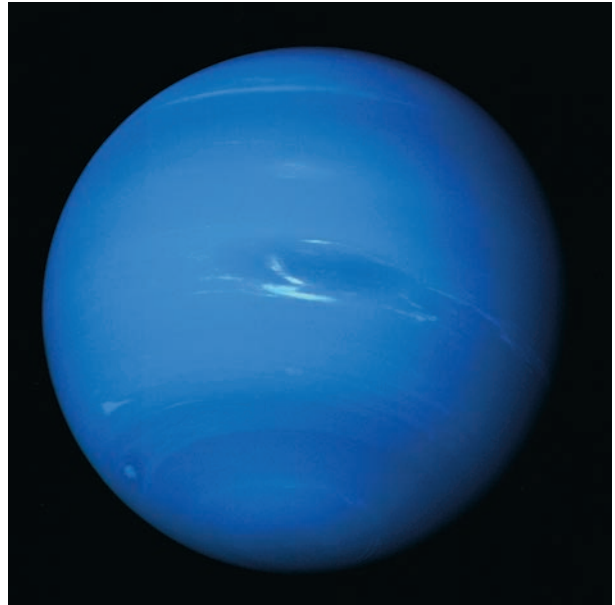We have: $W = G \dfrac{m_N m}{r_N^2} = \left(G \dfrac{m_N}{r^2} \right) m \Rightarrow g_N = G \dfrac{m_N}{r_N^2}$

Note that the radius of Neptune is $r_N = \frac{1}{2}(49,532$ km$)$ $= 24,766$ km

Thus $g_N = \left(6.67 \times 10^{-11} \dfrac{\text{N-m}^2}{\text{kg}^2} \right) \left(\dfrac{1.0247 \times 10^{26} \text{ kg}}{(24766 \text{ km})^2} \right)$

$\times \left(\dfrac{1 \text{ km}}{1000 \text{ m}} \right)^2 = 11.1$ m/s^2

$$\boxed{g_N = 11.1 \text{ m/s}^2}$$

Problem 12.30 At a point between the earth and the moon, the magnitude of the force exerted on an object by the earth's gravity equals the magnitude of the force exerted on the object by the moon's gravity. What is the distance from the center of the earth to that point to three significant digits? The distance from the center of the earth to the center of the moon is 383,000 km, and the radius of the earth is 6370 km. The radius of the moon is 1738 km, and the acceleration due to gravity at its surface is 1.62 m/s^2.

Solution: Let r_{Ep} be the distance from the Earth to the point where the gravitational accelerations are the same and let r_{Mp} be the distance from the Moon to that point. Then, $r_{Ep} + r_{Mp} = r_{EM} = 383,000$ km. The fact that the gravitational attractions by the Earth and the Moon at this point are equal leads to the equation

$$g_E \left(\frac{R_E}{r_{Ep}} \right)^2 = g_M \left(\frac{R_M}{r_{Mp}} \right)^2,$$

where $r_{EM} = 383,000$ km. Substituting the correct numerical values leads to the equation

$$9.81 \left(\frac{\text{m}}{\text{s}^2} \right) \left(\frac{6370 \text{ km}}{r_{Ep}} \right)^2 = 1.62 \left(\frac{\text{m}}{\text{s}^2} \right) \left(\frac{1738 \text{ km}}{r_{EM} - r_{Ep}} \right)^2,$$

where r_{Ep} is the only unknown. Solving, we get $r_{Ep} = 344,770$ km $= 345,000$ km.