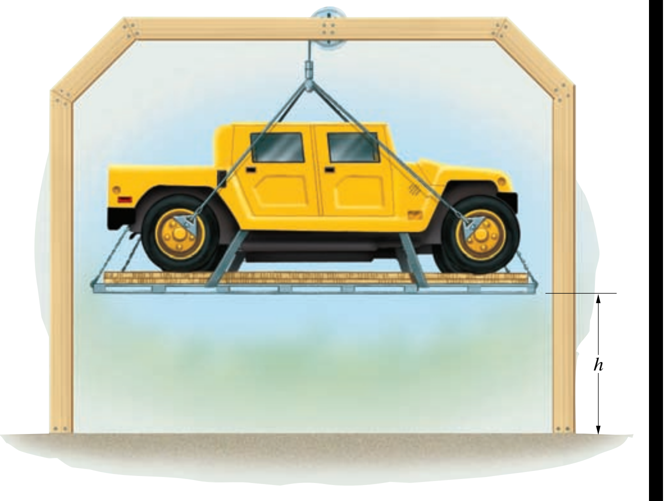
Problem 13.1 In Example 13.2, suppose that the vehicle is dropped from a height $h = 6\text{m}$. (a) What is the downward velocity 1 s after it is released? (b) What is its downward velocity just before it reaches the ground?



Solution: The equations that govern the motion are:

$a = -g = -9.81 \text{ m/s}^2$

$v = -gt$

$s = -\frac{1}{2}gt^2 + h$

(a) $v = -gt = -(9.81 \text{ m/s}^2)(1 \text{ s}) = -9.81 \text{ m/s}.$

The downward velocity is $\boxed{9.81 \text{ m/s.}}$

(b) We need to first determine the time at which the vehicle hits the ground

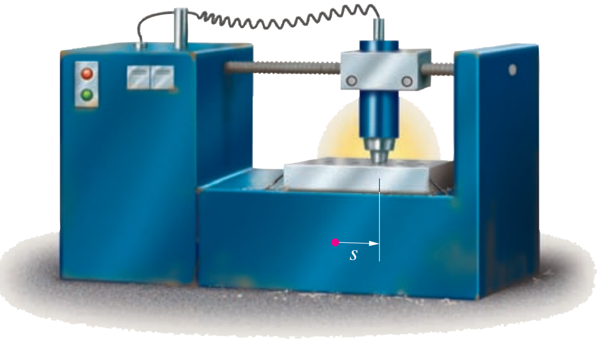
$$s = 0 = -\frac{1}{2}gt^2 + h \Rightarrow t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(6 \text{ m})}{9.81 \text{ m/s}^2}} = 1.106 \text{ s}$$

Now we can solve for the velocity

$$v = -gt = -(9.81 \text{ m/s}^2)(1.106 \text{ s}) = -10.8 \text{ m/s}.$$

The downward velocity is $\boxed{10.8 \text{ m/s.}}$

Problem 13.2 The milling machine is programmed so that during the interval of time from $t = 0$ to $t = 2$ s, the position of its head (in inches) is given as a function of time by $s = 4t - 2t^3$. What are the velocity (in in/s) and acceleration (in in/s^2) of the head at $t = 1$ s?



Solution: The motion is governed by the equations

$s = (4 \text{ in/s})t - (2 \text{ in/s}^2)t^2,$

$v = (4 \text{ in/s}) - 2(2 \text{ in/s}^2)t,$

$a = -2(2 \text{ in/s}^2).$

At $t = 1$ s, we have $\boxed{v = 0, a = -4 \text{ in/s}^2.}$

Problem 13.3 In an experiment to estimate the acceleration due to gravity, a student drops a ball at a distance of 1 m above the floor. His lab partner measures the time it takes to fall and obtains an estimate of 0.46 s.

(a) What do they estimate the acceleration due to gravity to be?

(b) Let s be the ball's position relative to the floor. Using the value of the acceleration due to gravity that they obtained, and assuming that the ball is released at $t = 0$, determine s (in m) as a function of time.

Solution: The governing equations are

$$a = -g$$

$$v = -gt$$

$$s = -\tfrac{1}{2}gt^2 + h$$

(a) When the ball hits the floor we have

$$0 = -\tfrac{1}{2}gt^2 + h \Rightarrow g = \frac{2h}{t^2} = \frac{2(1\text{ m})}{(0.46\text{ s})^2} = 9.45\text{ m/s}^2$$

$$\boxed{g = 9.45\text{ m/s}^2}$$

(b) The distance s is then given by

$$s = -\tfrac{1}{2}(9.45\text{ m/s}^2) + 1\text{ m.} \quad \boxed{s = -(4.73\text{ m/s}^2)t^2 + 1.0\text{ m.}}$$

Problem 13.4 The boat's position during the interval of time from $t = 2$ s to $t = 10$ s is given by $s = 4t + 1.6t^2 - 0.08t^3$ m.

(a) Determine the boat's velocity and acceleration at $t = 4$ s.

(b) What is the boat's maximum velocity during this interval of time, and when does it occur?

Solution:

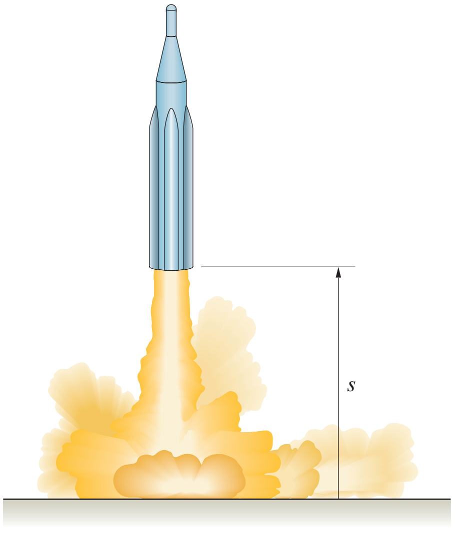
$$s = 4t + 1.6t^2 - 0.08t^3$$

$$v = \frac{ds}{dt} = 4 + 3.2t - 0.24t^2 \Rightarrow$$

$$a = \frac{dv}{dt} = 3.2 - 0.48t$$

$$\boxed{\begin{array}{l} a)\ v(4s) = 12.96\text{ m/s}^2 \\ \quad a(4s) = 1.28\text{ m/s}^2 \\ b)\ a = 3.2 - 0.48t = 0 \Rightarrow t = 6.67s \\ \quad v(6.67s) = 14.67\text{ m/s} \end{array}}$$

Problem 13.5 The rocket starts from rest at $t = 0$ and travels straight up. Its height above the ground as a function of time can be approximated by $s = bt^2 + ct^3$, where b and c are constants. At $t = 10$ s, the rocket's velocity and acceleration are $v = 229$ m/s and $a = 28.2$ m/s^2. Determine the time at which the rocket reaches supersonic speed (325 m/s). What is its altitude when that occurs?



Solution: The governing equations are

$$s = bt^2 + ct^3,$$

$$v = 2bt + 3ct^2,$$

$$a = 2b + 6ct.$$

Using the information that we have allows us to solve for the constants b and c.

$$(229 \text{ m/s}) = 2b(10 \text{ s}) + 3c(10 \text{ s})^2,$$

$$(28.2 \text{ m/s}^2) = 2b + 6c(10 \text{ s}).$$

Solving these two equations, we find $b = 8.80$ m/s^2, $c = 0.177$ m/s^3.

When the rocket hits supersonic speed we have

$$(325 \text{ m/s}) = 2(8.80 \text{ m/s}^2)t + 3(0.177 \text{ m/s}^3)t^2 \Rightarrow \boxed{t = 13.2 \text{ s.}}$$

The altitude at this time is

$$s = (8.80 \text{ m/s}^2)(13.2 \text{ s})^2 + (0.177 \text{ m/s}^3)(13.2 \text{ s})^3 \quad \boxed{s = 1940 \text{ m.}}$$

Problem 13.6 The position of a point during the interval of time from $t = 0$ to $t = 6$ s is given by $s = -\frac{1}{2}t^3 + 6t^2 + 4t$ m.

(a) What is the maximum velocity during this interval of time, and at what time does it occur?
(b) What is the acceleration when the velocity is a maximum?

Solution:

$$s = -\tfrac{1}{2}t^3 + 6t^2 + 4t \text{ m}$$

$$v = -\tfrac{3}{2}t^2 + 12t + 4 \text{ m/s}$$

$$a = -3t + 12 \text{ m/s}^2$$

Maximum velocity occurs where $a = \dfrac{dv}{dt} = 0$ (it could be a minimum)

This occurs at $t = 4$ s. At this point $\dfrac{da}{dt} = -3$ so we have a maximum.

(a) Max velocity is at $t = 4$ s. where $\underline{v = 28 \text{ m/s}}$ and
(b) $\underline{a = 0 \text{ m/s}^2}$

Problem 13.7 The position of a point during the interval of time from $t = 0$ to $t = 3$ seconds is $s = 12 + 5t^2 - t^3$ m.

(a) What is the maximum velocity during this interval of time, and at what time does it occur?
(b) What is the acceleration when the velocity is a maximum?

Solution:

(a) The velocity is $\dfrac{ds}{dt} = 10t - 3t^2$. The maximum occurs when $\dfrac{dv}{dt} = 10 - 6t = 0$, from which

$\boxed{t = \dfrac{10}{6} = 1.667}$ seconds.

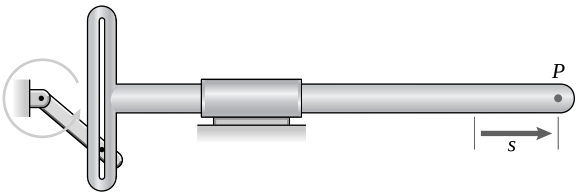
This is indeed a maximum, since $\dfrac{d^2v}{dt^2} = -6 < 0$. The maximum velocity is

$$\boxed{v = \left[10t - 3t^2\right]_{t=1.667} = 8.33 \text{ m/s}}$$

(b) The acceleration is $\boxed{\dfrac{dv}{dt} = 0}$ when the velocity is a maximum.

Problem 13.8 The rotating crank causes the position of point P as a function of time to be $s = 0.4 \sin(2\pi t)$ m.

(a) Determine the velocity and acceleration of P at $t = 0.375$ s.
(b) What is the maximum magnitude of the velocity of P?
(c) When the magnitude of the velocity of P is a maximum, what is the acceleration of P?



Solution:

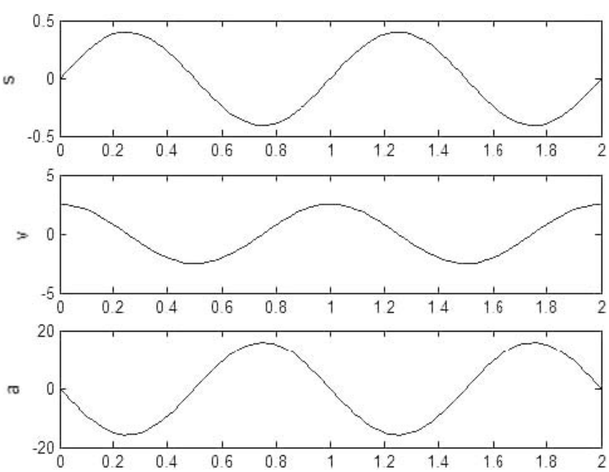
$$s = 0.4 \sin(2\pi t)$$

$$v = \frac{ds}{dt} = 0.8\pi \cos(2\pi t) \quad \Rightarrow \quad \boxed{\begin{array}{l} a)\ v(0.375s) = -1.777 \text{ m/s} \\ \quad a(0.375) = -11.2 \text{ m/s}^2 \\ b)\ v_{\max} = 0.8\pi = 2.513 \text{ m/s}^2 \\ c)\ v_{\max} \Rightarrow t = 0, n\pi \Rightarrow a = 0 \end{array}}$$

$$a = \frac{dv}{dt} = -1.6\pi^2 \sin(2\pi t)$$

Problem 13.9 For the mechanism in Problem 13.8, draw graphs of the position s, velocity v, and acceleration a of point P as functions of time for $0 \le t \le 2$ s. Using your graphs, confirm that the slope of the graph of s is zero at times for which v is zero, and the slope of the graph of v is zero at times for which a is zero.

Solution:



Problem 13.10 A seismograph measures the horizontal motion of the ground during an earthquake. An engineer analyzing the data determines that for a 10-s interval of time beginning at $t = 0$, the position is approximated by $s = 100\cos(2\pi t)$ mm. What are (a) the maximum velocity and (b) maximum acceleration of the ground during the 10-s interval?

Solution:

(a) The velocity is

$$\frac{ds}{dt} = -(2\pi)100\sin(2\pi t)\ \text{mm/s} = -0.2\pi\ \sin(2\pi t)\ \text{m/s}.$$

The velocity maxima occur at

$$\frac{dv}{dt} = -0.4\pi^2\cos(2\pi t) = 0,$$

from which

$$2\pi t = \frac{(2n-1)\pi}{2},\ \text{ or } t = \frac{(2n-1)}{4},$$

$$n = 1, 2, 3, \ldots M, \quad \text{where} \quad \frac{(2M-1)}{4} \leq 10 \text{ seconds.}$$

These velocity maxima have the absolute value

$$\left|\frac{ds}{dt}\right|_{t=\frac{(2n-1)}{4}} = [0.2\pi] = \underline{0.628\ \text{m/s}}.$$

(b) The acceleration is

$$\frac{d^2s}{dt^2} = -0.4\pi^2\cos(2\pi t).$$

The acceleration maxima occur at

$$\frac{d^3s}{dt^3} = \frac{d^2v}{dt^2} = 0.8\pi^3\sin(2\pi t) = 0,$$

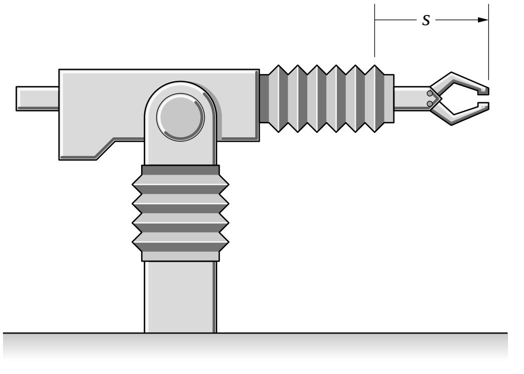
from which $2\pi t = n\pi$, or $t = \frac{n}{2}$, $n = 0, 1, 2, \ldots K$, where

$$\frac{K}{2} \leq 10 \text{ seconds.}$$

These acceleration maxima have the absolute value

$$\left|\frac{dv}{dt}\right|_{t=\frac{n\pi}{2}} = 0.4\pi^2 = \underline{3.95\ \text{m/s}^2}.$$

Problem 13.11 In an assembly operation, the robot's arm moves along a straight horizontal line. During an interval of time from $t = 0$ to $t = 1$ s, the position of the arm is given by $s = 30t^2 - 20t^3$ mm. (a) Determine the maximum velocity during this interval of time. (b) What are the position and acceleration when the velocity is a maximum?



Solution:

$$s = 30t^2 - 20t^3 \text{ mm}$$

$$v = 60t - 60t^2 \text{ mm/s}$$

$$a = 60 - 120t \text{ mm/s}^2$$

$$\frac{da}{dt} = -120 \text{ mm/s}^3$$

(a) Maximum velocity occurs when $\frac{dv}{dt} = a = 0$. This occurs at $0 = 60 - 120t$ or $t = 1/2$ second. (since $da/dt < 0$, we have a maximum). The velocity at this time is

$$v = (60)\left(\frac{1}{2}\right) - 60\left(\frac{1}{4}\right) \text{ mm/s}$$

$$v = 15 \text{ mm/s}$$

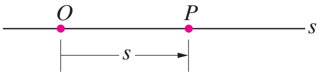
(b) The position and acceleration at this time are

$$s = 7.5 - 2.5 \text{ mm}$$

$$\underline{s = 5 \text{ mm}}$$

$$\underline{a = 0 \text{ mm/s}^2}$$

Problem 13.12 In Active Example 13.1, the acceleration (in $\mathrm{m/s^2}$) of point P relative to point O is given as a function of time by $a = 3t^2$. Suppose that at $t = 0$ the position and velocity of P are $s = 5$ m and $v = 2$ m/s. Determine the position and velocity of P at $t = 4$ s.



Solution: The governing equations are

$$a = (3\ \mathrm{m/s^4})t^2$$

$$v = \frac{1}{3}(3\ \mathrm{m/s^4})t^3 + (2\ \mathrm{m/s})$$

$$s = \frac{1}{12}(3\ \mathrm{m/s^4})t^4 + (2\ \mathrm{m/s})t + (5\ \mathrm{m})$$

At $t = 4$ s, we have $\boxed{s = 77\ \mathrm{m},\ v = 66\ \mathrm{m/s}.}$

Problem 13.13 The Porsche starts from rest at time $t = 0$. During the first 10 seconds of its motion, its velocity in km/h is given as a function of time by $v = 22.8t - 0.88t^2$, where t is in seconds. (a) What is the car's maximum acceleration in $\mathrm{m/s^2}$, and when does it occur? (b) What distance in km does the car travel during the 10 seconds?

Solution: First convert the numbers into meters and seconds

$$22.8\frac{\mathrm{km}}{\mathrm{hr}}\left(\frac{1000\ \mathrm{m}}{1\ \mathrm{km}}\right)\left(\frac{1\mathrm{hr}}{3600\ \mathrm{s}}\right) = 6.33\ \mathrm{m/s}$$

$$0.88\frac{\mathrm{km}}{\mathrm{hr}}\left(\frac{1000\ \mathrm{m}}{1\ \mathrm{km}}\right)\left(\frac{1\ \mathrm{hr}}{3600\ \mathrm{s}}\right) = 0.244\ \mathrm{m/s}$$

The governing equations are then

$$s = \frac{1}{2}(6.33\ \mathrm{m/s})(t^2/\mathrm{s}) - \frac{1}{3}(0.88\ \mathrm{m/s})(t^3/\mathrm{s}^2),$$

$$v = (6.33\ \mathrm{m/s})(t/\mathrm{s}) - (0.88\ \mathrm{m/s})(t^2/\mathrm{s}^2),$$

$$a = (6.33\ \mathrm{m/s^2}) - 2(0.88\ \mathrm{m/s})(t/\mathrm{s}^2),$$

$$\frac{d}{dt}a = -2(0.88\ \mathrm{m/s})(1/\mathrm{s}^2) = -1.76\ \mathrm{m/s^3}.$$

The maximum acceleration occurs at $t = 0$ (and decreases linearly from its initial value).

$$\boxed{a_{\max} = 6.33\ \mathrm{m/s^2}\ @\ t = 0}$$

In the first 10 seconds the car travels a distance

$$s = \left[\frac{1}{2}\left(22.8\frac{\mathrm{km}}{\mathrm{hr}}\right)\frac{(10\ \mathrm{s})^2}{s} - \frac{1}{3}\left(0.88\frac{\mathrm{km}}{\mathrm{hr}}\right)\frac{(10\ \mathrm{s})^3}{\mathrm{s}^2}\right]\left(\frac{1\ \mathrm{hr}}{3600\ \mathrm{s}}\right)$$

$$\boxed{s = 0.235\ \mathrm{km}.}$$

Problem 13.14 The acceleration of a point is $a = 20t$ m/s^2. When $t = 0$, $s = 40$ m and $v = -10$ m/s. What are the position and velocity at $t = 3$ s?

Solution: The velocity is

$$v = \int a\,dt + C_1,$$

where C_1 is the constant of integration. Thus

$$v = \int 20t\,dt + C_1 = 10t^2 + C_1.$$

At $t = 0$, $v = -10$ m/s, hence $C_1 = -10$ and the velocity is $v = 10t^2 - 10$ m/s. The position is

$$s = \int v\,dt + C_2,$$

where C_2 is the constant of integration.

$$s = \int (10t^2 - 10)\,dt + C_2 = \left(\frac{10}{3}\right)t^3 - 10t + C_2.$$

At $t = 0$, $s = 40$ m, thus $C_2 = 40$. The position is

$$s = \left(\frac{10}{3}\right)t^3 - 10t + 40 \text{ m}.$$

At $t = 3$ seconds,

$$\boxed{s = \left[\frac{10}{3}t^3 - 10t + 40\right]_{t=3} = 100 \text{ m.}}$$

The velocity at $t = 3$ seconds is

$$\boxed{v = \left[10t^2 - 10\right]_{t=3} = 80 \text{ m/s}}.$$

Problem 13.15 The acceleration of a point is $a = 60t - 36t^2$ m/s^2. When $t = 0$, $s = 0$ and $v = 20$ m/s. What are position and velocity as a function of time?

Solution: The velocity is

$$v = \int a\,dt + C_1 = \int (60t - 36t^2) + C_1 = 30t^2 - 12t^3 + C_1.$$

At $t = 0$, $v = 20$ m/s, hence $C_1 = 20$, and the velocity as a function of time is

$$\boxed{v = 30t^2 - 12t^3 + 20 \text{ m/s}}.$$

The position is

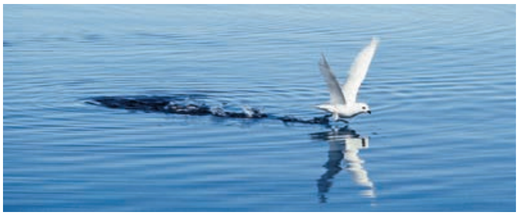
$$s = \int v\,dt + C_2 = \int (30t^2 - 12t^3 + 20) + C_2$$

$$= 10t^3 - 3t^4 + 20t + C_2.$$

At $t = 0$, $s = 0$, hence $C_2 = 0$, and the position is

$$\boxed{s = 10t^3 - 3t^4 + 20t \text{ m}}$$

Problem 13.16 As a first approximation, a bioengineer studying the mechanics of bird flight assumes that the snow petrel takes off with constant acceleration. Video measurements indicate that a bird requires a distance of 4.3 m to take off and is moving at 6.1 m/s when it does. What is its acceleration?



Solution: The governing equations are

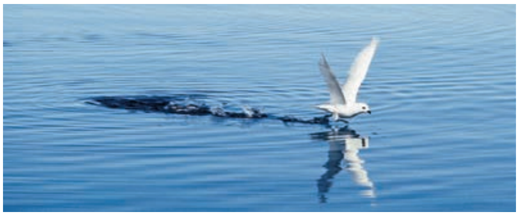
$$a = \text{constant}, \quad v = at, \quad s = \frac{1}{2}at^2.$$

Using the information given, we have

$$6.1 \text{ m/s} = at, \quad 4.3 \text{ m} = \frac{1}{2}at^2.$$

Solving these two equations, we find $t = 1.41$ s and $\boxed{a = 4.33 \text{ m/s}^2.}$

Problem 13.17 Progressively developing a more realistic model, the bioengineer next models the acceleration of the snow petrel by an equation of the form $a = C(1 + \sin \omega t)$, where C and ω are constants. From video measurements of a bird taking off, he estimates that $\omega = 18$/s and determines that the bird requires 1.42 s to take off and is moving at 6.1 m/s when it does. What is the constant C?



Solution: We find an expression for the velocity by integrating the acceleration

$$a = C(1 + \sin \omega t),$$

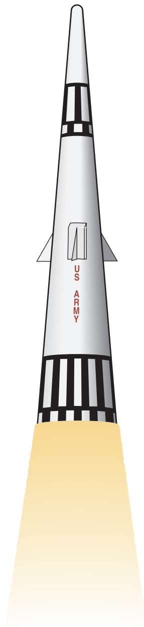
$$v = Ct + \frac{C}{\omega}(1 - \cos \omega t) = C\left(t + \frac{1}{\omega} - \frac{1}{\omega}\cos \omega t\right).$$

Using the information given, we have

$$6.1 \text{ m/s} = C\left(1.42 \text{ s} + \frac{\text{s}}{18} - \frac{\text{s}}{18}\cos[18(1.42)]\right)$$

Solving this equation, we find $\boxed{C = 4.28 \text{ m/s}^2.}$

Problem 13.18 Missiles designed for defense against ballistic missiles have attained accelerations in excess of 100 g's, or 100 times the acceleration due to gravity. Suppose that the missile shown lifts off from the ground and has a constant acceleration of 100 g's. How long does it take to reach an altitude of 3000 m? How fast is it going when it ranches that altitude?



Solution: The governing equations are

$$a = \text{constant}, \quad v = at, \quad s = \frac{1}{2}at^2$$

Using the given information we have

$$3000 \text{ m} = \frac{1}{2}100(9.81 \text{ m/s}^2)t^2 \Rightarrow \quad \boxed{t = 2.47 \text{ s}}$$

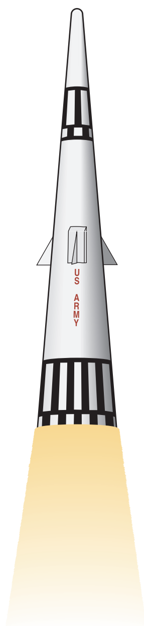
The velocity at that time is

$$v = 100(9.81 \text{ m/s}^2)(2.47 \text{ s}) = 243 \text{ m/s}. \quad \boxed{v = 2430 \text{ m/s.}}$$

Problem 13.19 Suppose that the missile shown lifts off from the ground and, because it becomes lighter as its fuel is expended, its acceleration (in g's) is given as a function of time in seconds by

$$a = \frac{100}{1 - 0.2t}.$$

What is the missile's velocity in kilometres per hour 1 s after liftoff?



Solution: We find an expression for the velocity by integrating the acceleration (valid only for $0 < t < 5$s).

$$v = \int_0^t a\,dt = \int_0^t \frac{100g}{1 - 0.2(t/s)}\,dt = \frac{100gs}{0.2} \ln\left(\frac{1}{1 - 0.2[t/s]}\right)$$

At time $t = 1$ s, we have

$$v = \frac{100(9.81 \text{ m/s}^2)(1 \text{ s})}{0.2} \ln\left(\frac{1}{1 - 0.2}\right) = 1095 \text{ m/s}.$$

$v = 3940$ km/h.

Problem 13.20 The airplane releases its drag parachute at time $t = 0$. Its velocity is given as a function of time by

$$v = \frac{80}{1 + 0.32t} \text{ m/s.}$$

What is the airplane's acceleration at $t = 3$ s?

Solution:

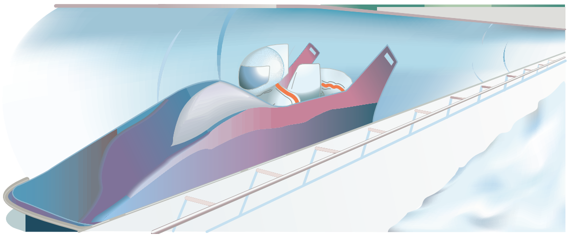
$$v = \frac{80}{1 + 0.32t}; \quad a = \frac{dv}{dt} = \frac{-25.6}{(1 + 0.32t)^2} \Rightarrow \boxed{a(3 \text{ s}) = -6.66 \text{ m/s}^2}$$

Problem 13.21 How far does the airplane in Problem 13.20 travel during the interval of time from $t = 0$ to $t = 10$ s?

Solution:

$$\boxed{v = \frac{80}{1 + 0.32t}; \quad s = \int_0^{10\,\text{s}} \frac{80}{1 + 0.32t}dt = 250 \ln\left(\frac{1 + 3.2}{1}\right) = 359 \text{ m}}$$

Problem 13.22 The velocity of a bobsled is $v = 10t$ m/s. When $t = 2$ s, the position is $s = 25$ m. What is its position at $t = 10$ s?



Solution: The equation for straight line displacement under constant acceleration is

$$s = \frac{a(t - t_0)^2}{2} + v(t_0)(t - t_0) + s(t_0).$$

Choose $t_0 = 0$. At $t = 2$, the acceleration is

$$a = \left[\frac{dv(t)}{dt}\right]_{t=2} = 10 \text{ m/s}^2,$$

the velocity is $v(t_0) = 10(2) = 20$ m/s, and the initial displacement is $s(t_0) = 25$ m. At $t = 10$ seconds, the displacement is

$$\boxed{s = \frac{10}{2}(10 - 2)^2 + 20(10 - 2) + 25 = 505 \text{ m}}$$

Problem 13.23 In September, 2003, Tony Schumacher started from rest and drove 402 km in 4.498 seconds in a National Hot Rod Association race. His speed as he crossed the finish line was 528 km/h. Assume that the car's acceleration can be expressed by a linear function of time $a = b + ct$.

(a) Determine the constants b and c.
(b) What was the car's speed 2 s after the start of the race?

Solution:

$$a = b + ct, \quad v = bt + \frac{ct^2}{2}, \quad s = \frac{bt^2}{2} + \frac{ct^3}{6}$$

Both constants of integration are zero.

(a) $$528 \text{ km/h} = b(4.498 \text{ s}) + \frac{c}{2}(4.498 \text{ s})^2$$

$$402 \text{ km} = \frac{b}{2}(4.498 \text{ s})^2 + \frac{c}{6}(4.498 \text{ s})^3$$

$$\Rightarrow \boxed{\begin{aligned} b &= 54 \text{ m/s}^2 \\ c &= -9.5 \text{ m/s}^3 \end{aligned}}$$

(b) $$\boxed{v = b(2 \text{ s}) + \frac{c}{2}(2 \text{ s})^2 = 89 \text{ m/s}}$$

Problem 13.24 The velocity of an object is $v = 200 - 2t^2$ m/s. When $t = 3$ seconds, its position is $s = 600$ m. What are the position and acceleration of the object at $t = 6$ s?

Solution: The acceleration is

$$\frac{dv(t)}{dt} = -4t \text{ m/s}^2.$$

At $t = 6$ seconds, the acceleration is $a = -24$ m/s^2. Choose the initial conditions at $t_0 = 3$ seconds. The position is obtained from the velocity:

$$\boxed{s(t - t_0) = \int_3^6 v(t)\,dt + s(t_0) = \left[200t - \frac{2}{3}t^3\right]_3^6 + 600 = 1070 \text{ m}}.$$

Problem 13.25 An inertial navigation system measures the acceleration of a vehicle from $t = 0$ to $t = 6$ s and determines it to be $a = 2 + 0.1t$ m/s^2. At $t = 0$, the vehicle's position and velocity are $s = 240$ m, $v = 42$ m/s, respectively. What are the vehicle's position and velocity at $t = 6$ s?

Solution:

$$a = 2 + 0.1t \text{ m/s}^2$$

$$v_0 = 42 \text{ m/s} \quad s_0 = 240 \text{ m}$$

Integrating

$$v = v_0 + 2t + 0.1t^2/2$$

$$s = v_0 t + t^2 + 0.1t^3/6 + s_0$$

Substituting the known values at $t = 6$ s, we get

$\underline{v = 55.8 \text{ m/s}}$

$\underline{s = 531.6 \text{ m}}$

Problem 13.26 In Example 13.3, suppose that the cheetah's acceleration is constant and it reaches its top speed of 120 km/h in 5 s. What distance can it cover in 10 s?

Solution: The governing equations while accelerating are

$$a = \text{constant}, \quad v = at, \quad s = \frac{1}{2}at^2.$$

Using the information supplied, we have

$$\left(\frac{120 \times 1000 \text{ m}}{3600 \text{ s}}\right) = a(5 \text{ s}) \Rightarrow a = 6.67 \text{ m/s}^2$$

The distance that he travels in the first 10 s (5 seconds accelerating and then the last 5 seconds traveling at top speed) is

$$s = \frac{1}{2}(6.67 \text{ m/s}^2)(5 \text{ s})^2 + \left(\frac{120 \times 1000 \text{ m}}{3600 \text{ s}}\right)(10 \text{ s} - 5 \text{ s}) = 250 \text{ m}.$$

$$\boxed{s = 250 \text{ m}.}$$

Problem 13.27 The graph shows the airplane's acceleration during its takeoff. What is the airplane's velocity when it rotates (lifts off) at $t = 30$ s?

Solution: Velocity = Area under the curve

$$\boxed{v = \frac{1}{2}(3 \text{ m/s}^2 + 9 \text{ m/s}^2)(5 \text{ s}) + (9 \text{ m/s}^2)(25 \text{ s}) = 255 \text{ m/s}}$$

Problem 13.28 Determine the distance traveled during its takeoff by the airplane in Problem 13.27.

Solution: for $0 \le t \le 5$ s

$$a = \left(\frac{6 \text{ m/s}^2}{5 \text{ s}}\right)t + (3 \text{ m/s}^2), \quad v = \left(\frac{6 \text{ m/s}^2}{5 \text{ s}}\right)\frac{t^2}{2} + (3 \text{ m/s}^2)t$$

$$s = \left(\frac{6 \text{ m/s}^2}{5 \text{ s}}\right)\frac{t^3}{6} + (3 \text{ m/s}^2)\frac{t^2}{2}$$

$v(5 \text{ s}) = 30 \text{ m/s}, \quad s(5 \text{ s}) = 62.5 \text{ m}$

for $5 \text{ s} \le t \le 30$ s

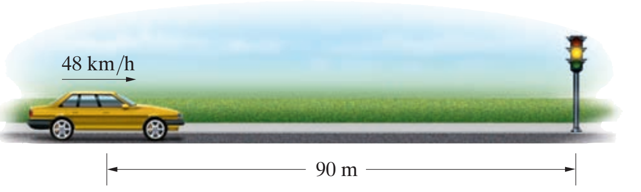
$a = 9 \text{ m/s}^2, \quad v = (9 \text{ m/s}^2)(t - 5 \text{ s}) + 30 \text{ m/s},$

$$s = (9 \text{ m/s}^2)\frac{(t - 5 \text{ s})^2}{2} + (30 \text{ m/s})(t - 5 \text{ s}) + 62.5 \text{ m}$$

$$\Rightarrow \boxed{s(30 \text{ s}) = 3625 \text{ m}}$$

Problem 13.29 The car is traveling at 48 km/h when the traffic light 90 m ahead turns yellow. The driver takes one second to react before he applies the brakes.

(a) After he applies the brakes, what constant rate of deceleration will cause the car to come to a stop just as it reaches the light?
(b) How long does it take the car to travel the 90 m?



Solution: for $0 \le t \le 1$ s

$a = 0,\ v = 48 \text{ km/h} = 13.33 \text{ m/s},\ s = (13.33 \text{ m/s})t$

$s(1\text{ s}) = 13.33$ m

for $t > 1$ s

$a = -c$ (constant), $v = -ct + 13.33 \text{ m/s}$, $s = -c\dfrac{t^2}{2} + (13.33 \text{ m/s})t + 13.33 \text{ m}$

At the stop we have

$$90 \text{ m} = -c\frac{t^2}{2} + (13.33 \text{ m/s})t + 13.33 \text{ m}$$

$$0 = -ct + 13.33 \text{ m/s}$$

$$\Rightarrow \boxed{\begin{array}{ll} a) & c = 1.17 \text{ m/s}^2 \\ b) & t = 11.41 \text{ s} \end{array}}$$

Problem 13.30 The car is traveling at 48 km/h when the traffic light 90 m ahead turns yellow. The driver takes 1 s to react before he applies the accelerator. If the car has a constant acceleration of 2 m/s^2 and the light remains yellow for 5 s, will the car reach the light before it turns red? How fast is the car moving when it reaches the light?

Solution: First, convert the initial speed into m/s.

$$48 \text{ km/h} = 13.33 \text{ m/s}.$$

At the end of the 5 s, the car will have traveled a distance

$$d = (13.33 \text{ m/s})(1 \text{ s}) + \left[\frac{1}{2}(2 \text{ m/s}^2)(5 \text{ s} - 1 \text{ s})^2 + (13.33 \text{ m/s})(5 \text{ s} - 1 \text{ s})\right] = 82.65 \text{ m}.$$

When the light turns red, the driver will still be 7.35 m from the light.
$\boxed{\text{No.}}$

To find the time at which the car does reach the light, we solve

$$90 \text{ m} = (13.33 \text{ m/s})(1 \text{ s}) + \left[\frac{1}{2}(2 \text{ m/s}^2)(t - 1 \text{ s})^2 + (13.33 \text{ m/s})(t - 1 \text{ s})\right]$$

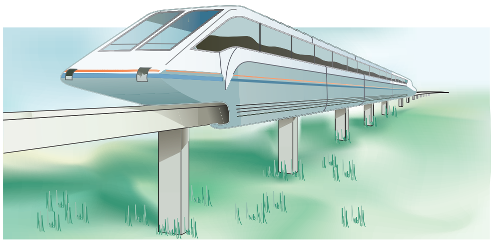
$$\Rightarrow t = 5.34 \text{ s}.$$

The speed at this time is

$$v = 13.33 \text{ m/s} + (2 \text{ m/s}^2)(5.34 \text{ s} - 1 \text{ s}) = 22.01 \text{ m/s}.$$

$$\boxed{v = 79.2 \text{ km/h.}}$$

Problem 13.31 A high-speed rail transportation system has a top speed of 100 m/s. For the comfort of the passengers, the magnitude of the acceleration and deceleration is limited to 2 m/s^2. Determine the time required for a trip of 100 km.



Strategy: A graphical approach can help you solve this problem. Recall that the change in the position from an initial time t_0 to a time t is equal to the area defined by the graph of the velocity as a function of time from t_0 to t.

Solution: Divide the time of travel into three intervals: The time required to reach a top speed of 100 m/s, the time traveling at top speed, and the time required to decelerate from top speed to zero. From symmetry, the first and last time intervals are equal, and the distances traveled during these intervals are equal. The initial time is obtained from $v(t_1) = at_1$, from which $t_1 = 100/2 = 50$ s. The distance traveled during this time is $s(t_1) = at_1^2/2$ from which $s(t_1) = 2(50)^2/2 = 2500$ m. The third time interval is given by $v(t_3) = -at_3 + 100 = 0$, from which $t_3 = 100/2 = 50$ s. *Check*. The distance traveled is $s(t_3) = -\frac{a}{2}t_3^2 + 100t_3$, from which $s(t_3) = 2500$ m. *Check*. The distance traveled at top speed is $s(t_2) = 100000 - 2500 - 2500 = 95000$ m $= 95$ km. The time of travel is obtained from the distance traveled at zero acceleration: $s(t_2) = 95000 = 100t_2$, from which $t_2 = 950$. The total time of travel is $t_{\text{total}} = t_1 + t_2 + t_3 = 50 + 950 + 50 = 1050$ s

$\boxed{= 17.5 \text{ minutes}}$.

A plot of velocity versus time can be made and the area under the curve will be the distance traveled. The length of the constant speed section of the trip can be adjusted to force the length of the trip to be the required 100 km.

Problem 13.32 The nearest star, Proxima Centauri, is 4.22 light years from the Earth. Ignoring relative motion between the solar system and Proxima Centauri, suppose that a spacecraft accelerates from the vicinity of the Earth at 0.01 g (0.01 times the acceleration due to gravity at sea level) until it reaches one-tenth the speed of light, coasts until it is time to decelerate, then decelerates at 0.01 g until it comes to rest in the vicinity of Proxima Centauri. How long does the trip take? (Light travels at 3×10^8 m/s.)

Solution: The distance to Proxima Centauri is

$$d = (4.22 \text{ light - year})(3 \times 10^8 \text{ m/s})(365.2422 \text{ day})\left(\frac{86400 \text{ s}}{1 \text{ day}}\right)$$

$$= 3.995 \times 10^{16} \text{ m}.$$

Divide the time of flight into the three intervals. The time required to reach 0.1 times the speed of light is

$$t_1 = \frac{v}{a} = \frac{3 \times 10^7 \text{ m/s}}{0.0981 \text{ m/s}^2} = 3.0581 \times 10^8 \text{ seconds}.$$

The distance traveled is

$$s(t_1) = \frac{a}{2}t_1^2 + v(0)t + s(0),$$

where $v(0) = 0$ and $s(0) = 0$ (from the conditions in the problem), from which $s(t_1) = 4.587 \times 10^{15}$ m. From symmetry, $t_3 = t_1$, and $s(t_1) = s(t_3)$. The length of the middle interval is $s(t_2) = d - s(t_1) - s(t_3) = 3.0777 \times 10^{16}$ m. The time of flight at constant velocity is

$$t_2 = \frac{3.0777 \times 10^{16} \text{ m}}{3 \times 10^7} = 1.026 \times 10^9 \text{ seconds}.$$

The total time of flight is $t_{\text{total}} = t_1 + t_2 + t_3 = 1.63751 \times 10^9$ seconds. In solar years:

$$t_{\text{total}} = (1.63751 \times 10^9 \text{ sec})\left(\frac{1 \text{ solar years}}{365.2422 \text{ days}}\right)\left(\frac{1 \text{ days}}{86400 \text{ sec}}\right)$$

$$= 51.9 \text{ solar years}$$

Problem 13.33 A race car starts from rest and accelerates at $a = 5 + 2t$ m/s^2 for 10 seconds. The brakes are then applied, and the car has a constant acceleration $a = -30$ m/s^2 until it comes to rest. Determine (a) the maximum velocity, (b) the total distance traveled; (c) the total time of travel.

Solution:

(a) For the first interval, the velocity is

$$v(t) = \int (5 + 2t)\,dt + v(0) = 5t + t^2$$

since $v(0) = 0$. The velocity is an increasing monotone function; hence the maximum occurs at the end of the interval, $t = 10$ s, from which

$\boxed{v_{\max} = 150 \text{ m/s}}$.

(b) The distance traveled in the first interval is

$$s(10) = \int_0^{10} (5t + t^2)\,dt = \left[\frac{5}{2}t^2 + \frac{1}{3}t^3\right]_0^{10} = 583.33 \text{ m}.$$

The time of travel in the second interval is

$$v(t_2 - 10) = 0 = a(t_2 - 10) + v(10),\ t_2 \geq 10 \text{ s},$$

from which

$$(t_2 - 10) = -\frac{150}{-30} = 5, \quad \text{and}$$

(c) the total time of travel is $t_2 = 15$. The total distance traveled is

$$s(t_2 - 10) = \frac{a}{2}(t_2 - 10)^2 + v(10)(t_2 - 10) + s(10),$$

from which (b)

$$\boxed{s(5) = \frac{-30}{2}5^2 + 150(5) + 583.33 = 958.33 \text{ m}}$$

Problem 13.34 When $t = 0$, the position of a point is $s = 6$ m and its velocity is $v = 2$ m/s. From $t = 0$ to $t = 6$ s, the acceleration of the point is $a = 2 + 2t^2$ m/s^2. From $t = 6$ s until it comes to rest, its acceleration is $a = -4$ m/s^2.

(a) What is the total time of travel?
(b) What total distance does the point move?

Solution: For the first interval the velocity is

$$v(t) = \int (2 + 2t^2)\,dt + v(0) = \left[2t + \frac{2}{3}t^3\right] + 2 \text{ m/s}.$$

The velocity at the end of the interval is $v(6) = 158$ m/s. The displacement in the first interval is

$$s(t) = \int \left(2t + \frac{2}{3}t^3 + 2\right)\,dt + 6 = \left[t^2 + \frac{1}{6}t^4 + 2t\right] + 6.$$

The displacement at the end of the interval is $s(6) = 270$ m. For the second interval, the velocity is $v(t - 6) = a(t - 6) + v(6) = 0$, $t \geq 6$, from which

$$(t - 6) = -\frac{v(6)}{a} = -\frac{158}{-4} = 39.5.$$

The total time of travel is

(a) $\boxed{t_{\text{total}} = 39.5 + 6 = 45.5}$ seconds.

(b) The distance traveled is

$$s(t - 6) = \frac{-4}{2}(t - 6)^2 + v(6)(t - 6) + s(6)$$

$$= -2(39.5)^2 + 158(39.5) + 270,$$

from which the total distance is $\boxed{s_{\text{total}} = 3390 \text{ m}}$

Problem 13.35 Zoologists studying the ecology of the Serengeti Plain estimate that the average adult cheetah can run 100 km/h and that the average springbuck can run 65 km/h. If the animals run along the same straight line, start at the same time, and are each assumed to have constant acceleration and reach top speed in 4 s, how close must the a cheetah be when the chase begins to catch a springbuck in 15 s?

Solution: The top speeds are $V_c = 100$ km/h $= 27.78$ m/s for the cheetah, and $V_s = 65$ km/h $= 18.06$ m/s. The acceleration is $a_c = \dfrac{V_c}{4} = 6.94$ m/s^2 for the cheetah, and $a_s = \dfrac{V_s}{4} = 4.513$ m/s^2 for the springbuck. Divide the intervals into the acceleration phase and the chase phase. For the cheetah, the distance traveled in the first is $s_c(t) = \dfrac{6.94}{2}(4)^2 = 55.56$ m. The total distance traveled at the end of the second phase is $s_{\text{total}} = V_c(11) + 55.56 = 361.1$ m. For the springbuck, the distance traveled during the acceleration phase is $s_s(t) = \dfrac{4.513}{2}(4)^2 = 36.11$ m. The distance traveled at the end of the second phase is $s_s(t) = 18.06(11) + 36.1 = 234.7$ m. The permissible separation between the two at the beginning for a successful chase is $\boxed{d = s_c(15) - s_s(15) = 361.1 - 234.7 = 126.4 \text{ m}}$

Problem 13.36 Suppose that a person unwisely drives 120 km/h in a 88 km/h zone and passes a police car going 88 km/h in the same direction. If the police officers begin constant acceleration at the instant they are passed and increase their speed to 129 km/h in 4 s, how long does it take them to be even with the pursued car?

Solution: The conversion from mi/h to m/s is

$$\frac{\text{km}}{\text{h}} = \frac{1000 \text{ m}}{3600 \text{ s}} = 0.278 \text{ m/s}$$

The acceleration of the police car is

$$a = \frac{(129-88)(0.278) \text{ m/s}}{4 \text{ s}} = 2.85 \text{ m/s}^2.$$

The distance traveled during acceleration is

$$s(t_1) = \frac{2.85}{2}(4)^2 + 88(0.278)(4) = 121 \text{ m}.$$

The distance traveled by the pursued car during this acceleration is

$$s_c(t_1) = 120(0.278)\,t_1 = 33.36(4) = 133.4 \text{ m}.$$

The separation between the two cars at 4 seconds is

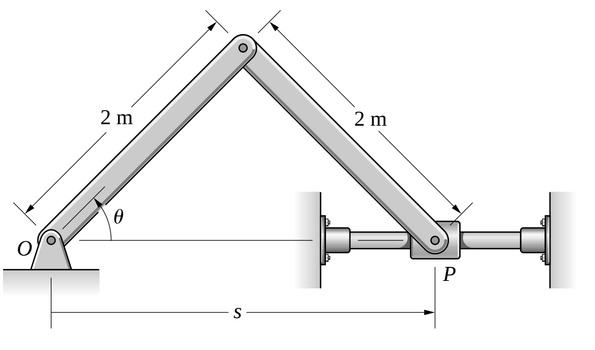
$$d = 133.4 - 121 = 12.4 \text{ m}.$$

This distance is traversed in the time

$$t_2 = \frac{12.4}{(129-88)(0.278)} = 1.09.$$

The total time is $\boxed{t_{\text{total}} = 1.09 + 4 = 5.09}$ seconds.

Problem 13.37 If $\theta = 1$ rad and $\dfrac{d\theta}{dt} = 1$ rad/s, what is the velocity of P relative to O?



Strategy: You can write the position of P relative to O as $s = (2 \text{ m})\cos\theta + (2 \text{ m})\cos\theta$ and then take the derivative of this expression with respect to time to determine the velocity.

Solution: The distance s from point O is

$$s = (2 \text{ m})\cos\theta + (2 \text{ m})\cos\theta.$$

The derivative is

$$\frac{ds}{dt} = -4\sin\theta\frac{d\theta}{dt}.$$

For $\theta = 1$ radian and $\dfrac{d\theta}{dt} = 1$ radian/second,

$$\boxed{\frac{ds}{dt} = v(t) = -4(\sin(1 \text{ rad})) = -4(0.841) = -3.37 \text{ m/s}}$$

Problem 13.38 In Problem 13.37, if $\theta = 1$ rad, $d\theta/dt = -2$ rad/s and $d^2\theta/dt^2 = 0$, what are the velocity and acceleration of P relative to O?

Solution: The velocity is

$$\boxed{\frac{ds}{dt} = -4\sin\theta\frac{d\theta}{dt} = -4(\sin(1\text{ rad}))(-2) = 6.73 \text{ m/s}}.$$

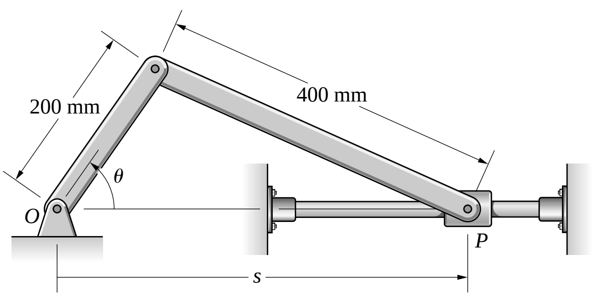
The acceleration is

$$\frac{d^2s}{dt^2} = -4\cos\theta\left(\frac{d\theta}{dt}\right)^2 - 4\sin\theta\left(\frac{d^2\theta}{dt^2}\right),$$

from which

$$\boxed{\frac{d^2s}{dt^2} = a = -4\cos(1\text{ rad})(4) = -8.64 \text{ m/s}^2}$$

Problem 13.39 If $\theta = 1$ rad and $\dfrac{d\theta}{dt} = 1$ rad/s, what is the velocity of P relative to O?



Solution: The acute angle formed by the 400 mm arm with the horizontal is given by the sine law:

$$\frac{200}{\sin\alpha} = \frac{400}{\sin\theta},$$

from which

$$\sin\alpha = \left(\frac{200}{400}\right)\sin\theta.$$

For $\theta = 1$ radian, $\alpha = 0.4343$ radians. The position relative to O is.

$$s = 200\cos\theta + 400\cos\alpha.$$

The velocity is

$$\frac{ds}{dt} = v(t) = -200\sin\theta\left(\frac{d\theta}{dt}\right) - 400\sin\alpha\left(\frac{d\alpha}{dt}\right).$$

From the expression for the angle

$$\alpha, \cos\alpha\left(\frac{d\alpha}{dt}\right) = 0.5\cos\theta\left(\frac{d\theta}{dt}\right),$$

from which the velocity is

$$v(t) = (-200\sin\theta - 200\tan\alpha\cos\theta)\left(\frac{d\theta}{dt}\right).$$

Substitute: $\boxed{v(t) = -218.4 \text{ mm/s}}$.

Problem 13.40 In Active Example 13.4, determine the time required for the plane's velocity to decrease from 50 m/s to 10 m/s.

Solution: From Active Example 13.4 we know that the acceleration is given by

$$a = -(0.004/\mathrm{m})v^2.$$

We can find an expression for the velocity as a function of time by integrating

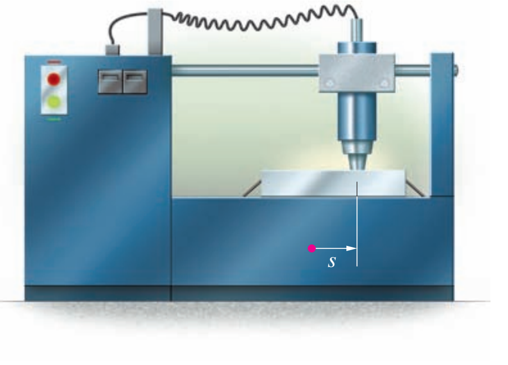
$$a = \frac{dv}{dt} = -(0.004/\mathrm{m})v^2 \Rightarrow \frac{dv}{v^2} = -(0.004/\mathrm{m})t$$

$$\int_{50\ \mathrm{m/s}}^{10\ \mathrm{m/s}} \frac{dv}{v^2} = \left(-\frac{1}{v}\right)_{50\ \mathrm{m/s}}^{10\ \mathrm{m/s}} = \left(-\frac{1\ \mathrm{s}}{10\ \mathrm{m}} + \frac{1\ \mathrm{s}}{50\ \mathrm{m}}\right) = \left(-\frac{4\ \mathrm{s}}{50\ \mathrm{m}}\right) = -(0.004/\mathrm{m})t$$

$$t = \left(-\frac{4\ \mathrm{s}}{50\ \mathrm{m}}\right)\left(\frac{-1\ \mathrm{m}}{0.004}\right) = 20\ \mathrm{s}.$$

$$\boxed{t = 20\ \mathrm{s}.}$$

Problem 13.41 An engineer designing a system to control a router for a machining process models the system so that the router's acceleration (in cm/s^2) during an interval of time is given by $a = -0.4v$, where v is the velocity of the router in cm/s. When $t = 0$, the position is $s = 0$ and the velocity is $v = 2$ cm/s. What is the position at $t = 3$ s?



Solution: We will first find the velocity at t = 3.

$$a = \frac{dv}{dt} = -\left(\frac{0.4}{\mathrm{s}}\right)v,$$

$$\int_{2\ \mathrm{cm/s}}^{v_2} \frac{dv}{v} = \int_0^{3\ s}\left(\frac{-0.4}{\mathrm{s}}\right)dt,$$

$$\ln\left(\frac{v_2}{2\ \mathrm{cm/s}}\right) = \left(\frac{-0.4}{\mathrm{s}}\right)(3\ \mathrm{s}) = -1.2$$

$$v_2 = (2\ \mathrm{cm/s})e^{-1.2} = 0.602\ \mathrm{cm/s}.$$

Now we can find the position

$$a = \frac{v\,dv}{ds} = \left(\frac{-0.4}{\mathrm{s}}\right)v,$$

$$\int_{2\ \mathrm{cm/s}}^{0.602\ \mathrm{cm/s}} \frac{v\,dv}{v} = \left(\frac{-0.4}{\mathrm{s}}\right)\int_0^{s_2} ds$$

$$(0.602\ \mathrm{cm/s}) - (2\ \mathrm{cm/s}) = \left(\frac{-0.4}{s}\right)s_2$$

$$s_2 = \frac{-1.398\ \mathrm{cm/s}}{-0.4/\ \mathrm{s}} = 3.49\ \mathrm{cm}$$

$$\boxed{s_2 = 3.49\ \mathrm{cm}}$$

Problem 13.42 The boat is moving at 10 m/s when its engine is shut down. Due to hydrodynamic drag, its subsequent acceleration is $a = -0.05v^2$ m/s^2, where v is the velocity of the boat in m/s. What is the boat's velocity 4 s after the engine is shut down?

Solution:

$$a = \frac{dv}{dt} = -(0.05\ \text{m}^{-1})v^2$$

$$\int_{10\,\text{m/s}}^{v} \frac{dv}{v^2} = -(0.05\ \text{m}^{-1})\int_0^t dt \Rightarrow -\left.\frac{1}{v}\right|_{10\,\text{m/s}}^{v} = -(0.05\ \text{m}^{-1})t$$

$$v = \frac{10\ \text{m/s}}{1 + (0.5\ \text{s}^{-1})t}$$

$$\boxed{v(4\ \text{s}) = 3.33\ \text{m/s}}$$

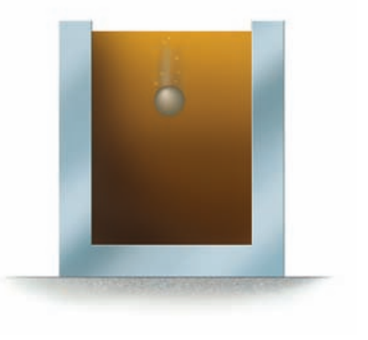
Problem 13.43 In Problem 13.42, what distance does the boat move in the 4 s following the shutdown of its engine?

Solution: From Problem 13.42 we know

$$v = \frac{ds}{dt} = \frac{10\ \text{m/s}}{1 + (0.5\ \text{s}^{-1})t} \Rightarrow s(4\ \text{s}) = \int_0^{4\ \text{s}} \frac{10\ \text{m/s}}{1 + (0.5\ \text{s}^{-1})t}dt$$

$$\boxed{s(4\ \text{s}) = (20\ \text{m})\ln\left[\frac{2 + (1\ \text{s}^{-1})(4\ \text{s})}{2}\right] = 21.97\ \text{m}}$$

Problem 13.44 A steel ball is released from rest in a container of oil. Its downward acceleration is $a = 2.4 - 0.6v$ cm/s^2, where v is the ball's velocity in cm/s. What is the ball's downward velocity 2 s after it is released?



Solution:

$$a = \frac{dv}{dt} = (2.4\ \text{cm/s}) - (0.6\ \text{s}^{-1})v$$

$$\int_0^v \frac{dv}{(2.4\ \text{cm/s}) - (0.6\ \text{s}^{-1})v} = \int_0^t dt$$

$$-\frac{5}{3}\ln\left(\frac{v + 4\ \text{cm/s}}{4\ \text{cm/s}}\right) = t \Rightarrow v = (4\ \text{cm/s})\left(1 - e^{-(0.6\ \text{s}^{-1})t}\right)$$

$$\boxed{v(2\ \text{s}) = 2.795\ \text{cm/s}}$$

Problem 13.45 In Problem 13.44, what distance does the ball fall in the first 2 s after its release?

Solution: From 13.44 we know

$$v = \frac{ds}{dt} = (4\ \text{cm/s})\left(1 - e^{-(0.6\ \text{s}^{-1})t}\right)$$

$$s(2\ \text{s}) = \int_0^t (4\ \text{cm/s})\left(1 - e^{-(0.6\ \text{s}^{-1})t}\right)dt$$

$$= \frac{20\ \text{cm}}{3}\left(e^{(-0.6\ \text{s}^{-1})t} - 1\right) + (4\ \text{cm/s})t$$

$$\boxed{s(2\ \text{s}) = 3.34\,\text{cm.}}$$

Problem 13.46 The greatest ocean depth yet discovered is the Marianas Trench in the western Pacific Ocean. A steel ball released at the surface requires 64 minutes to reach the bottom. The ball's downward acceleration is $a = 0.9g - cv$, where $g = 9.81$ m/s^2 and the constant $c = 3.02$ s^{-1}. What is the depth of the Marianas Trench in kilometers?

Solution:

$$a = \frac{dv}{dt} = 0.9g - cv.$$

Separating variables and integrating,

$$\int_0^v \frac{dv}{0.9g - cv} = \int_0^t dt = t.$$

Integrating and solving for v,

$$v = \frac{ds}{dt} = \frac{0.9g}{c}(1 - e^{-ct}).$$

Integrating,

$$\int_0^s ds = \int_0^t \frac{0.9g}{c}(1 - e^{-ct})\, dt.$$

We obtain

$$s = \frac{0.9g}{c}\left(t + \frac{e^{-ct}}{c} - \frac{1}{c}\right).$$

At $t = (64)(60) = 3840$ s, we obtain

$\underline{s = 11{,}225 \text{ m}}$.

Problem 13.47 The acceleration of a regional airliner during its takeoff run is $a = 14 - 0.0003v^2$ m/s^2, where v is its velocity in m/s. How long does it take the airliner to reach its takeoff speed of 200 m/s?

Solution:

$$a = \frac{dv}{dt} = (14 \text{ m/s}^2) - (0.0003 \text{ m}^{-1})v^2$$

$$\int_0^{200\,\text{m/s}} \frac{dv}{(14 \text{ m/s}^2) - (0.0003\,\text{m}^{-1})v^2} = \int_0^t dt$$

$$\boxed{t = 25.1 \text{ s}}$$

Problem 13.48 In Problem 13.47, what distance does the airliner require to take off?

Solution:

$$a = v\frac{dv}{ds} = (14 \text{ m/s}^2) - (0.0003 \text{ m}^{-1})v^2$$

$$\int_0^{200 \text{ m/s}} \frac{v\,dv}{(14 \text{ m/s}^2) - (0.0003\,\text{m}^{-1})v^2} = \int_0^s ds$$

$$\boxed{s = 3243 \text{ m}}$$

Problem 13.49 A sky diver jumps from a helicopter and is falling straight down at 30 m/s when her parachute opens. From then on, her downward acceleration is approximately $a = g - cv^2$, where $g = 9.81\ \text{m/s}^2$ and c is a constant. After an initial "transient" period she descends at a nearly constant velocity of 5 m/s.

(a) What is the value of c, and what are its SI units?
(b) What maximum deceleration is the sky diver subjected to?
(c) What is her downward velocity when she has fallen 2 meters from the point at which her parachute opens?

Solution: Assume $c > 0$.

(a) After the initial transient, she falls at a constant velocity, so that the acceleration is zero and $cv^2 = g$, from which

$$\boxed{c = \frac{g}{v^2} = \frac{9.81\ \text{m/s}^2}{(5)^2\ \text{m}^2/\text{s}^2} = 0.3924\ \text{m}^{-1}}$$

(b) The maximum acceleration (in absolute value) occurs when the parachute first opens, when the velocity is highest:

$$\boxed{a_{\max} = |g - cv^2| = |g - c(30)^2| = 343.4\ \text{m/s}^2}$$

(c) Choose coordinates such that distance is measured positive downward. The velocity is related to position by the chain rule:

$$\frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = v\frac{dv}{ds} = a,$$

from which

$$\frac{v\,dv}{g - cv^2} = ds.$$

Integrate:

$$\left(-\frac{1}{2c}\right)\ln|g - cv^2| = s + C.$$

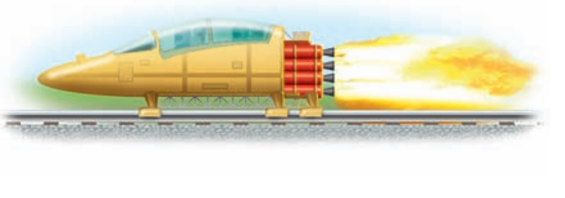
When the parachute opens $s = 0$ and $v = 30$ m/s, from which

$$C = -\left(\frac{1}{2c}\right)\ln|g - 900c| = -7.4398.$$

The velocity as a function of distance is $\ln|g - cv^2| = -2c(s + C)$. For $s = 2$ m,

$$\boxed{v = 14.4\ \text{m/s}}$$

Problem 13.50 The rocket sled starts from rest and accelerates at $a = 30 + 2t\ \text{m/s}^2$ until its velocity is 400 m/s. It then hits a water brake and its acceleration is $a = -0.003v^2\ \text{m/s}^2$ until its velocity decreases to 100 m/s. What total distance does the sled travel?



Solution: Acceleration Phase

$$a = 30 + 2t\ \text{m/s}^2$$

$$v = 30t + t^2\ \text{m/s}$$

$$s = 15t^2 + t^3/3\ \text{m}$$

When $v = 400$ m/s, acceleration ends. At this point, $t = 10$ s and $s = 1833$ m. Deceleration Phase starts at $s_1 = 1833$ m, $v_1 = 400$ m/s. Let us start a new clock for the deceleration phase. $v_f = 100$ m/s

$$a = v\frac{dv}{ds} = -0.003v^2$$

$$\int_{s_1}^{s_f} ds = -\frac{1}{(0.003)}\int_{v_1}^{v_f}\frac{v\,dv}{v^2}$$

$$s_f - 1833\ \text{m} = -\frac{1}{(0.003)}[\ln(100) - \ln(400)]$$

$$\underline{s_f = 2300\ \text{m}}$$

Problem 13.51 In Problem 13.50, what is the sled's total time of travel?

Solution: From the solution to Problem 13.50, the acceleration takes 10 s. At $t = 10$ s, the velocity is 400 m/s. We need to find out how long it takes to decelerate from 400 m/s to 100 m/s and add this to the 10 s required for acceleration. The deceleration is given as

$$a = \frac{dv}{dt} = -0.003v^2 \text{ m/s}^2$$

$$-0.003\int_0^{t_d} dt = \int_{400}^{100} \frac{dv}{v^2}$$

$$-0.003t_d = -\frac{1}{v}\bigg|_{400}^{100} = -\left(\frac{1}{100} - \frac{1}{400}\right)$$

$$0.003t_d = \frac{3}{400}$$

$$t_d = 2.5 \text{ s}$$

$$\underline{t = 10 + t_d = 12.5 \text{ s}}$$

Problem 13.52 A car's acceleration is related to its position by $a = 0.01s$ m/s^2. When $s = 100$ m, the car is moving at 12 m/s. How fast is the car moving when $s = 420$ m?

Solution:

$$a = v\frac{dv}{ds} = 0.01s \text{ m/s}^2$$

$$\int_{12}^{v_f} v\,dv = 0.01\int_{100}^{420} s\,ds$$

$$\left[\frac{v^2}{2}\right]_{12 \text{ m/s}}^{v_f} = 0.01\left[\frac{s^2}{2}\right]_{100 \text{ m}}^{420 \text{ m}}$$

$$\frac{v_f^2}{2} = \frac{12^2}{2} + 0.01\frac{(420^2 - 100^2)}{2}$$

$$v_f = 42.5 \text{ m/s}$$

Problem 13.53 Engineers analyzing the motion of a linkage determine that the velocity of an attachment point is given by $v = A + 4s^2$ m/s, where A is a constant. When $s = 2$ m, its acceleration is measured and determined to be $a = 320$ m/s^2. What is its velocity of the point when $s = 2$ m?

Solution: The velocity as a function of the distance is

$$v\frac{dv}{ds} = a.$$

Solve for a and carry out the differentiation.

$$a = v\frac{dv}{ds} = (A + 4s^2)(8s).$$

When $s = 2$ m, $a = 320$ m/s^2, from which $A = 4$.

The velocity at $s = 2$ m is

$$\boxed{v = 4 + 4(2^2) = 20 \text{ m/s}}$$

Problem 13.54 The acceleration of an object is given as a function of its position in feet by $a = 2\ s^2(\text{m/s}^2)$. When $s = 0$, its velocity is $v = 1$ m/s. What is the velocity of the object when $s = 2$ m?

Solution: We are given

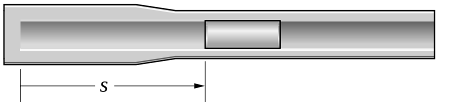
$$a = \frac{vdv}{ds} = \left(\frac{2}{\text{m-s}^2}\right) s^2,$$

$$\int_{1\ \text{m/s}}^{v} vdv = \left(\frac{2}{\text{m-s}^2}\right) \int_0^{2\text{m}} s^2 ds$$

$$\frac{v^2}{2} - \frac{(1\ \text{m/s})^2}{2} = \left(\frac{2}{\text{m-s}^2}\right) \frac{(2\ \text{m})^3}{3}$$

$$\boxed{v = 3.42\ \text{m/s.}}$$

Problem 13.55 Gas guns are used to investigate the properties of materials subjected to high-velocity impacts. A projectile is accelerated through the barrel of the gun by gas at high pressure. Assume that the acceleration of the projectile is given by $a = c/s$, where s is the position of the projectile in the barrel in meters and c is a constant that depends on the initial gas pressure behind the projectile. The projectile starts from rest at $s = 1.5$ m and accelerates until it reaches the end of the barrel at $s = 3$ m. Determine the value of the constant c necessary for the projectile to leave the barrel with a velocity of 200 m/s.



Solution:

$$a = v\frac{dv}{ds} = \frac{c}{s}, \Rightarrow \int_0^{200\,\text{m/s}} vdv = \int_{1.5\,\text{m}}^{3\,\text{m}} \frac{c}{s} ds$$

$$\Rightarrow \frac{(200\ \text{m/s})^2}{2} = c \ln\left(\frac{3\ \text{m}}{1.5\ \text{m}}\right)$$

$$\boxed{c = 28.85 \times 10^3\ \text{m}^2/\text{s}^2}$$

Problem 13.56 If the propelling gas in the gas gun described in Problem 13.55 is air, a more accurate modeling of the acceleration of the projectile is obtained by assuming that the acceleration of the projectile is given by $a = c/s^\gamma$, where $\gamma = 1.4$ is the ratio of specific heats for air. (This means that an isentropic expansion process is assumed instead of the isothermal process assumed in Problem 13.55.) Determine the value of the constant c necessary for the projectile to leave the barrel with a velocity of 200 m/s.

Solution:

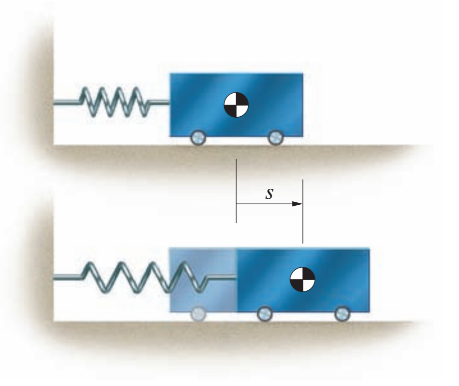
$$a = v\frac{dv}{ds} = \frac{c}{s^{1.4}}, \ \Rightarrow \int_0^{200\,\text{m/s}} vdv = \int_{1.5\,\text{m}}^{3\,\text{m}} \frac{c}{s^{1.4}} ds$$

$$\frac{(200\ \text{m/s})^2}{2} = -2.5c\left((3m)^{-0.4} - (1.5m)^{-0.4}\right)$$

$$\boxed{c = 38.86 \times 10^3\ \text{m}^{2.4}/\text{s}^2}$$

Problem 13.57 A spring-mass oscillator consists of a mass and a spring connected as shown. The coordinate s measures the displacement of the mass relative to its position when the spring is unstretched. If the spring is linear, the mass is subjected to a deceleration proportional to s. Suppose that $a = -4s$ m/s^2, and that you give the mass a velocity $v = 1$ m/s in the position $s = 0$.

(a) How far will the mass move to the right before the spring brings it to a stop?
(b) What will be the velocity of the mass when it has returned to the position $s = 0$?



Solution: The velocity of the mass as a function of its position is given by $v\,dv/ds = a$. Substitute the given acceleration, separate variables, and integrate: $v\,dv = -4s\,ds$, from which $v^2/2 = -2s^2 + C$. The initial velocity $v(0) = 1$ m/s at $s = 0$, from which $C = 1/2$. The velocity is $v^2/2 = -2s^2 + 1/2$.

(a) The velocity is zero at the position given by

$$0 = -2(s_1)^2 + \frac{1}{2},$$

from which $s_1 = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$ m.

Since the displacement has the same sign as the velocity,

$\boxed{s_1 = +1/2 \text{ m}}$

is the distance traveled before the spring brings it to a stop.

(b) At the return to $s = 0$, the velocity is $v = \pm\sqrt{\frac{2}{2}} = \pm 1$ m/s. From the physical situation, the velocity on the first return is negative (opposite the sign of the initial displacement),

$\boxed{v = -1 \text{ m/s}}$.

Problem 13.58 In Problem 13.57, suppose that at $t = 0$ you release the mass from rest in the position $s = 1$ m. Determine the velocity of the mass as a function of s as it moves from the initial position to $s = 0$.

Solution: From the solution to Problem 13.57, the velocity as a function of position is given by

$$\frac{v^2}{2} = -2s^2 + C.$$

At $t = 0$, $v = 0$ and $s = 1$ m, from which $C = 2(1)^2 = 2$. The velocity is given by

$$v(s) = \pm(-4s^2 + 4)^{\frac{1}{2}} = \pm 2\sqrt{1 - s^2} \text{ m/s}.$$

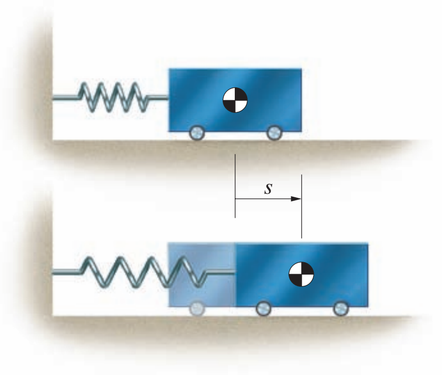
From the physical situation, the velocity is negative (opposite the sign of the initial displacement):

$\boxed{v = -2\sqrt{1 - s^2} \text{ m/s}}$.

[*Note*: From the initial conditions, $s^2 \leq 1$ always.]

Problem 13.59 A spring-mass oscillator consists of a mass and a spring connected as shown. The coordinate s measures the displacement of the mass relative to its position when the spring is unstretched. Suppose that the nonlinear spring subjects the mass to an acceleration $a = -4s - 2s^3$ m/s^2 and that you give the mass a velocity $v = 1$ m/s in the position $s = 0$.

(a) How far will the mass move to the right before the springs brings it to a stop?
(b) What will be the velocity of the mass when it has returned to the position $s = 0$?



Solution:

(a) Find the distance when the velocity is zero.

$$a = \frac{v\,dv}{ds} = \left(\frac{-4}{\text{s}^2}\right) s + \left(\frac{-2}{\text{m}^2\text{s}^2}\right) s^3$$

$$\int_{1\text{ m/s}}^{0} v\,dv = \int_0^d \left[\left(\frac{-4}{\text{s}^2}\right) s + \left(\frac{-2}{\text{m}^2\text{s}^2}\right) s^3\right] ds$$

$$-\frac{(1\text{ m/s})^2}{2} = -\left(\frac{4}{\text{s}^2}\right)\frac{d^2}{2} - \left(\frac{2}{\text{m}^2\text{s}^2}\right)\frac{d^4}{4}$$

Solving for d we find $\boxed{d = 0.486\ m.}$

(b) When the cart returns to the position $s = 0$, we have

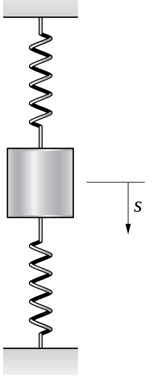
$$a = \frac{v\,dv}{ds} = \left(\frac{-4}{\text{s}^2}\right) s + \left(\frac{-2}{\text{m}^2\text{s}^2}\right) s^3$$

$$\int_{1\text{ m/s}}^{v} v\,dv = \int_0^0 \left[\left(\frac{-4}{\text{s}^2}\right) s + \left(\frac{-2}{\text{m}^2\text{s}^2}\right) s^3\right] ds = 0$$

$$\frac{v^2}{2} - \frac{(1\text{ m/s})^2}{2} = 0, \Rightarrow v = \pm 1\text{ m/s}.$$

The cart will be moving to the left, so we choose $\boxed{v = -1\text{ m/s.}}$

Problem 13.60 The mass is released from rest with the springs unstretched. Its downward acceleration is $a = 32.2 - 50s$ m/s^2, where s is the position of the mass measured from the position in which it is released. (a) How far does the mass fall? (b) What is the maximum velocity of the mass as it falls?



Solution: The acceleration is given by

$$a = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = v\frac{dv}{ds} = 32.2 - 50s\ \text{m/s}^2.$$

Integrating, we get

$$\int_0^v v\,dv = \int_0^s (32.2 - 50s)\,ds \quad \text{or} \quad \frac{v^2}{2} = 32.2s - 25s^2.$$

(a) The mass falls until $v = 0$. Setting $v = 0$, we get $0 = (32.2 - 25s)s$. We find $v = 0$ at $s = 0$ and at $s = 1.288$ m. Thus, the mass falls 1.288 m before coming to rest.

(b) From the integration of the equation of motion, we have $v^2 = 2(32.2s - 25s^2)$. The maximum velocity occurs where $\frac{dv}{ds} = 0$. From the original equation for acceleration, we have $a = v\frac{dv}{ds} = (32.2 - 50s)$ m/s^2. Since we want maximum velocity, we can assume that $v \neq 0$ at this point. Thus, $0 = (32.2 - 50s)$, or $s = (32.2/50)$ m when $v = v_{MAX}$. Substituting this value for s into the equation for v, we get

$$v_{MAX}^2 = 2\left(\frac{(32.2)^2}{50} - \frac{(25)(32.2)^2}{50^2}\right),$$

or $v_{MAX} = 4.55$ m/s

Problem 13.61 Suppose that the mass in Problem 13.60 is in the position $s = 0$ and is given a downward velocity of 10 m/s.

(a) How far does the mass fall?
(b) What is the maximum velocity of the mass as it falls?

Solution:

$$a = v\frac{dv}{ds} = (32.2 \text{ m/s}^2) - (50 \text{ s}^{-2})s$$

$$\int_{10 \text{ m/s}}^{v} v\,dv = \int_0^s [(32.2 \text{ m/s}^2) - (50 \text{ s}^{-2})s]ds$$

$$\frac{v^2}{2} - \frac{(10 \text{ m/s})^2}{2} = (32.2 \text{ m/s}^2)\, s - (50 \text{ s}^{-2})\frac{s^2}{2}$$

$$v^2 = (10 \text{ m/s})^2 + (64.4 \text{ m/s}^2)s - (50 \text{ s}^{-2})s^2$$

(a) The mass falls until $v = 0$

$$\boxed{0 = (10 \text{ m/s})^2 + (64.4 \text{ m/s}^2)s - (50 \text{ s}^{-2})s^2 \Rightarrow s = 2.20 \text{ m}}$$

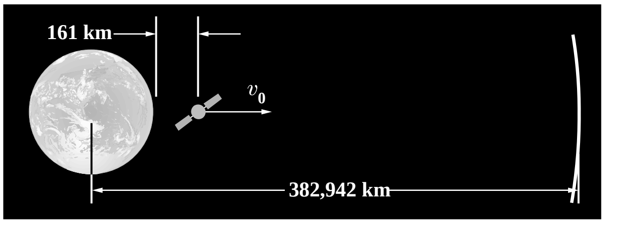
(b) The maximum velocity occurs when $a = 0$

$$0 = (32.2 \text{ m/s}^2) - (50 \text{ s}^{-2})s \Rightarrow s = 0.644 \text{ m}$$

$$v^2 = (10 \text{ m/s})^2 + (64.4 \text{ m/s}^2)(0.644 \text{ m}) - (50 \text{ s}^{-2})(0.644 \text{ m})^2$$

$$\boxed{v = 10.99 \text{ m/s}}$$

Problem 13.62 If a spacecraft is 161 km above the surface of the earth, what initial velocity v_0 straight away from the earth would be required for the vehicle to reach the moon's orbit 382,942 km from the center of the earth? The radius of the earth is 6372 km. Neglect the effect of the moon's gravity. (See Example 13.5.)



Solution: For computational convenience, convert the acceleration due to Earth's gravity into the units given in the problem, namely miles and hours:

$$g = \left(\frac{9.81 \text{ m}}{1\ s^2}\right)\left(\frac{1 \text{ km}}{1000 \text{ m}}\right)\left(\frac{3600^2 \text{ s}^2}{1 \text{ h}^2}\right) = 127137.6 \text{ km/h}^2.$$

The velocity as a function of position is given by

$$v\frac{dv}{ds} = a = -\frac{gR_E^2}{s^2}.$$

Separate variables,

$$v\,dv = -gR_E^2\frac{ds}{s^2}.$$

Integrate:

$$v^2 = -2gR_E^2\left(-\frac{1}{2}\right) + C.$$

Suppose that the velocity at the distance of the Moon's orbit is zero. Then

$$0 = 2(127137.6)\left(\frac{6372^2}{382942}\right) + C,$$

from which $C = -26960164 \text{ km}^2/\text{h}^2$. At the 161 km altitude, the equation for the velocity is

$$v_0^2 = 2\ g\left(\frac{R_E^2}{R_E + 161}\right) + C.$$

From which

$$\boxed{v_0 = \sqrt{1553351991} = 39{,}413 \text{ km/h}}$$

Converting:

$$\boxed{v_0 = \left(\frac{39413 \text{ km}}{1 \text{ h}}\right)\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 10{,}948 \text{ m/s}}.$$

Check: Use the result of Example 13.5

$$v_0 = \sqrt{2gR_E^2\left(\frac{1}{s_0} - \frac{1}{\text{H}}\right)},$$

(where $\text{H} > s_0$ always), and $\text{H} = 382{,}942$,

from which $v_0 = 39{,}413$ km/h. *check*.

Problem 13.63 The moon's radius is 1738 km. The magnitude of the acceleration due to gravity of the moon at a distance s from the center of the moon is

$$\frac{4.89 \times 10^{12}}{s^2} \text{ m/s}^2.$$

Suppose that a spacecraft is launched straight up from the moon's surface with a velocity of 2000 m/s.

(a) What will the magnitude of its velocity be when it is 1000 km above the surface of the moon?
(b) What maximum height above the moon's surface will it reach?

Solution: Set $G = 4.89 \times 10^{12}$ m^3/s^2, $r_0 = 1.738 \times 10^6$ m, $v_0 = 2000$ m/s

$$a = v\frac{dv}{ds} = -\frac{G}{s^2} \Rightarrow \int_{v_0}^{v} v\,dv = -\int_{r_0}^{r} \frac{G}{s^2} ds \Rightarrow \frac{v^2}{2} - \frac{v_0{}^2}{2} = G\left(\frac{1}{r} - \frac{1}{r_0}\right)$$

$$v^2 = v_0{}^2 + 2G\left(\frac{r_0 - r}{r r_0}\right)$$

(a) $\boxed{v(r_0 + 1.0 \times 10^6 \text{ m}) = 1395 \text{ m/s}}$

(b) The maximum velocity occurs when $v = 0$

$$r = \frac{2G}{2G - r_0 v_0{}^2} = 6010 \text{ km} \quad \Rightarrow \boxed{h = r - r_0 = 4272 \text{ km}}$$

Problem 13.64* The velocity of an object subjected only to the earth's gravitational field is

$$v = \left[v_0^2 + 2gR_E^2\left(\frac{1}{s} - \frac{1}{s_0}\right)\right]^{1/2},$$

where s is the object's position relative to the center of the earth, v_0 is the velocity at position s_0, and R_E is the earth's radius. Using this equation, show that the object's acceleration is given as a function of s by $a = -gR_E^2/s^2$.

Solution:

$$v = v_0^2 + 2gR_E^2\left(\frac{1}{s} - \frac{1}{s_0}\right)^{1/2}$$

$$a = \frac{dv}{dt} = v\frac{dv}{ds}$$

Rewrite the equation given as $v^2 = v_0^2 + \dfrac{2gR_E^2}{s} - \dfrac{2gR_E^2}{s_0}$

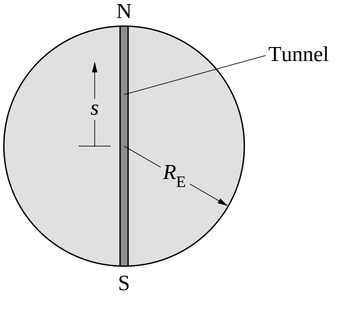
Take the derivative with respect to s.

$$2v\frac{dv}{ds} = -\frac{2gR_E^2}{s^2}$$

Thus

$$\underline{a = v\frac{dv}{ds} - \frac{gR_E^2}{s^2}}$$

Problem 13.65 Suppose that a tunnel could be drilled straight through the earth from the North Pole to the South Pole and the air was evacuated. An object dropped from the surface would fall with the acceleration $a = -gs/R_E$, where g is the acceleration of gravity at sea level, R_E is radius of the earth, and s is the distance of the object from the center of the earth. (The acceleration due to gravitation is equal to zero at the center of the earth and increases linearly with the distance from the center.) What is the magnitude of the velocity of the dropped object when it reaches the center of the earth?



Solution: The velocity as a function of position is given by

$$v\frac{dv}{ds} = -\frac{gs}{R_E}.$$

Separate variables and integrate:

$$v^2 = -\left(\frac{g}{R_E}\right)s^2 + C.$$

At $s = R_E$, $v = 0$, from which $C = gR_E$. Combine and reduce:

$$v^2 = gR_E\left(1 - \frac{s^2}{R_E^2}\right)$$

At the center of the earth $s = 0$, and the velocity is $\boxed{v = \sqrt{gR_E}}$

Problem 13.66 Determine the time in seconds required for the object in Problem 13.65 to fall from the surface of the earth to the center. The earth's radius is 6370 km.

Solution: From Problem 13.65, the acceleration is

$$a = v\frac{dv}{ds} = -\frac{g}{R_E}s$$

$$\int_0^v v\,du = -\int_{R_E}^{s}\left(\frac{g}{R_E}\right)s\,ds$$

$$v^2 = \left(\frac{g}{R_E}\right)(R_E^2 - s^2)$$

Recall that $v = ds/dt$

$$v = \frac{ds}{dt} = \pm\sqrt{\frac{g}{R_E}}\sqrt{R_E^2 - s^2}$$

$$\int_{R_E}^{0}\frac{ds}{\sqrt{R_E^2 - s^2}} = \pm\sqrt{\frac{g}{R_E}}\int_0^{t_f} dt$$

$$\sqrt{\frac{g}{R_E}}t_f = \pm\sin^{-1}\left(\frac{s}{R_E}\right)\bigg|_{R_E}^{0} = \pm\sin^{-1}(1)$$

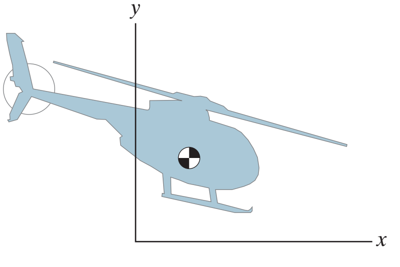
$$t_f = \pm\sqrt{\frac{R_E}{g}}\frac{\pi}{2} = \pm 1266\text{ s} = \pm 21.1\text{ min}$$

Problem 13.67 In a second test, the coordinates of the position (in m) of the helicopter in Active Example 13.6 are given as functions of time by

$x = 4 + 2t,$

$y = 4 + 4t + t^2.$

(a) What is the magnitude of the helicopter's velocity at $t = 3$ s?
(b) What is the magnitude of the helicopter's acceleration at $t = 3$ s?



Solution: We have

$$x = (4\text{ m}) + (2\text{ m/s})t, \quad y = (4\text{ m}) + (4\text{ m/s})t + (1\text{ m/s}^2)t^2,$$

$$v_x = (2\text{ m/s}), \quad v_y = (4\text{ m/s}) + 2(1\text{ m/s}^2)t,$$

$$a_x = 0, \quad a_y = 2(1\text{ m/s}^2).$$

At $t = 3$ s, we have

$$x = 10\text{ m}, \quad v_x = 2\text{ m/s}, \quad a_x = 0,$$

$$y = 25\text{ m}, \quad v_y = 10\text{ m/s}, \quad a_y = 2\text{ m/s}^2.$$

Thus

$$v = \sqrt{v_x^2 + v_y^2} = 10.2\text{ m/s} \quad \boxed{v = 10.2\text{ m/s.}}$$

$$a = \sqrt{0^2 + (2\text{ m/s}^2)^2} = 2\text{ m/s}^2 \quad \boxed{a = 2\text{ m/s}^2.}$$

Problem 13.68 In terms of a particular reference frame, the position of the center of mass of the F-14 at the time shown ($t = 0$) is $\mathbf{r} = 10\mathbf{i} + 6\mathbf{j} + 22\mathbf{k}$ (m). The velocity from $t = 0$ to $t = 4$ s is $\mathbf{v} = (52 + 6t)\mathbf{i} + (12 + t^2)\mathbf{j} - (4 + 2t^2)\mathbf{k}$ (m/s). What is the position of the center of mass of the plane at $t = 4$ s?

Solution:

$$\mathbf{r}_0 = 10\mathbf{i} + 6\mathbf{j} + 22\mathbf{k} \text{ m}$$

$$\mathbf{v} = (52 + 6t)\mathbf{i} + (12 + t^2)\mathbf{j} - (4 + 2t^2)\mathbf{k} \text{ m/s}$$

$$x_4 = \int_0^4 v_x\, dt = 52t + 3t^2 + x_0$$

$$x_4 = (52)(4) + 3(4)^2 + 10 \text{ m} = 266.0 \text{ m}$$

$$y_4 = \int_0^4 v_y\, dt = 12t + t^3/3 + y_0$$

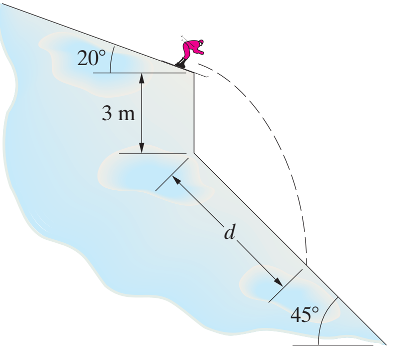
$$y_4 = 12(4) + (4)^3/3 + 6 \text{ m} = 75.3 \text{ m}$$

$$z_4 = \int_0^4 v_z\, dt = -(4t + 2t^3/3) + z_0$$

$$z_4 = -4(4) - 2(4)^3/3 + 22 = -36.7 \text{ m}$$

$$\mathbf{r}|_{t=4s} = 266\mathbf{i} + 75.3\mathbf{j} - 36.7\mathbf{k} \text{ (m)}$$

Problem 13.69 In Example 13.7, suppose that the angle between the horizontal and the slope on which the skier lands is 30° instead of 45°. Determine the distance d to the point where he lands.



Solution: The skier leaves the 20° surface at 10 m/s.

The equations are

$a_x = 0$, $a_y = -9.81 \text{ m/s}^2$,

$v_x = (10 \text{ m/s}) \cos 20^\circ$, $v_y = -(9.81 \text{ m/s}^2)t - (10 \text{ m/s}) \sin 20^\circ$

$s_x = (10 \text{ m/s}) \cos 20^\circ t$, $s_y = -\frac{1}{2}(9.81 \text{ m/s}^2)t^2 - (10 \text{ m/s}) \sin 20^\circ t$

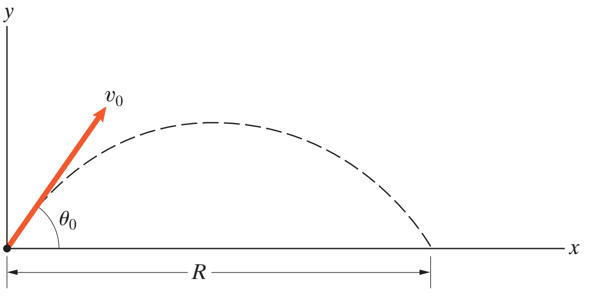
When he hits the slope, we have

$s_x = d \cos 30^\circ = (10 \text{ m/s}) \cos 20^\circ t$,

$s_y = (-3 \text{ m}) - d \sin 30^\circ = -\frac{1}{2}(9.81 \text{ m/s}^2)t^2 - (10 \text{ m/s}) \sin 20^\circ t$

Solving these two equations together, we find $t = 1.01$ s, $\boxed{d = 11.0 \text{ m.}}$

Problem 13.70 A projectile is launched from ground level with initial velocity $v_0 = 20$ m/s. Determine its range R if (a) $\theta_0 = 30^\circ$; (b) $\theta_0 = 45^\circ$ (c) $\theta_0 = 60^\circ$.



Solution: Set $g = 9.81$ m/s^2, $v_0 = 20$ m/s

$$a_y = -g,\ v_y = -gt + v_0 \sin\theta_0,\ s_y = -\frac{1}{2}gt^2 + v_0 \sin\theta_0 t$$

$$a_x = 0,\ v_x = v_0 \cos\theta_0,\ s_x = v_0 \cos\theta_0 t$$

When it hits the ground, we have

$$0 = -\frac{1}{2}gt^2 + v_0 \sin\theta_0 t \Rightarrow t = \frac{2v_0 \sin\theta_0}{g}$$

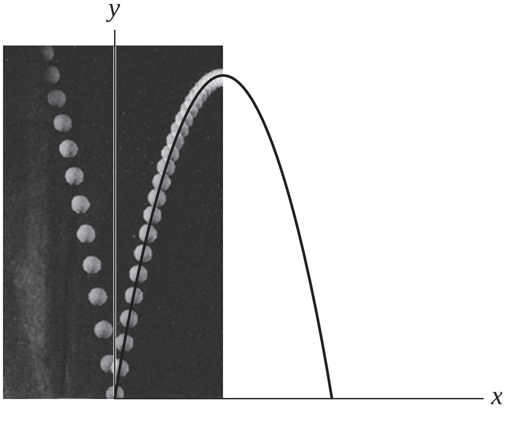
$$R = v_0 \cos\theta_0 t \Rightarrow R = \frac{{v_0}^2 \sin 2\theta_0}{g}$$

$$\Rightarrow \boxed{\begin{array}{ll} a) & \theta_0 = 30^\circ \Rightarrow R = 35.3 \text{ m} \\ b) & \theta_0 = 45^\circ \Rightarrow R = 40.8 \text{ m} \\ c) & \theta_0 = 60^\circ \Rightarrow R = 35.3 \text{ m} \end{array}}$$

Problem 13.71 Immediately after the bouncing golf ball leaves the floor, its components of velocity are $v_x = 0.662$ m/s and $v_y = 3.66$ m/s.

(a) Determine the horizontal distance from the point where the ball left the floor to the point where it hits the floor again.

(b) The ball leaves the floor at $x = 0$, $y = 0$. Determine the ball's y coordinate as a function of x. (The parabolic function you obtain is shown superimposed on the photograph of the ball.)



Solution: The governing equations are

$$a_x = 0, \quad a_y = -g,$$

$$v_x = v_{x0}, \quad v_y = -gt + v_{y0},$$

$$x = v_{x0}t, \quad y = -\frac{1}{2}gt^2 + v_{y0}t$$

(a) When it hits the ground again

$$0 = -\frac{1}{2}gt^2 + v_{y0}t \Rightarrow t = \frac{2v_{y0}}{g} \Rightarrow x = v_{x0}t = v_{x0}\left(\frac{2y_0}{g}\right) = \frac{2v_{x0}v_{y0}}{g}$$

$$x = \frac{2(0.662 \text{ m/s})(3.66 \text{ m/s})}{9.81 \text{ m/s}^2} \quad \boxed{x = 0.494 \text{ m.}}$$

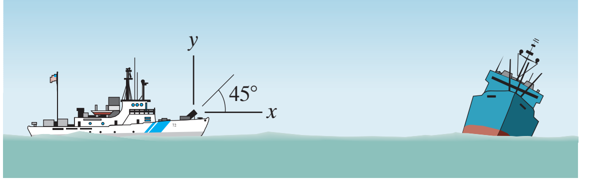
(b) At any point of the flight we have

$$t = \frac{x}{v_{x0}},\ y = -\frac{1}{2}g\left(\frac{x}{v_{x0}}\right)^2 + v_{y0}\left(\frac{x}{v_{x0}}\right)$$

$$y = -\frac{1}{2}\left(\frac{9.81 \text{ m/s}^2}{[0.662 \text{ m/s}]^2}\right)x^2 + \frac{3.66 \text{ m/s}}{0.662 \text{ m/s}}x$$

$$\boxed{y = -\left(\frac{11.2}{\text{m}}\right)x^2 + 5.53x.}$$

Problem 13.72 Suppose that you are designing a mortar to launch a rescue line from coast guard vessel to ships in distress. The light line is attached to a weight fired by the mortar. Neglect aerodynamic drag and the weight of the line for your preliminary analysis. If you want the line to be able to reach a ship 91 m away when the mortar is fired at 45° above the horizontal, what muzzle velocity is required?



Solution: From 13.70 we know that

$$R = \frac{{v_0}^2 \sin 2\theta_0}{g} \Rightarrow v_0 = \sqrt{\frac{Rg}{\sin 2\theta_0}}$$

$$\boxed{v_0 = \sqrt{\frac{(91)(9.81\ \text{m/s}^2)}{\sin(90^\circ)}} = 29.9\ \text{m/s}}$$

Problem 13.73 In Problem 13.72, what maximum height above the point where it was fired is reached by the weight?

Solution: From Problem 13.70 we have

$$v_y = -gt + v_0 \sin\theta_0, \quad s_y = -\frac{1}{2}gt^2 + v_0 \sin\theta_0 t$$

When we reach the maximum height,

$$0 = -gt + v_0 \sin\theta_0 \Rightarrow t = \frac{v_0 \sin\theta_0}{g}$$

$$h = -\frac{1}{2}gt^2 + v_0 \sin\theta_0 t \Rightarrow h = -\frac{1}{2}g\left(\frac{v_0 \sin\theta_0}{g}\right)^2 + v_0 \sin\theta_0 \left(\frac{v_0 \sin\theta_0}{g}\right)$$

$$h = \frac{{v_0}^2 \sin^2\theta_0}{2g}$$

Putting in the numbers we have

$$\boxed{h = \frac{(29.9\ \text{m/s})^2 \sin^2(45^\circ)}{2(9.81\ \text{m/s}^2)} = 22.78\ \text{m}}$$

Problem 13.74 When the athlete releases the shot, it is 1.82 m above the ground and its initial velocity is $v_0 = 13.6$ m/s. Determine the horizontal distance the shot travels from the point of release to the point where it hits the ground.

Solution: The governing equations are

$a_x = 0,$

$v_x = v_0 \cos 30^\circ,$

$s_x = v_0 \cos 30^\circ t,$

$a_y = -g$

$v_y = -gt + v_0 \sin 30^\circ$

$s_y = -\frac{1}{2}gt^2 + v_0 \sin 30^\circ t + h$

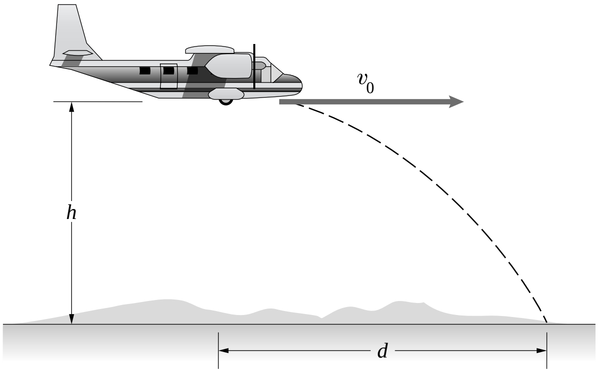
When it hits the ground, we have

$s_x = (13.6 \text{ m/s}) \cos 30^\circ t,$

$s_y = 0 = -\frac{1}{2}(9.81 \text{ m/s}^2)t^2 + (13.6 \text{ m/s}) \sin 30^\circ t + 1.82 \text{ m}.$

Solving these two equations, we find $t = 1.62$ s, $\boxed{s_x = 19.0 \text{ m.}}$

Problem 13.75 A pilot wants to drop survey markers at remote locations in the Australian outback. If he flies at a constant velocity $v_0 = 40$ m/s at altitude $h = 30$ m and the marker is released with zero velocity relative to the plane, at what horizontal d from the desired impact point should the marker be released?



Solution: We want to find the horizontal distance traveled by the marker before it strikes the ground (y goes to zero for $t > 0$.)

$a_x = 0 \qquad a_y = -g$

$v_x = v_{x0} \qquad v_y = v_{y0} - gt$

$x = x_0 + v_{x0}t \quad y = y_0 + v_{y0}t - gt^2/2$

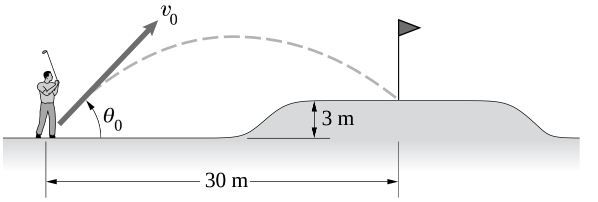
From the problem statement, $x_0 = 0$, $v_{y0} = 0$, $v_{x0} = 40$ m/s, and $y_0 =$ 30 m The equation for y becomes

$y = 30 - (9.81)t^2/2$

Solving with $y = 0$, we get $t_f = 2.47$ s.
Substituting this into the equation for x, we get

$\underline{x_f = 40t_f = 98.9 \text{ m}}$

Problem 13.76 If the pitching wedge the golfer is using gives the ball an initial angle $\theta_0 = 50°$, what range of velocities v_0 will cause the ball to land within 3 m of the hole? (Assume the hole lies in the plane of the ball's trajectory).



Solution: Set the coordinate origin at the point where the golfer strikes the ball. The motion in the horizontal (x) direction is given by $a_x = 0$, $V_x = V_0 \cos\theta_0$, $x = (V_0 \cos\theta_0)t$. The motion in the vertical (y) direction is given by

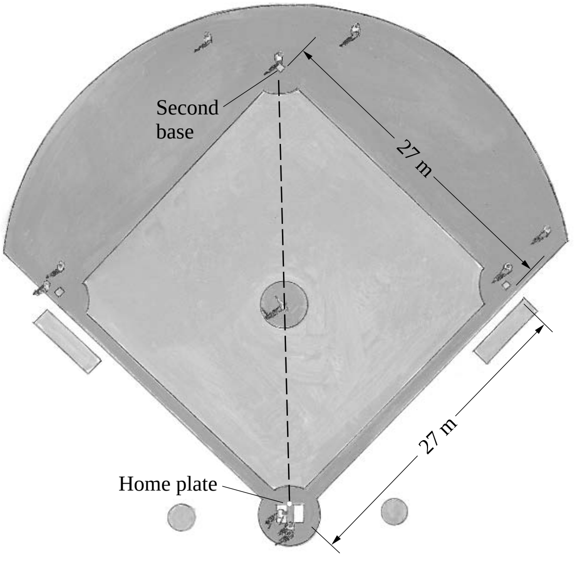
$$a_y = -g, \quad V_y = V_0 \sin\theta_0 - gt, \quad y = (V_0 \sin\theta_0)t - \frac{gt^2}{2}.$$

From the x equation, we can find the time at which the ball reaches the required value of x (27 or 33 metres). This time is

$$t_f = x_f/(V_0 \cos\theta_0).$$

We can substitute this information the equation for Y with $Y_f = 3\,\text{m}$ and solve for V_0. The results are: For hitting (27,3) metre, $V_0 = 31.2$ m/s. For hitting (33,3) metre, $V_0 = 34.2$ m/s.

Problem 13.77 A batter strikes a baseball 0.9 m above home plate and pops it up. The second baseman catches it 1.8 m above second base 3.68 s after it was hit. What was the ball's initial velocity, and what was the angle between the ball's initial velocity vector and the horizontal?



Solution: The equations of motion $g = 9.81$ m/s^2

$$a_x = 0 \qquad a_y = -g$$

$$v_x = v_0 \cos\theta_0 \qquad v_y = -gt + v_0 \sin\theta_0$$

$$s_x = v_0 \cos\theta_0 t \qquad s_y = -\frac{1}{2}gt^2 + v_0 \sin\theta_0 t + 0.9\text{ m}$$

When the second baseman catches the ball we have

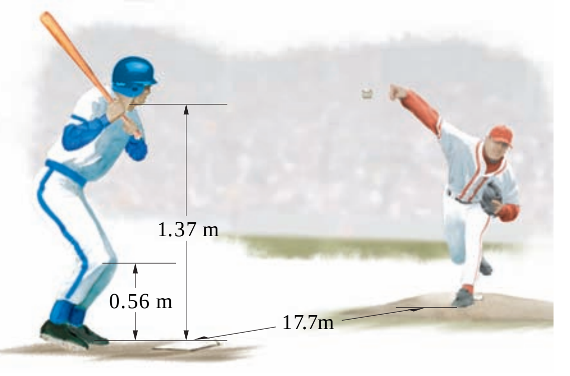
$$38.2\text{ m} = v_0 \cos\theta_0 (3.68\text{ s})$$

$$1.8\text{ m} = \frac{1}{2} - (9.81\text{ m/s}^2)(3.68\text{ s})^2 + v_0 \sin\theta_0 (3.68\text{ s})$$

Solving simultaneously we find

$$\boxed{v_0 = 21\,\text{m/s}, \qquad \theta_0 = 60.4°}$$

Problem 13.78 A baseball pitcher releases a fastball with an initial velocity $v_0 = 144.8$ km/h. Let θ be the initial angle of the ball's velocity vector above the horizontal. When it is released, the ball is 1.83 m above the ground and 17.7 m from the batter's plate. The batter's strike zone extends from 0.56 m above the ground to 1.37 m above the ground. Neglecting aerodynamic effects, determine whether the ball will hit the strike zone (a) if $\theta = 1^\circ$; (b) if $\theta = 2^\circ$.



Solution: The initial velocity is $v_0 = 144.8$ km/h $= 40.3$ m/s. The velocity equations are

(1) $\dfrac{dv_x}{dt} = 0$, from which $v_x = v_0 \cos\theta$.

(2) $\dfrac{dv_y}{dt} = -g$, from which $v_y = -gt + v_0 \sin\theta$.

(3) $\dfrac{dx}{dt} = v_0 \cos\theta$, from which $x(t) = v_0 \cos\theta t$, since the initial position is zero.

(4) $\dfrac{dy}{dt} = -gt + v_0 \sin\theta$, from which

$$y(t) = -\frac{g}{2}t^2 + v_0 \sin\theta t + 1.83,$$

since the initial position is $y(0) = 1.83$ m. At a distance $d = 17.7$ m, the height is h. The time of passage across the home plate is $x(t_p) = d = v_0 \cos\theta t_p$, from which

$$t_p = \frac{d}{v_0 \cos\theta}.$$

Substitute:

$$y(t_p) = h = -\frac{g}{2}\left(\frac{d}{v_0 \cos\theta}\right)^2 + d\tan\theta + 1.83.$$

For $\theta = 1^\circ$, $\boxed{h = 1.2 \text{ m}}$, <u>Yes</u>, the pitcher hits the strike zone.

For $\theta = 2^\circ$, $\boxed{h = 1.5 \text{ m}}$ <u>No</u>, the pitcher misses the strike zone.

Problem 13.79 In Problem 13.78, assume that the pitcher releases the ball at an angle $\theta = 1^\circ$ above the horizontal, and determine the range of velocities v_0 (in m/s) within which he must release the ball to hit the strike zone.

Solution: From the solution to Problem 13.78,

$$h = -\frac{g}{2}\left(\frac{d}{v_0 \cos\theta}\right)^2 + d\tan\theta + 1.83,$$

where $d = 17.7$ m, and $1.37 \geq h \geq 0.56$ m. Solve for the initial velocity:

$$v_0 = \sqrt{\frac{gd^2}{2\cos^2\theta(d\tan\theta + 1.83 - h)}}.$$

For $h = 1.37$, $v_0 = 44.74$ m/s. For $h = 0.56$ m, $v_0 = 31.15$ m/s. The pitcher will hit the strike zone for velocities of release of

$$\boxed{31.15 \leq v_0 \leq 44.74 \text{ m/s}},$$

and a release angle of $\theta = 1^\circ$. *Check*: The range of velocities in miles per hour is 112.1 km/h $\leq v_0 \leq$ 161.1 km/h, which is within the range of major league pitchers, although the 160.9 km upper value is achievable only by a talented few (Nolan Ryan, while with the Houston Astros, would occasionally in a game throw a 168.9 km fast ball, as measured by hand held radar from behind the plate).

Problem 13.80 A zoology student is provided with a bow and an arrow tipped with a syringe of sedative and is assigned to measure the temperature of a black rhinoceros (*Diceros bicornis*). The range of his bow when it is fully drawn and aimed 45° above the horizontal is 100 m. A truculent rhino charges straight toward him at 30 km/h. If he fully draws his bow and aims 20° above the horizontal, how far away should the rhino be when the student releases the arrow?

Solution: The strategy is (a) to determine the range and flight time of the arrow when aimed 20° above the horizontal, (b) to determine the distance traveled by the rhino during this flight time, and then (c) to add this distance to the range of the arrow. Neglect aerodynamic drag on the arrow. The equations for the trajectory are: Denote the constants of integration by V_x, V_y, C_x, C_y, and the velocity of the arrow by V_A.

(1) $\dfrac{dv_x}{dt} = 0$, from which $v_x = V_x$. At $t = 0$, $V_x = V_A \cos\theta$.

(2) $\dfrac{dv_y}{dt} = -g$, from which $v_y = -gt + V_y$. At $t = 0$, $V_y = V_A \sin\theta$.

(3) $\dfrac{dx}{dt} = v_x = V_A \cos\theta$, from which $x(t) = V_A \cos\theta t + C_x$. At $t = 0$, $x(0) = 0$, from which $C_x = 0$.

(4) $\dfrac{dy}{dt} = v_y = -gt + V_A \sin\theta$, from which

$$y = -\frac{g}{2}t^2 + V_A \sin\theta t + C_y.$$

At $t = 0$, $y = 0$, from which $C_y = 0$. The time of flight is given by

$$y(t_{\text{flight}}) = 0 = \left(-\frac{g}{2}t_{\text{flight}} + V_A \sin\theta\right) t_{\text{flight}},$$

from which

$$t_{\text{flight}} = \frac{2V_A \sin\theta}{g}.$$

The range is given by

$$x(t_{\text{flight}}) = R = V_A \cos\theta t_{\text{flight}} = \frac{2V_A^2 \cos\theta \sin\theta}{g}.$$

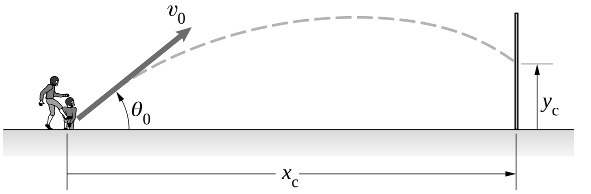
The maximum range (100 meters) occurs when the arrow is aimed 45° above the horizon. Solve for the arrow velocity: $V_A = \sqrt{gR_{\max}} = 31.3$ m/s. The time of flight when the angle is 20° is

$$t_{\text{flight}} = \frac{2V_A \sin\theta}{g} = 2.18 \text{ s},$$

and the range is $R = V_A \cos\theta t_{\text{flight}} = 64.3$ m. The speed of the rhino is 30 km/h $= 8.33$ m/s. The rhino travels a distance $d = 8.33(2.18) = 18.2$ m. The required range when the arrow is released is

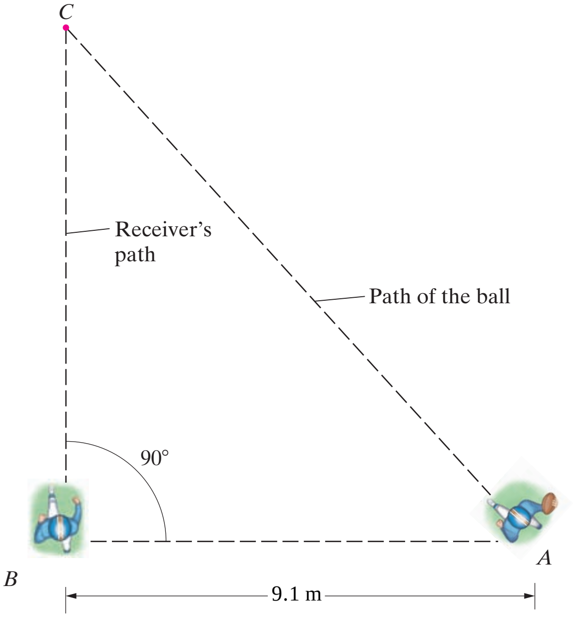
$\boxed{d + R = 82.5 \text{ m}}$.

Problem 13.81 The crossbar of the goalposts in American football is $y_c = 3.05$ m above the ground. To kick a field goal, the ball must make the ball go between the two uprights supporting the crossbar and be above the crossbar when it does so. Suppose that the kicker attempts a 36.58 m field goal, and kicks the ball with an initial velocity $v_0 = 21.3$ m/s and $\theta_0 = 40°$. By what vertical distance does the ball clear the crossbar?



Solution: Set the coordinate origin at the point where the ball is kicked. The x (horizontal) motion of the ball is given by $a_x = 0$, $V_x = V_0 \cos\theta_0$, $x = (V_0 \cos\theta_0)t$. The y motion is given by $a_y = -g$, $V_y = V_0 \sin\theta_0 - gt$, $y = (V_0 \sin\theta_0)t - \dfrac{gt^2}{2}$. Set $x = x_c = 36.58$ m and find the time t_c at which the ball crossed the plane of the goal posts. Substitute this time into the y equation to find the y coordinate Y_B of the ball as it passes over the crossbar. Substituting in the numbers ($g = 9.81$ m/s^2), we get $t_c = 2.24$ s and $y_B = 6.11$ m. Thus, the ball clears the crossbar by 3.07 m.

Problem 13.82 An American football quarterback stands at A. At the instant the quarterback throws the football, the receiver is at B running at 6.1 m/s toward C, where he catches the ball. The ball is thrown at an angle of 45° above the horizontal, and it is thrown and caught at the same height above the ground. Determine the magnitude of the ball's initial velocity and the length of time it is in the air.



Solution: Set x as the horizontal motion of the football, y as the vertical motion of the football and z as the horizontal motion of the receiver. Set $g = 9.81$ m/s^2, $\theta_0 = 45°$. We have

$$a_z = 0, \; v_z = 6.1 \text{ m/s}, \; s_z = (6.1 \text{ m/s})t$$

$$a_y = -g, \; v_y = -gt + v_0 \sin\theta_0, \; s_y = -\frac{1}{2}gt^2 + v_0 \sin\theta_0 t$$

$$a_x = 0, \; v_x = v_0 \cos\theta_0, \; s_x = v_0 \cos\theta_0 t$$

When the ball is caught we have

$$s_z = (6.1 \text{ m/s})t$$

$$0 = -\frac{1}{2}gt^2 + v_0 \sin\theta_0 t$$

$$s_x = v_0 \cos\theta_0 t$$

$$s_x{}^2 = s_z{}^2 + (9.1 \text{ m})^2$$

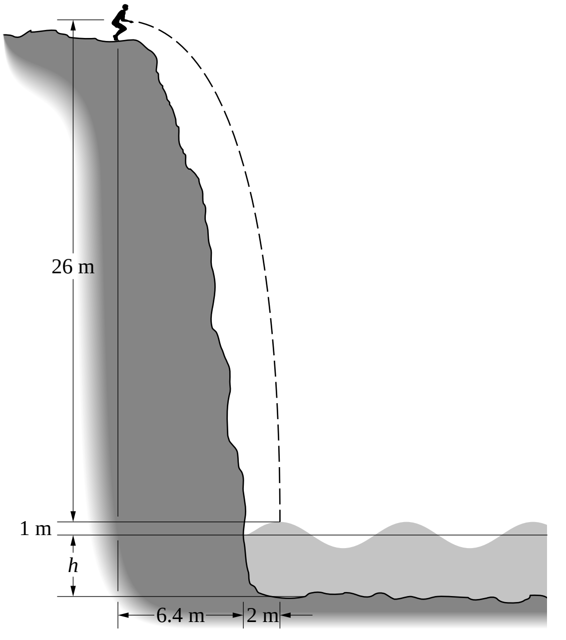
We can solve these four equations for the four unknowns s_x, s_z, v_0, t

We find $\boxed{t = 1.67 \text{ s}, \quad v_0 = 11.6 \text{ m/s}}$

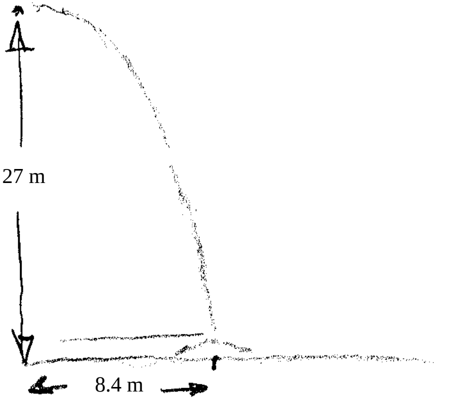
Problem 13.83 The cliff divers of Acapulco, Mexico must time their dives that they enter the water at the crest (high point) of a wave. The crests of the waves are 1 m above the mean water depth $h = 4$ m. The horizontal velocity of the waves is equal to $\sqrt{gh}$. The diver's aiming point is 2 m out from the base of the cliff. Assume that his velocity is horizontal when he begins the dive.

(a) What is the magnitude of the driver's velocity when he enters the water?

(b) How far from his aiming point must a wave crest be when he dives in order for him to enter the water at the crest?



Solution:



$t = 0$, $v_{y\Delta} = 0$, $y = 27$ m, $x_0 = 0$

$a_y = -g = -9.81$ m/s

$V_y = \cancel{V_{y0}}^{0} - gt$

$y = y_0 - gt^{2/2}$

$y = 1$ m at t_{IMPACT}

for an ideal dive to hit the crest of the wave

$\underline{t_1 = t_{\text{IMPACT}} = 2.30 \text{ s}}$

$\underline{V_y(t_1) = 22.59 \text{ m/s}}$

$a_x = 0$

$V_x = V_{x_0}$

$X_I = V_{x_0} t_1 + X_0$

At impact $X_I = 8.4$ m.

For impact to occur as planned, then

$V_x = 8.4/t_1 = 3.65$ m/s = constant

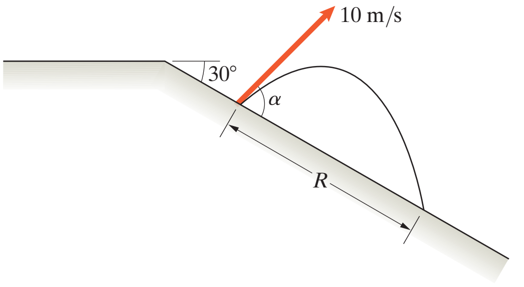
The velocity at impact is

$(a)\ |V| = \sqrt{(V_x)^2 + [V_y(t_1)]^2} = 22.9$ m/s

The wave moves at $\sqrt{gh} = 6.26$ m/s.

The wave crest travels 2.30 seconds while the diver is in their $s = \sqrt{gh} t_1 = 14.4$ m.

Problem 13.84 A projectile is launched at 10 m/s from a sloping surface. The angle $\alpha = 80^\circ$. Determine the range R.



Solution: Set $g = 9.81\ \text{m/s}^2$, $v_0 = 10$ m/s.

The equations of motion are

$$a_x = 0,\ \ v_x = v_0 \cos(80^\circ - 30^\circ),\ \ s_x = v_0 \cos 50^\circ t$$

$$a_y = -g,\ \ v_y = -gt + v_0 \sin(80^\circ - 30^\circ)t,\ \ s_y = -\frac{1}{2}gt^2 + v_0 \sin 50^\circ t$$

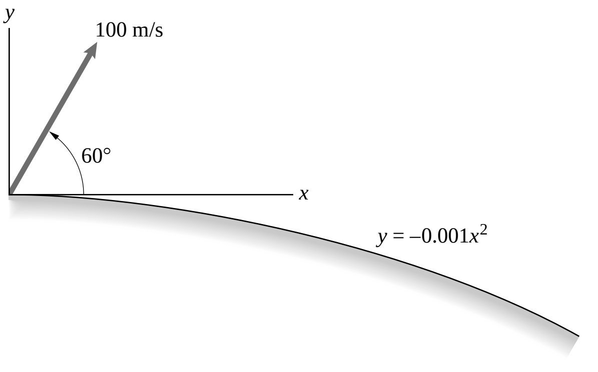
When the projectile hits we have

$$R \cos 30^\circ = v_0 \cos 50^\circ t$$

$$\Rightarrow \boxed{t = 2.32\ \text{s},\quad R = 17.21\ \text{m}}$$

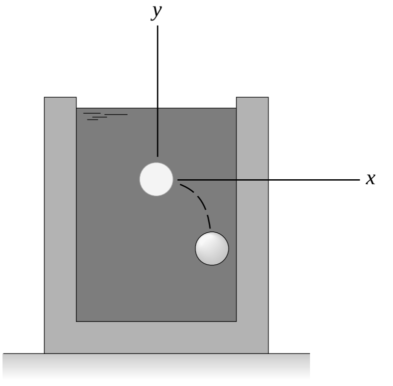
$$-R \sin 30^\circ = -\frac{1}{2}gt^2 + v_0 \sin 50^\circ t$$

Problem 13.85 A projectile is launched at 100 m/s at 60° above the horizontal. The surface on which it lands is described by the equation shown. Determine the point of impact.



Solution: The motion in the x direction is $a_x = 0$, $v_x = V_0 \cos\theta_0$, $x = (V_0 \cos\theta_0)t$, and the motion in the y direction is given by $a_y = -g$, $v_y = (V_0 \sin\theta_0) - gt$, $y = (V_0 \sin\theta_0)t - gt^2/2$. We know that $V_0 = 100$ m/s and $\theta_0 = 60^\circ$. The equation of the surface upon which the projectile impacts is $y = -0.001x^2$. Thus, the time of impact, t_I, can be determined by substituting the values of x and y from the motion equations into the equation for the surface. Hence, we get $(V_0 \sin\theta_0)t_I - g\dfrac{t_I^2}{2} = -0.001(V_0 \cos\theta_0)^2 t_I^2$. Evaluating with the known values, we get $t_I = 6.37$ s Substituting this value into the motion equations reveals that impact occurs at $(x, y) = (318.4, -101.4)$ m.

Problem 13.86 At $t = 0$, a steel ball in a tank of oil is given a horizontal velocity $\mathbf{v} = 2\mathbf{i}$ (m/s). The components of the ball's acceleration in m/s^2 are $a_x = -1.2v_x$, $a_y = -8 - 1.2v_y$, $a_z = -1.2v_z$. What is the velocity of the ball at $t = 1$ s?



Solution: Assume that the effect of gravity is included in the given accelerations. The equations for the path are obtained from:

(1) $\dfrac{dv_x}{dt} = a_x = -1.2v_x$. Separate variables and integrate:

$$\frac{dv_x}{v_x} = -1.2\,dt,$$

from which $\ln(v_x) = -1.2t + V_x$. At $t = 0$, $v_x(0) = 2$, from which

$$\ln\left(\frac{v_x}{2}\right) = -1.2t.$$

Inverting: $v_x(t) = 2e^{-1.2t}$.

(2) $\dfrac{dv_y}{dt} = a_y = -8 - 1.2v_y$. Separate variables and integrate:

$$\frac{dv_y}{\dfrac{8}{1.2} + v_y} = -1.2\,dt,$$

from which

$$\ln\left(\frac{8}{1.2} + v_y\right) = -1.2t + V_y.$$

At $t = 0$, $v_y(0) = 0$, from

$$\ln\left(1 + \frac{1.2}{8}v_y\right) = -1.2t.$$

Inverting: $v_y(t) = \dfrac{8}{1.2}(e^{-1.2t} - 1)$.

(3) $\dfrac{dv_z}{dt} = a_z = -1.2v_z$, from which $\ln(v_z) = -1.2t + V_z$. Invert to obtain $v_z(t) = V_z e^{-1.2t}$. At $t = 0$, $v_z(0) = 0$, hence $V_z = 0$ and $v_z(t) = 0$. At $t = 1$ second,

$\boxed{v_x(1) = 2e^{-1.2} = 0.6024 \text{ m/s}}$, and

$\boxed{v_y(1) = -\left(\dfrac{8}{12}\right)(1 - e^{-1.2}) = -4.66 \text{ m/s}}$, or

$\boxed{\mathbf{v} = 0.602\mathbf{i} - 4.66\mathbf{j} \text{ (m/s)}}$.

Problem 13.87 In Problem 13.86, what is the position of the ball at $t = 1$ s relative to its position at $t = 0$?

Solution: Use the solution for the velocity components from Problem 13.86. The equations for the coordinates:

(1) $\frac{dx}{dt} = v_x = 2e^{-1.2t}$, from which

$$x(t) = -\left(\frac{2}{1.2}\right)e^{-1.2t} + C_x.$$

At $t = 0$, $x(0) = 0$, from which

$$x(t) = \left(\frac{2}{1.2}\right)(1 - e^{-1.2t}).$$

(2) $\frac{dy}{dt} = \left(\frac{8}{1.2}\right)(e^{-1.2t} - 1)$, from which

$$y(t) = -\left(\frac{8}{1.2}\right)\left(\frac{e^{-1.2t}}{1.2} + t\right) + C_y.$$

At $t = 0$, $y(0) = 0$, from which

$$y(t) = -\left(\frac{8}{1.2}\right)\left(\frac{e^{-1.2t}}{1.2} + t - \frac{1}{1.2}\right).$$

(3) Since $v_z(0) = 0$ and $z(0) = 0$, then $z(t) = 0$. At $t = 1$,

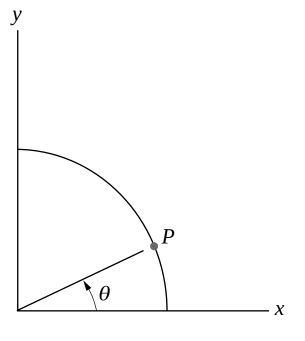
$$\boxed{x(1) = \left(\frac{2}{1.2}\right)(1 - e^{-1.2}) = 1.165 \text{ m}}.$$

$$\boxed{y(1) = -\left(\frac{8}{1.2}\right)\left(\frac{e^{-1.2}}{1.2} + 1 - \frac{1}{1.2}\right) = -2.784 \text{ m}}, \quad \text{or}$$

$$\boxed{\mathbf{r} = 1.165\mathbf{i} - 2.784\mathbf{j} \text{ (m)}}.$$

Problem 13.88 The point P moves along a circular path with radius R. Show that the magnitude of its velocity is $|v| = R|d\theta/dt|$.

Strategy: Use Eqs. (13.23).



Solution:

$$x = R\cos\theta$$

$$y = R\sin\theta$$

$$v_x = -R\sin\theta\left(\frac{d\theta}{dt}\right)$$

$$v_y = R\cos\theta\left(\frac{d\theta}{dt}\right)$$

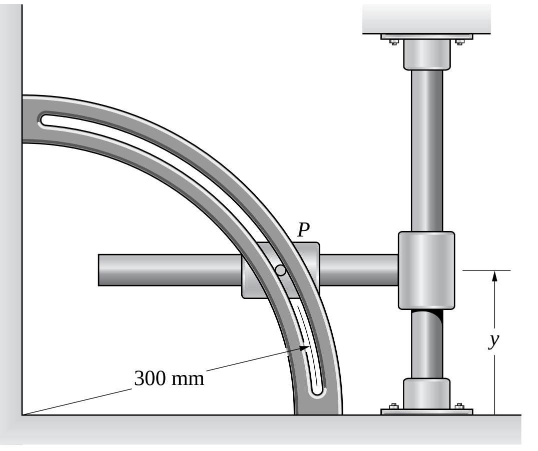
$$|\mathbf{V}| = \sqrt{V_x^2 + V_y^2}$$

$$|\mathbf{V}| = \sqrt{R^2\sin^2\theta\left(\frac{d\theta}{dt}\right)^2 + R^2\cos^2\theta\left(\frac{d\theta}{dt}\right)^2}$$

$$|\mathbf{V}| = \sqrt{R^2\left(\frac{d\theta}{dt}\right)^2(\sin^2\theta + \cos^2\theta)}$$

$$|\mathbf{V}| = R\left|\frac{d\theta}{dt}\right|$$

Problem 13.89 If $y = 150$ mm, $\dfrac{dy}{dt} = 300$ mm/s, and $\dfrac{d^2y}{dt^2} = 0$, what are the magnitudes of the velocity and acceleration of point P?



Solution: The equation for the location of the point P is $R^2 = x^2 + y^2$, from which $x = (R^2 - y^2)^{\frac{1}{2}} = 0.2598$ m, and

$$\frac{dx}{dt} = -\left(\frac{y}{x}\right)\left(\frac{dy}{dt}\right) = -0.1732 \text{ m/s},$$

$$\frac{d^2x}{dt^2} = -\frac{1}{x}\left(\frac{dy}{dt}\right)^2 + \frac{y}{x^2}\left(\frac{dx}{dt}\right)\left(\frac{dy}{dt}\right) - \left(\frac{y}{x}\right)\left(\frac{d^2y}{dt^2}\right)$$

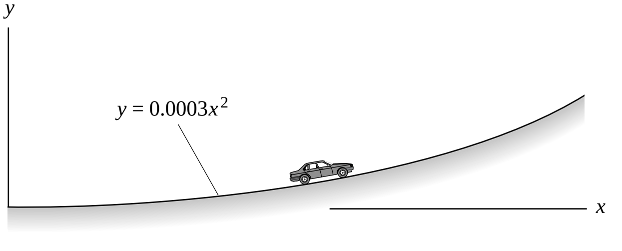
$$= -0.4619 \text{ m/s}^2.$$

The magnitudes are:

$$\boxed{|v_P| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = 0.3464 \text{ m/s}}$$

$$\boxed{|a_p| = \sqrt{\left(\frac{d^2x}{dt^2}\right)^2 + \left(\frac{d^2y}{dt^2}\right)^2} = 0.4619 \text{ m/s}^2}$$

Problem 13.90 A car travels at a constant speed of 100 km/h on a straight road of increasing grade whose vertical profile can be approximated by the equation shown. When the car's horizontal coordinate is $x = 400$ m, what is the car's acceleration?



Solution: Denote $C = 0.0003$ and $V = 100$ km/h $= 27.78$ m/s. The magnitude of the constant velocity is

$$V = \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}$$

The equation for the road is $y = Cx^2$ from which

$$\frac{dy}{dt} = 2Cx\left(\frac{dx}{dt}\right).$$

Substitute and solve:

$$\left|\frac{dx}{dt}\right| = \frac{V}{\sqrt{(2Cx)^2 + 1}} = 27.01 \text{ m/s}.$$

$\dfrac{dx}{dt}$ is positive (car is moving to right in sketch). The acceleration is

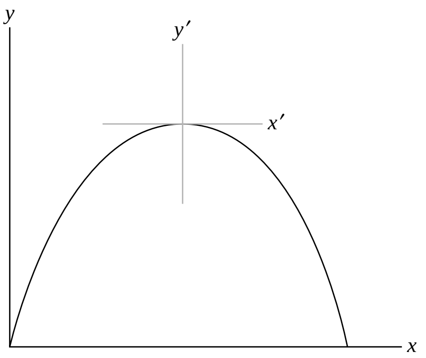
$$\boxed{\begin{aligned}\frac{d^2x}{dt^2} &= \frac{d}{dt}\left(\frac{V}{\sqrt{(2Cx)^2+1}}\right) = \frac{-4C^2Vx}{((2Cx)^2+1)^{\frac{3}{2}}}\left(\frac{dx}{dt}\right) \\ &= -0.0993 \text{ m/s}^2\end{aligned}}.$$

$$\boxed{\frac{d^2y}{dt^2} = 2C\left(\frac{dx}{dt}\right)^2 + 2Cx\left(\frac{d^2x}{dt^2}\right) = 0.4139 \text{ m/s}^2}, \quad \text{or}$$

$$\boxed{\mathbf{a} = -0.099\mathbf{i} + 0.414\mathbf{j} (\text{m/s}^2)}$$

Problem 13.91 Suppose that a projectile has the initial conditions shown in Fig. 13.12. Show that in terms of the $x'y'$ coordinate system with its origin at the highest point of the trajectory, the equation describing the trajectory is

$$y' = -\frac{g}{2v_0^2\cos^2\theta_0}(x')^2.$$



Solution: The initial conditions are $t = 0$, $x(0) = 0$, $y(0) = 0$, $v_x(0) = v_0\cos\theta_0$, and $v_y(0) = v_0\sin\theta_0$. The accelerations are $a_x(t) = 0$, $a_y(t) = -g$. The path of the projectile in the x, y system is obtained by solving the differential equations subject to the initial conditions:

$$x(t) = (v_0\cos\theta_0)t,\ y(t) = -\frac{g}{2}t^2 + (v_0\sin\theta_0)t$$

Eliminate t from the equations by substituting

$$t = \frac{x}{v_0\cos\theta_0}$$

to obtain

$$y(x) = -\frac{gx^2}{2v_0^2\cos^2\theta_0} + x\tan\theta_0.$$

At the peak,

$$\left|\frac{dy}{dx}\right|_{\text{peak}} = 0,$$

from which

$$x_p = \frac{v_0^2\cos\theta_0\sin\theta_0}{g},$$

and $\quad y_p = \dfrac{v_0^2\sin^2\theta_0}{2g}.$

The primed coordinates: $y' = y - y_p \quad x' = x - x_p$. Substitute and reduce:

$$y' = -\frac{g(x'+x_p)^2}{2v_0^2\cos^2\theta_0} + (x'+x_p)\tan\theta_0 - y_p.$$

$$y' = -\frac{g}{2v_0^2\cos^2\theta_0}((x')^2 + x_p^2 + 2x'x_p) + (x'+x_p)\tan\theta_0$$

$$-\frac{v_0^2\sin^2\theta_0}{2g}.$$

Substitute $x_p = \dfrac{v_0^2\cos\theta_0\sin\theta_0}{g}$,

$$y' = \frac{-g(x')^2}{2v_0^2\cos^2\theta_0} - \frac{v_0^2\sin^2\theta_0}{2g} - x'\tan\theta_0 + x'\tan\theta_0 + \frac{v_0^2\sin^2\theta_0}{g}$$

$$-\frac{v_0^2\sin^2\theta_0}{2g}.$$

$$\boxed{y' = -\frac{g}{2v_0^2\cos^2\theta_0}(x')^2}$$

Problem 13.92 The acceleration components of a point are $a_x = -4\cos 2t$, $a_y = -4\sin 2t$, $a_z = 0$. At $t = 0$, its position and velocity are $\mathbf{r} = \mathbf{i}$, $\mathbf{v} = 2\mathbf{j}$. Show that (a) the magnitude of the velocity is constant; (b) the velocity and acceleration vectors are perpendicular; (c) the magnitude of the acceleration is constant and points toward the origin; (d) the trajectory of a point is a circle with its center at the origin.

Solution: The equations for the path are

(1) $\dfrac{dv_x}{dt} = a_x = -4\cos(2t)$, from which $v_x(t) = -2\sin(2t) + V_x$. At $t = 0$, $v_x(0) = 0$, from which $V_x = 0$. $\dfrac{dx}{dt} = v_x = -2\sin(2t)$, from which $x(t) = \cos(2t) + C_x$. At $t = 0$, $x(0) = 1$, from which $C_x = 0$.

(2) $\dfrac{dv_y}{dt} = a_y = -4\sin(2t)$, from which $v_y(t) = 2\cos(2t) + V_y$. At $t = 0$, $v_y(0) = 2$, from which $V_y = 0$. $\dfrac{dy}{dt} = v_y = 2\cos(2t)$, from which $y(t) = \sin(2t) + C_y$. At $t = 0$, $y(0) = 0$, from which $C_y = 0$.

(3) For $a_z = 0$ and zero initial conditions, it follows that $v_z(t) = 0$ and $z(t) = 0$.

(a) The magnitude of the velocity is

$$|\mathbf{v}| = \sqrt{(-2\sin(2t))^2 + (2\cos(2t))^2} = 2 = \text{const.}$$

(b) The velocity is $\mathbf{v}(t) = -\mathbf{i}2\sin(2\mathbf{t}) + \mathbf{j}2\cos(2t)$. The acceleration is $\mathbf{a}(t) = -\mathbf{i}4\cos(2t) - \mathbf{j}4\sin(2t)$. If the two are perpendicular, the dot product should vanish: $\mathbf{a}(t)\cdot\mathbf{v}(t) = (-2\sin(2t))(-4\cos(2t)) + (2\cos(2t))(-4\sin(2t)) = 0$, and it does

(c) The magnitude of the acceleration:

$$\boxed{|\mathbf{a}| = \sqrt{(-4\cos(2t))^2 + (-4\sin(2t))^2} = 4 = \text{const}}.$$

The unit vector parallel to the acceleration is

$$\mathbf{e} = \frac{\mathbf{a}}{|\mathbf{a}|} = -\mathbf{i}\cos(2t) - \mathbf{j}\sin(2\mathbf{t}),$$

which always points to the origin.

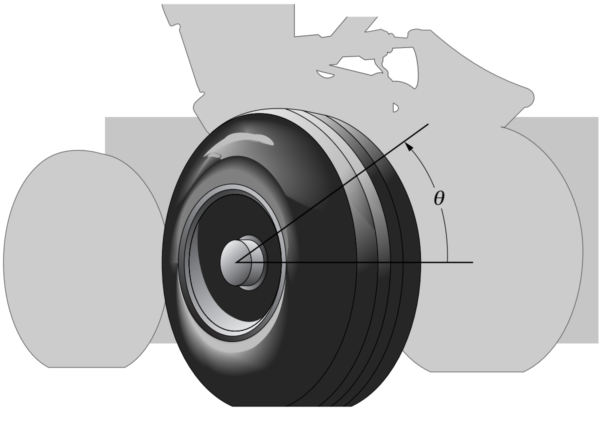
(d) The trajectory path is $x(t) = \cos(2t)$ and $y(t) = \sin(2t)$. These satisfy the condition for a circle of radius 1:

$$\boxed{1 = x^2 + y^2}$$

Problem 13.93 When an airplane touches down at $t = 0$, a stationary wheel is subjected to a constant angular acceleration $\alpha = 110$ rad/s^2 until $t = 1$ s.

(a) What is the wheel's angular velocity at $t = 1$ s?

(b) At $t = 0$, the angle $\theta = 0$. Determine θ in radians and in revolutions at $t = 1$ s.



Solution:

$\alpha = 110$ rad/s^2

$\omega = \alpha t + \omega_0$

$\theta = (\frac{1}{2}\alpha t^2) + \omega_0 t + \theta_0$

From the problem statement, $\omega_0 = \theta_0 = 0$

(a) At $t = 1$ s,

$\underline{\omega = (110)(1) + 0 = 110 \text{ rad/s}}$

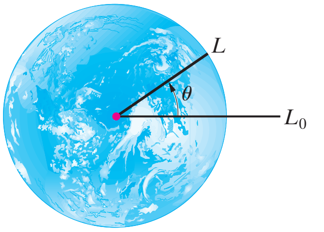
(b) At $t = 1$ s,

$\underline{\theta = 110(1)^2/2 = 55 \text{ radians}}$ (8.75 revolutions)

Problem 13.94 Let L be a line from the center of the earth to a fixed point on the equator, and let L_0 be a fixed reference direction. The figure views the earth from above the north pole.

(a) Is $d\theta/dt$ positive or negative? (Remember that the sun rises in the east.)

(b) Determine the approximate value of $d\theta/dt$ in rad/s and use it to calculate the angle through which the earth rotates in one hour.



Solution:

(a) $\boxed{\dfrac{d\theta}{dt} > 0.}$

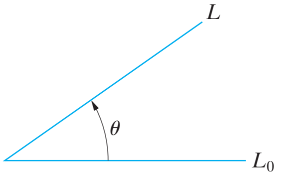
(b) $\dfrac{d\theta}{dt} \approx \dfrac{2\pi \text{ rad}}{24(3600)\ s} = 7.27 \times 10^{-5}$ rad/s.

In one hour $\Delta\theta \approx (7.27 \times 10^{-5} \text{ rad/s})(1 \text{ hr}) \left(\dfrac{3600 \text{ s}}{1 \text{ hr}}\right)$

$= 0.262 \text{ rad} = 15^\circ$

$$\boxed{\frac{d\theta}{dt} \approx 7.27 \times 10^{-5} \text{ rad/s}, \ \ \Delta\theta \approx 15^\circ.}$$

Problem 13.95 The angular acceleration of the line L relative to the line L_0 is given as a function of time by $\alpha = 2.5 - 1.2t$ rad/s^2. At $t = 0, \theta = 0$ and the angular velocity of L relative to L_0 is $\omega = 5$ rad/s. Determine θ and ω at $t = 3$ s.



Solution:

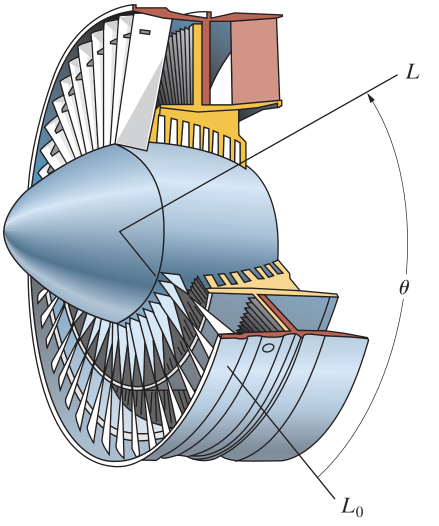
$\alpha = 2.5 - 1.2t$

$\omega = 2.5t - 0.6t^2 + 5$

$\theta = 1.25t^2 - 0.2t^3 + 5t$

$$\Rightarrow \boxed{\begin{array}{l} \theta(3) = 1.25(3)^2 - 0.2(3)^3 + 5(3) = 20.85 \text{ rad} \\ \omega(3) = 2.5(3) - 0.6(3)^2 + 5 = 7.1 \text{ rad/s} \end{array}}$$

Problem 13.96 In Active Example 13.8, suppose that the angular acceleration of the rotor is $\alpha = -0.00002\omega^2$, where ω is the angular velocity of the rotor in rad/s. How long does it take the rotor to slow from 10,000 rpm to 1000 rpm?



Solution: Let $\alpha = k\omega^2$, where $k = -0.0002$.
Then

$$a = \frac{d\omega}{dt} = k\omega^2,$$

$$\int_{\omega_1}^{\omega_2} \frac{d\omega}{\omega^2} = \int_0^t k dt,$$

$$-\frac{1}{\omega_2} + \frac{1}{\omega_1} = kt \Rightarrow t = \frac{1}{k}\left(\frac{1}{\omega_1} - \frac{1}{\omega_2}\right)$$

Convert the numbers to rad/s

$$\omega_1 = 10000 \text{ rpm}\left(\frac{2\pi \text{ rad}}{\text{rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right) = 1047 \text{ rad/s}$$

$$\omega_2 = 1000 \text{ rpm}\left(\frac{2\pi \text{rad}}{\text{rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right) = 104.7 \text{ rad/s}$$

Using the numbers in the problem

$$t = \frac{1}{-0.0002}\left(\frac{1}{1047 \text{ rad/s}} - \frac{1}{104.7 \text{ rad/s}}\right)$$

$$\boxed{t = 430 \text{ s.}}$$

Problem 13.97 The astronaut is not rotating. He has an orientation control system that can subject him to a constant angular acceleration of 0.1 rad/s^2 about the vertical axis is either direction. If he wants to rotate 180° about the vertical axis (that is, rotate so that he is facing toward the left) and not be rotating in his new orientation, what is the minimum time in which he could achieve the new orientation?

Solution: He could achieve the rotation in minimum time by accelerating until he has turned 90° and then decelerating for the same time while he rotates the final 90°.

Thus the time needed to turn 90° ($\pi/2$ rad) is

$$\alpha = 0.1 \text{ rad/s}^2,$$

$$\omega = \alpha t$$

$$\theta = \frac{1}{2}\alpha t^2 \Rightarrow t = \sqrt{\frac{2\theta}{\alpha}} = \sqrt{\frac{2(\pi/2)}{0.1 \text{ rad/s}^2}} = 5.605 \text{ s.}$$

The total maneuver time is $2t$. $\boxed{2t = 11.2 \text{ s.}}$

Problem 13.98 The astronaut is not rotating. He has an orientation control system that can subject him to a constant angular acceleration of 0.1 rad/s^2 about the vertical axis in either direction. Refer to problem 13.97. For safety, the control system will not allow his angular velocity to exceed 15° per second. If he wants to rotate 180° about the vertical axis (that is, rotate so that he is facing toward the left) and not be rotating in his new orientation, what is the minimum time in which he could achieve the new orientation?

Solution: He could achieve the rotation in minimum time by accelerating until he has reached the angular velocity limit, then coasting until he has turned 90°. He would then continue to coast until he needed to decelerate to a stop at the 180° position. The maneuver is symmetric in the spin up and the spin down phases.

We will first find the time needed to reach the angular velocity limit and also find the angle through which he has rotated in this time.

$\alpha = 0.1 \text{ rad/s}^2$

$$\omega = \alpha t_1 \Rightarrow t_1 = \frac{\omega}{\alpha} = \frac{(15/180)\pi \text{ rad/s}}{0.1 \text{ rad/s}^2} = 2.62 \text{ s}.$$

$$\theta_1 = \frac{1}{2}\alpha t^2 = \frac{1}{2}(0.1 \text{ rad/s}^2)(2.62 \text{ s})^2 = 0.343 \text{ rad}.$$

Now we need to find the coast time (constant angular velocity)

$$\frac{\pi}{2} \text{ rad} - 0.343 \text{ rad} = \left(\frac{15^\circ}{180^\circ}\right)\pi \text{ rad/s } t_2 \Rightarrow t_2 = 4.69 \text{ s}$$

Thus the total time to complete a 90° turn is $t = t_1 + t_2 = 2.62 \text{ s} + 4.69 \text{ s} = 7.31 \text{ s}$.

The time for the full 180° turn is $2t$ $\boxed{2t = 14.6 \text{ s.}}$

Problem 13.99 The rotor of an electric generator is rotating at 200 rpm when the motor is turned off. Due to frictional effects, the angular acceleration of the rotor after the motor is turned off is $\alpha = -0.01\omega$ rad/s^2, where ω is the angular velocity in rad/s.

(a) What is the rotor's angular velocity one minute after the motor is turned off?
(b) After the motor is turned off, how many revolutions does the rotor turn before it comes to rest?

Strategy: To do part (b), use the chain rule to write the angular acceleration as

$$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta}\frac{d\theta}{dt} = \frac{d\omega}{d\theta}\omega.$$

Solution: Let $\alpha = k\omega$, where $k = -0.01 \text{ s}^{-1}$. Note that 200 rpm = 20.9 rad/s.

(a) One minute after the motor is turned off

$$\alpha = \frac{d\omega}{dt} = k\omega \Rightarrow \int_{\omega_0}^{\omega}\frac{d\omega}{\omega} = \int_0^t k\,dt \Rightarrow \ln\left(\frac{\omega}{\omega_0}\right) = kt$$

$$\omega = \omega_0 e^{kt} = (20.9 \text{ rad/s})e^{(-0.01/\text{s})(60 \text{ s})} = 11.5 \text{ rad/s}.$$

$$\boxed{\omega = 11.5 \text{ rad/s (110 rpm).}}$$

(b) When the rotor comes to rest

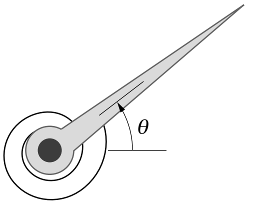
$$\alpha = \omega\frac{d\omega}{d\theta} = k\omega \Rightarrow \int_{\omega_0}^{0} d\omega = \int_0^\theta k\,d\theta \Rightarrow -\omega_0 = k\theta$$

$$\theta = \frac{1}{k}(-\omega_0) = \frac{1}{-0.01 \text{ s}^{-1}}(-20.9 \text{ rad/s})$$

$$\theta = 2094 \text{ rad}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 333 \text{ rev}.$$

$$\boxed{\theta = 333 \text{ rev.}}$$

Problem 13.100 The needle of a measuring instrument is connected to a *torsional spring* that gives it an angular acceleration $\alpha = -4\theta$ rad/s^2, where θ is the needle's angular position in radians relative to a reference direction. The needle is given an angular velocity $\omega = 2$ rad/s in the position $\theta = 0$.



(a) What is the magnitude of the needle's angular velocity when $\theta = 30°$?
(b) What maximum angle θ does the needle reach before it rebounds?

Solution:

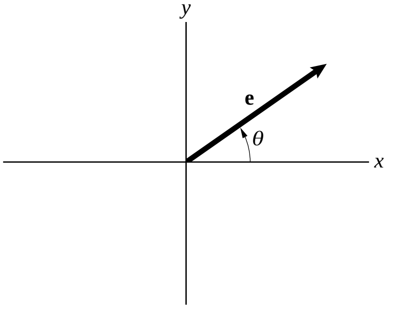
$$\alpha = \omega\frac{d\omega}{d\theta} = -4\theta \Rightarrow \int_2^{\omega}\omega d\omega = -4\int_0^{\theta}\theta d\theta \Rightarrow \frac{\omega^2}{2} - \frac{2^2}{2} = -2\theta^2$$

$$\omega = 2\sqrt{1-\theta^2}$$

(a) $\boxed{\omega = 2\sqrt{1-(\pi/6)^2} = 1.704 \text{ rad/s}}$

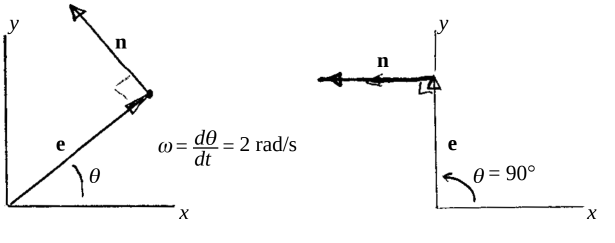
(b) Maximum angle means $\omega = 0$. $\boxed{\theta = 1 \text{ rad} = 57.3°}$

Problem 13.101 The angle θ measures the direction of the unit vector **e** relative to the x axis. The angular velocity of **e** is $\omega = d\theta/dt = 2$ rad/s, constant. Determine the derivative $d\mathbf{e}/dt$ when $\theta = 90°$ in two ways:



(a) Use Eq. (13.33).
(b) Express the vector **e** in terms of its x and y components and take the time derivative of **e**.

Solution:



(a) $$\frac{d\mathbf{e}}{dt} = \frac{d\theta}{dt}\mathbf{n} = \omega\mathbf{n}$$

when $\theta = 90°$, $n = -\mathbf{i}$

$$\frac{d\mathbf{e}}{dt} = -2\mathbf{i} \text{ rad/s when } \theta = 90°$$

(b) $$\mathbf{e} = (1)\cos\theta\mathbf{i} + (1)\sin\theta\mathbf{j}$$

$$\frac{d\mathbf{e}}{dt} = -\sin\theta\left(\frac{d\theta}{dt}\right)\mathbf{i} + \cos\theta\left(\frac{d\theta}{dt}\right)\mathbf{j}$$

Evaluating at $\theta = 90°$

$$\frac{d\mathbf{e}}{dt} = -\frac{d\theta}{dt}\mathbf{i} = -2\mathbf{i} \text{ rad/s}$$

Problem 13.102 The angle θ measures the direction of the unit vector $\mathbf{e}$ relative to the x axis. The angle θ is given as a function of time by $\theta = 2t^2$ rad. What is the vector $d\mathbf{e}/dt$ at $t = 4$ s?

Solution: By definition:

$$\frac{d\mathbf{e}}{dt} = \left(\frac{d\theta}{dt}\right)\mathbf{n},$$

where

$$\mathbf{n} = \mathbf{i}\cos\left(\theta + \frac{\pi}{2}\right) + \mathbf{j}\sin\left(\theta + \frac{\pi}{2}\right)$$

is a unit vector in the direction of positive θ. The angular rate of change is

$$\left[\frac{d\theta}{dt}\right]_{t=4} = [4t]_{t=4} = 16 \text{ rad/s}.$$

The angle is $\theta = [\text{mod}(2t^2, 2\pi)]_{t=4} = \text{mod}(32, 2\pi) = 0.5841$ rad, where $\text{mod}(x, y)$ ("modulus") is a standard function that returns the remainder of division of the first argument by the second. From which,

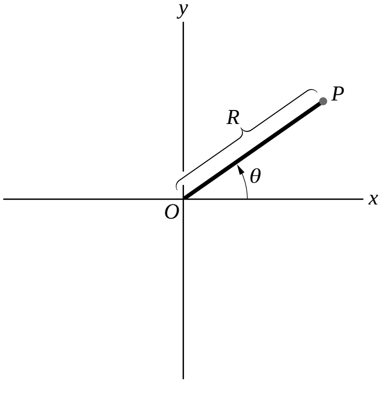
$$\boxed{\begin{aligned}\left[\frac{d\mathbf{e}}{dt}\right]_{t=4} &= 16\left(\mathbf{i}\cos\left(0.5841 + \frac{\pi}{2}\right) + \mathbf{j}\sin\left(0.5841 + \frac{\pi}{2}\right)\right)\\ &= -8.823\mathbf{i} + 13.35\mathbf{j}\end{aligned}}$$

Problem 13.103 The line OP is of constant length R. The angle $\theta = \omega_0 t$, where ω_0 is a constant.

(a) Use the relations $v_x = \dfrac{dx}{dt}$ and $v_y = \dfrac{dy}{dt}$ to determine the velocity of P relative to O.

(b) Use Eq. (13.33) to determine the velocity of P relative to O, and confirm that your result agrees with the result of (a).

Strategy: In part (b), write the position vector of P relative to O as $\mathbf{r} = R\mathbf{e}$ where $\mathbf{e}$ is a unit vector that points from O toward P.



Solution:

(a) The point P is described by $\mathbf{P} = \mathbf{i}x + \mathbf{j}y$. Take the derivative:

$$\frac{d\mathbf{P}}{dt} = \mathbf{i}\left(\frac{dx}{dt}\right) + \mathbf{j}\left(\frac{dy}{dt}\right).$$

The coordinates are related to the angle θ by $x = R\cos\theta$, $y = R\sin\theta$. Take the derivative and note that R is a constant and $\theta = \omega_0 t$, so that

$$\frac{d\theta}{dt} = \omega_0 : \frac{dx}{dt} = -R\sin\theta\left(\frac{d\theta}{dt}\right),$$

$$\frac{dy}{dt} = R\cos\theta\left(\frac{d\theta}{dt}\right).$$

Substitute into the derivative of the vector $\mathbf{P}$,

$$\boxed{\begin{aligned}\frac{d\mathbf{P}}{dt} &= R\left(\frac{d\theta}{dt}\right)(-\mathbf{i}\sin\theta + \mathbf{j}\cos\theta)\\ &= R\omega_0(-\mathbf{i}\sin(\omega_0 t) + \mathbf{j}\cos(\omega_0 t))\end{aligned}}$$

which is the velocity of the point P relative to the origin O.

(b) Note that $\mathbf{P} = R\mathbf{e}$, and $\dfrac{d\mathbf{P}}{dt} = R\dfrac{d\mathbf{e}}{dt}$ when R is constant. Use the definition (Eq. (13.33)),

$$\frac{d\mathbf{e}}{dt} = \left(\frac{d\theta}{dt}\right)\mathbf{n},$$

where $\mathbf{n}$ is a unit vector in the direction of positive θ, (i.e., perpendicular to $\mathbf{e}$). Thus

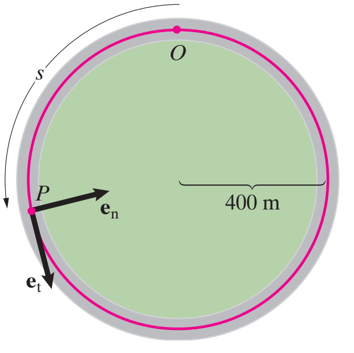
$$\mathbf{n} = \mathbf{i}\cos\left(\theta + \frac{\pi}{2}\right) + \mathbf{j}\sin\left(\theta + \frac{\pi}{2}\right).$$

Use the trigonometric sum-of-angles identities to obtain: $\mathbf{n} = -\mathbf{i}\sin\theta + \mathbf{j}\cos\theta$. Substitute,

$$\boxed{\frac{d\mathbf{P}}{dt} = R\omega_0(-\mathbf{i}\sin(\omega_0 t) + \mathbf{j}\cos(\omega_0 t))}$$

The results are the same.

Problem 13.104 In Active Example 13.9, determine the motorcycle's velocity and acceleration in terms of normal and tangential components at $t = 5$s.



Solution: We are given the tangential acceleration

$$a_t = (2 \text{ m/s}^2) + (0.2 \text{ m/s}^3)t$$

We can find the velocity at $t = 5$ s.

$$v = \int_0^{5\text{ s}} ([2 \text{ m/s}^2] + [0.2 \text{ m/s}^3]t)dt$$

$$= [2 \text{ m/s}^2][5 \text{ s}] + \frac{1}{2}[0.2 \text{ m/s}^3][5 \text{ s}]^2$$

$$= 1.2 \text{ m/s}.$$

The velocity is then $\boxed{\mathbf{v} = (12.5 \text{ m/s})\, \mathbf{e}_t}$

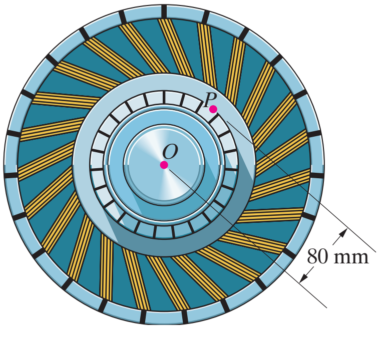
The accelerations are

$$a_t = (2 \text{ m/s}^2) + (0.2 \text{ m/s}^3)(5 \text{ s}) = 3 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{(12.5 \text{ m/s})^2}{400 \text{ m}} = 0.391 \text{ m/s}^2.$$

$$\boxed{\mathbf{a} = (3 \text{ m/s}^2)\mathbf{e}_t + (0.391 \text{ m/s}^2)\mathbf{e}_n}$$

Problem 13.105 The armature starts from rest at $t = 0$ and has constant angular acceleration $\alpha = 2$ rad/s^2. At $t = 4$ s, what are the velocity and acceleration of point P relative to point O in terms of normal and tangential components?



Solution: We can find the angular velocity at $t = 4$s.

$$\alpha = 2 \text{ rad/s}^2$$

$$\omega = (2 \text{ rad/s}^2)(4 \text{ s}) = 8 \text{ rad/s}.$$

Then

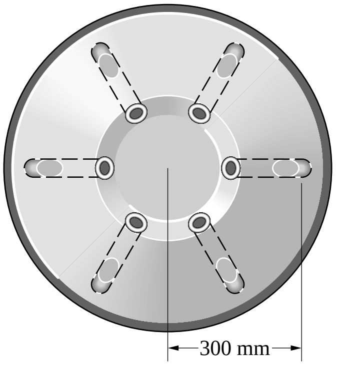
$$v = r\omega = (0.08 \text{ m})\ (8 \text{ rad/s}) = 0.64 \text{ m/s},$$

$$a_t = r\alpha = (0.08 \text{ m})\ (2 \text{ rad/s}^2) = 0.16 \text{ m/s}^2,$$

$$a_n = \frac{v^2}{r} = \frac{(0.64 \text{ m/s})^2}{0.08 \text{ m}} = 5.12 \text{ m/s}^2$$

$$\boxed{\begin{aligned} &\mathbf{v} = (0.64 \text{ m/s})\mathbf{e}_t, \\ &\mathbf{a} = (0.16 \text{ m/s}^2)\mathbf{e}_t + (5.12 \text{ m/s}^2)\mathbf{e}_n. \end{aligned}}$$

Problem 13.106 Suppose you want to design a medical centrifuge to subject samples to normal accelerations of 1000 g's. (a) If the distance from the center of the centrifuge to the sample is 300 mm, what speed of rotation in rpm is necessary? (b) If you want the centrifuge to reach its design rpm in 1 min, what constant angular acceleration is necessary?



Solution:

(a) The normal acceleration at a constant rotation rate is $a_n = R\omega^2$, giving

$$\omega = \sqrt{\frac{a_n}{R}} = \sqrt{\frac{(1000)9.81}{0.3}} = 180.83 \text{ rad/s}.$$

The speed in rpm is

$$\boxed{N = \omega\left(\frac{\text{rad}}{\text{s}}\right)\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right)\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 1730 \text{ rpm}}.$$

(b) The angular acceleration is

$$\boxed{\alpha = \frac{\omega}{t} = \frac{180.83}{60} = 3.01 \text{ rad/s}^2}$$

Problem 13.107 The medical centrifuge shown in Problem 13.106 starts from rest at $t = 0$ and is subjected to a constant angular acceleration $\alpha = 3$ rad/s^2. What is the magnitude of the total acceleration to which the samples are subjected at $t = 1$ s?

Solution: $\alpha = 3, \quad \omega = 3t, \quad \theta = 1.5t^2$

$$a_t = (0.3 \text{ m})(3 \text{ rad/s}^2) = 0.9 \text{ m/s}^2$$

$$a_n = (0.3 \text{ m})(3 \text{ rad/s})^2 = 2.7 \text{ m/s}^2$$

$$\boxed{a = \sqrt{(0.9)^2 + (2.7)^2} \text{ m/s}^2 = 2.85 \text{ m/s}^2}$$

Problem 13.108 A centrifuge used to subject engineering components to high acceleration has a radius of 8 m. It starts from rest at $t = 0$, and during its two-minute acceleration phase it is programmed so that its angular acceleration is given as a function of time in seconds by $\alpha = 0.192 - 0.0016t$ rad/s^2. At $t = 120$ s, what is the magnitude of the acceleration a component is subjected to?

Solution: We will first calculate the angular velocity

$$\omega = \int_0^{120\text{ s}} ([0.192 \text{ rad/s}^2] - [0.0016 \text{ rad/s}^3]t)\, dt$$

$$= [0.192 \text{ rad/s}][120 \text{ s}] - \frac{1}{2}[0.0016 \text{ rad/s}^3][120 \text{ s}]^2$$

$$= 11.52 \text{ rad/s}$$

The normal and tangential components of acceleration are

$$a_t = r\alpha = (8 \text{ m})([0.192 \text{ rad/s}^2] - [0.0016 \text{ rad/s}^3][120 \text{ s}]) = 0$$

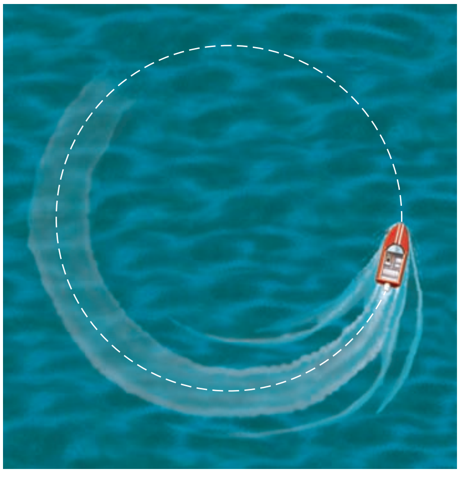
$$a_n = r\omega^2 = (8 \text{ m})(11.52 \text{ rad/s})^2 = 1060 \text{ m/s}^2$$

Since the tangential component is zero, then the total acceleration is the same as the normal acceleration

$$\boxed{a = 1060 \text{ m/s}^2 (108g\text{'s}).}$$

Problem 13.109 A powerboat being tested for maneuverability is started from rest at $t = 0$ and driven in a circular path 12 m in radius. The tangential component of the boat's acceleration as a function of time is $a_t = 0.4t$ m/s^2.

(a) What are the boat's velocity and acceleration in terms of normal and tangential components at $t = 4$ s?
(b) What distance does the boat move along its circular path from $t = 0$ to $t = 4$ s?



Solution:

(a) $a_t = 0.4t$ m/s^2 $\quad a_n = +v^2/r$

$v = 0.2t^2$ m/s

At $t = 4$ s,

$\mathbf{a} = 0.4t\mathbf{e}_t + v^2/r\mathbf{e}_n$

$\mathbf{a} = 1.6\mathbf{e}_t + 0.853\mathbf{e}_n$

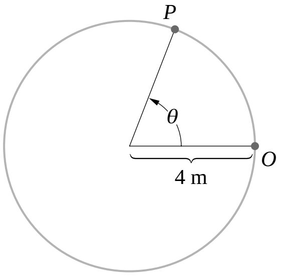
$\mathbf{v} = 3.2\mathbf{e}_t$ m/s

(b) $s = 0.2t^3/3$

$\underline{s|_{4s} = 4.27 \text{ m}}$

Problem 13.110 The angle $\theta = 2t^2$ rad.

(a) What are the velocity and acceleration of point P in terms of normal and tangential components at $t = 1$ s?
(b) What distance along the circular path does point P move from $t = 0$ to $t = 1$ s?



Solution:

$$\theta = 2t^2$$

$$\frac{d\theta}{dt} = 4t = \omega$$

$$\frac{d^2\theta}{dt^2} = 4\frac{\text{rad}}{\text{s}^2} = \alpha$$

$$s = r\theta = 4\theta = 8t^2$$

$$v_t = 16t \text{ m/s}$$

$$v = r\omega = 4(4t) = 16t$$

$$a_t = \frac{dv}{dt} = 16 \text{ m/s}^2$$

(a) $\underline{\mathbf{v} = 16(1)\mathbf{e}_t \text{ m/s} = 16\ e_t\ (\text{m/s})}$

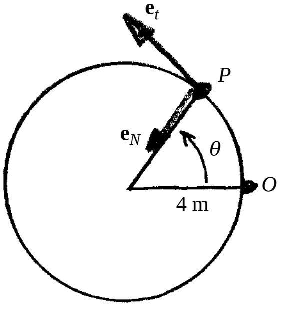
$$\mathbf{a} = R\alpha\mathbf{e}_t + R\omega^2\mathbf{e}_N$$

$$\mathbf{a} = (4)(4)\mathbf{e}_t + (4)(4^2)\mathbf{e}_N\ (\text{m/s}^2)$$

$$\underline{\mathbf{a} = 16\mathbf{e}_t + 64\mathbf{e}_N\ (\text{m/s}^2)}$$

(b) $s = R\theta = 8t^2$

$$= 8(1)^2 = 8 \text{ m}$$



Problem 13.111 The angle $\theta = 2t^2$ rad. What are the velocity and acceleration of point P in terms of normal and tangential components when P has gone one revolution around the circular path starting at $t = 0$?

Solution: From the solution to Problem 13.110,

$$\theta = 2t^2 \text{ rad}$$

$$\omega = 4t \text{ rad/s}$$

$$\alpha = 4 \text{ rad/s}^2$$

$$s = 8t^2 \text{ m}$$

$$v_t = 16t \text{ m/s}$$

$$a_t = 16 \text{ m/s}^2$$

We want to know **v** and **a** when $\theta = 2\pi$. Substituting into the first eqn, we find that $\theta = 2\pi$ when $t = t_1 = 1.77$ seconds. From the solution to Problem 13.110,

$\mathbf{v}_t = 16t\mathbf{e}_t$ and

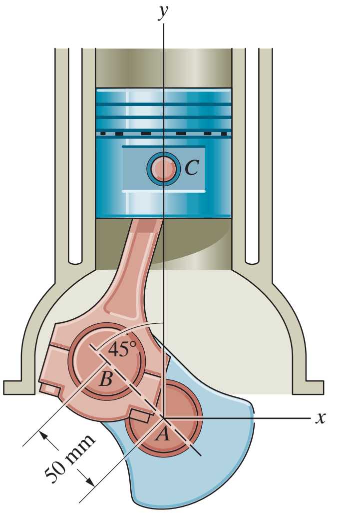
$$\mathbf{a} = R\alpha\mathbf{e}_t + R\omega^2\mathbf{e}_N$$

Substituting in the time t_1, we get

$$\underline{\mathbf{v}_t = 28.4\mathbf{e}_t\ (\text{m/s})}$$

$$\underline{\mathbf{a} = 16\mathbf{e}_t + 201.1\mathbf{e}_N\ (\text{m/s}^2)}$$

Problem 13.112 At the instant shown, the crank AB is rotating with a constant counterclockwise angular velocity of 5000 rpm. Determine the velocity of point B (a) in terms of normal and tangential components; (b) in terms of cartesian components.



Solution:

$$\omega = (5000 \text{ rpm})\left(\frac{2\pi \text{ rad}}{\text{rev}}\right)\left(\frac{\text{min}}{60 \text{ sec}}\right) = 524 \text{ rad/s}$$

(a) $\boxed{\mathbf{V}_B = 0.05 \text{ m } (524 \text{ rad/s})\mathbf{e}_t = (26.2\,\mathbf{e}_t) \text{ m/s}}$

(b) $\boxed{\mathbf{V}_B = (26.2 \text{ m/s})(-\cos 45^\circ\,\mathbf{i} - \sin 45^\circ\,\mathbf{j}) = (-18.53\mathbf{i} - 18.53\mathbf{j}) \text{ m/s}}$

Problem 13.113 The crank AB in Problem 13.112 is rotating with a constant counterclockwise angular velocity of 5000 rpm. Determine the acceleration of point B (a) in terms of normal and tangential components; (b) in terms of cartesian components.

Solution:

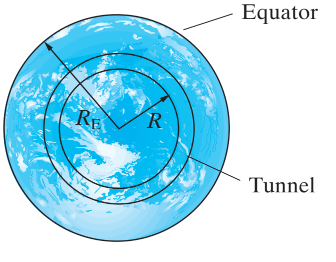
$$\omega = (5000 \text{ rpm})\left(\frac{2\pi \text{ rad}}{\text{rev}}\right)\left(\frac{\text{min}}{60 \text{ sec}}\right) = 524 \text{ rad/s}$$

$$a_t = 0,\ a_n = 0.05 \text{ m } (524 \text{ rad/s})^2 = 13728.8 \approx 13729$$

(a) $\boxed{\mathbf{a}_P = (13729\,\mathbf{e}_n) \text{ m/s}^2}$

(b) $\boxed{\begin{aligned}\mathbf{a}_p &= (13729 \text{ m/s}^2)(\cos 45^\circ\,\mathbf{i} - \sin 45^\circ\,\mathbf{j})\\ &= (9708\mathbf{i} - 9708\mathbf{j}) \text{ m/s}^2\end{aligned}}$

Problem 13.114 Suppose that a circular tunnel of radius R could be dug beneath the equator. In principle, a satellite could be placed in orbit about the center of the earth within the tunnel. The acceleration due to gravity in the tunnel would be gR/R_E, where g is the acceleration due to gravity at sea level and R_E is the earth's radius. Determine the velocity of the satellite and show that the time required to complete one orbit is independent of the radius R. (See Example 13.10.)



Solution: To be in orbit we must have

$$a_n = \frac{v^2}{R} = \frac{gR}{R_E} \Rightarrow \boxed{v = R\sqrt{\frac{g}{R_E}}}$$

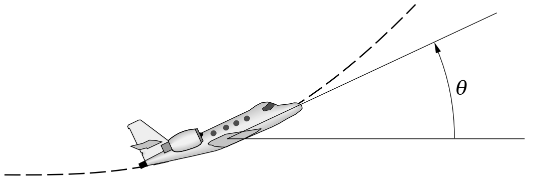
The velocity of the satellite is given by the distance it travels in one orbit divided by the time needed to complete that orbit.

$$v = \frac{2\pi R}{t} \Rightarrow t = \frac{v}{2\pi R} = \frac{R\sqrt{\frac{g}{R_E}}}{R} = \sqrt{\frac{g}{R_E}}.$$

Notice that the time does not depend on the radius R.

Problem 13.115 At the instant shown, the magnitude of the airplane's velocity is 130 m/s, its tangential component of acceleration is $a_t = -4$ m/s^2, and the rate of change of its path angle is $d\theta/dt = 5°/s$.

(a) What are the airplane's velocity and acceleration in terms of normal and tangential components?
(b) What is the instantaneous radius of curvature of the airplane's path?



Solution:

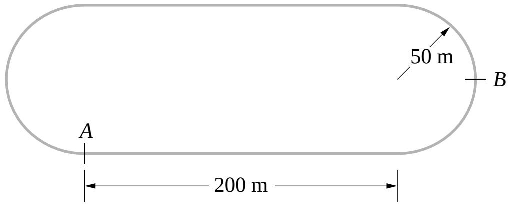
$$\omega = (5°/s)\left(\frac{\pi \text{ rad}}{180°}\right) = \left(\frac{\pi}{36}\right) \text{rad/s}$$

$$a_{Pt} = -4 \text{ m/s}^2,\ a_n = (130 \text{ m/s})\omega = 11.34 \text{ m/s}^2$$

(a) $$\boxed{\begin{aligned}\mathbf{v}_p &= (130\mathbf{e}_t) \text{ m/s}\\ \mathbf{a}_p &= (-4\mathbf{e}_t + 11.34\mathbf{e}_n) \text{ m/s}^2\end{aligned}}$$

(b) $$\boxed{\rho = \frac{v^2}{a_n} = \frac{(130 \text{ m/s})^2}{11.34 \text{ m/s}^2} = 1490 \text{ m}}$$

Problem 13.116 In the preliminary design of a sun-powered car, a group of engineering students estimates that the car's acceleration will be 0.6 m/s^2. Suppose that the car starts from rest at A and the tangential component of its acceleration is $a_t = 0.6$ m/s^2. What are the car's velocity and acceleration in terms of normal and tangential components when it reaches B?



Solution:

$$a_t = v\frac{dv}{ds} = 0.6 \text{ m/s}^2 \Rightarrow \int_0^v v\,dv = \int_0^s (0.6 \text{ m/s}^2)ds$$

$$v^2 = 2(0.6 \text{ m/s}^2)s$$

At point B

$$S_B = \left(200 + \frac{50\pi}{2}\right) m \Rightarrow v_B = 18.28 \text{ m/s}, \quad a_{Bn} = \frac{v_B^2}{50 \text{ m}} = 6.68 \text{ m/s}^2$$

Thus

$$\boxed{\begin{aligned}\mathbf{v}_B &= (18.28\mathbf{e}_t) \text{ m/s}\\ \mathbf{a}_B &= (0.6\mathbf{e}_t + 6.68\mathbf{e}_n) \text{ m/s}^2\end{aligned}}$$

Problem 13.117 After subjecting the car design described in Problem 13.116 to wind tunnel testing, the students estimate that the tangential component of the car's acceleration will be $a_t = 0.6 - 0.002v^2$ m/s^2, where v is the car's velocity in m/s. If the car starts from rest at A, what are its velocity and acceleration in terms of normal and tangential components when it reaches B?

Solution: At point B $S_B = \left(200 + \frac{50\pi}{2}\right)$ m

$$a_t = v\frac{dv}{ds} = 0.6 - 0.002v^2 \Rightarrow \int_0^{v_B} \frac{v\,dv}{0.6 - 0.002v^2} = \int_0^{s_B} ds$$

$$v_B = 14.20 \text{ m/s}, \quad a_{Bn} = \frac{v_B^2}{50 \text{ m}} = 4.03 \text{ m/s}^2$$

$$a_t = 0.6 - 0.002(14.20 \text{ m/s})^2 = 0.197 \text{ m/s}^2$$

Thus

$$\boxed{\begin{aligned}\mathbf{v}_B &= (14.20\mathbf{e}_t) \text{ m/s}\\ \mathbf{a}_B &= (0.197\mathbf{e}_t + 4.03\mathbf{e}_n) \text{ m/s}^2\end{aligned}}$$

Problem 13.118 Suppose that the tangential component of acceleration of the car described in Problem 13.117 is given in terms of the car's position by $a_t = 0.4 - 0.001s$ m/s^2, where s is the distance the car travels along the track from point A. What are the car's velocity and acceleration in terms of normal and tangential components at point B?

Solution:

$$a_t = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = 0.4 - 0.001s \text{ m/s}^2$$

$$v\frac{dv}{ds} = 0.4 - 0.001s$$

$$\int_0^v v\,dv = \int_0^{S_B} (0.4 - 0.001s)\,ds$$

$$\frac{v^2}{2} = \left[0.4s - \frac{0.001s^2}{2}\right]\Bigg|_0^{S_B}$$

From Fig. P13.116, $S_B = 200 + 2\pi\rho/4$ where $\rho = 50$ m, so $S_B = 278.5$ m

Solving for v,

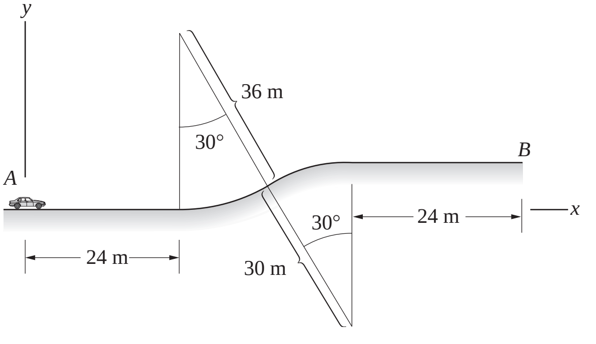
$v = 12.05$ m/s

$\underline{\mathbf{v} = 12.05\mathbf{e}_t \text{ (m/s)}}$

$\mathbf{a} = (0.4 - 0.001s_B)\mathbf{e}_t + v^2/\rho\mathbf{e}_N$ (m/s^2)

Solving, $\underline{\mathbf{a} = 0.121\mathbf{e}_t + 2.905\mathbf{e}_N \text{ (m/s}^2)}$

Problem 13.119 A car increases its speed at a constant rate from 64 km/h at A to 96 km/h at B. What is the magnitude of its acceleration 2 s after the car passes point A?



Solution: Use the chain rule to obtain

$$v\frac{dv}{ds} = a,$$

where a is constant. Separate variables and integrate: $v^2 = 2as + C$. At

$$s = 0, \quad v(0) = 64\left(\frac{1000}{3600}\right) = 17.78 \text{ m/s},$$

from which $C = 316.13$. The acceleration is

$$a = \frac{v^2 - C}{2s}$$

The distance traveled from A to B is

$$s = 2(24) + (30)\left(\frac{\pi}{180}\right)(36 + 30) = 82.6 \text{ m},$$

and the speed in

$$[v(s)]_{s=82.6} = 96\left(\frac{1000}{3600}\right) = 26.67 \text{ m/s},$$

from the constant acceleration is

$$a = \frac{(26.67)^2 - 316.13}{2(82.6)} = 2.39 \text{ m/s}^2.$$

The velocity is as a function of time is $v(t) = v(0) + at = 17.78 + 2.39t$ m/s. The distance from A is

$$s(t) = 17.78t + \frac{2.39}{2}t^2.$$

At a point 2 seconds past A, the distance is $s(2) = 40.34$ m, and the velocity is $v(2) = 22.56$ m/s. The first part of the hill ends at 43, so that at this point the car is still in the first part of the hill. The tangential acceleration is $a_t = 2.39$ m/s^2. The normal acceleration is

$$a_n = \frac{v^2}{R} = \frac{(22.56)^2}{36} = 14.1 \text{ m/s}^2.$$

The magnitude of the acceleration is

$$\boxed{|\mathbf{a}| = \sqrt{2.39^2 + 14.1^2} = 14.3 \text{ m/s}^2}$$

Note: This is a large acceleration–the driver (and passengers) would no doubt be uncomfortable.

Problem 13.120 The car increases its speed at a constant rate from 64 km/h at A to 96 km/h at B. Determine the magnitude of its acceleration when it has traveled along the road a distance (a) 36 m from A and (b) 48 m from A.

Solution: Use the solution in Problem 13.119.

(a) The velocity at a distance 36 m from A is

$$v(36) = \sqrt{2as + C} = \sqrt{(2)(2.39)(36) + 316.13}$$

$$= 22.1 \text{ m/s}.$$

At 36 m the car is in the first part of the hill. The tangential acceleration is $a_t = 2.39$ m/s^2 from Problem 13.119. The normal acceleration is

$$a_n = \frac{(v(36))^2}{R} = \frac{(22.1)^2}{36} = 13.6 \text{ m/s}^2.$$

The magnitude of the acceleration is

$$\boxed{|\mathbf{a}| = \sqrt{2.39^2 + 13.6^2} = 13.81 \text{ m/s}^2}$$

(b) The velocity at distance 48 m from A is

$$v(48) = \sqrt{2(2.39)(48) + C} = 23.4 \text{ m/s}.$$

At 48 m the car is on the second part of the hill. The tangential acceleration is unchanged: $a_t = 2.39$ m/s^2. The normal acceleration is

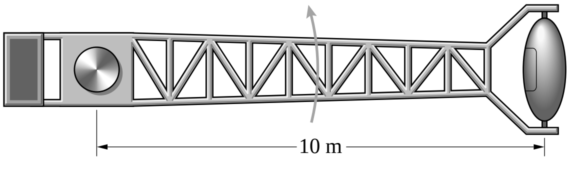
$$a_n = \frac{(v(48))^2}{R} = \frac{23.4^2}{30} = 18.3 \text{ m/s}^2.$$

The magnitude of the acceleration is

$$\boxed{|\mathbf{a}| = \sqrt{2.392^2 + 18.32^2} = 18.5 \text{ m/s}^2}$$

[*Note*: The car will "lift off" from the road.]

Problem 13.121 Astronaut candidates are to be tested in a centrifuge with 10-m radius that rotates in the horizontal plane. Test engineers want to subject the candidates to an acceleration of 5 g's, or five times the acceleration due to gravity. Earth's gravity effectively exerts an acceleration of 1 g in the vertical direction. Determine the angular velocity of the centrifuge in revolutions per second so that the magnitude of the total acceleration is 5 g's.



Solution:

$$a_n{}^2 + g^2 = (5g)^2 \Rightarrow a_n = \sqrt{24}g$$

$$a_n = r\omega^2 \Rightarrow \omega = \sqrt{a_n/r}$$

$$\boxed{\omega = \sqrt{\frac{\sqrt{24}(9.81 \text{ m/s}^2)}{10\text{m}}} = 2.19 \text{ rad/s}}$$

Problem 13.122 In Example 13.11, what is the helicopter's velocity in turns of normal and tangential components at $t = 4$ s?

Solution: In Example 13.11 we find the x and y components of acceleration and velocity at $t = 4$ s.

$a_x = 2.4$ m/s^2, $a_y = 0.36$ m/s^2,

$v_x = 4.80$ m/s, $v_y = 4.32$ m/s

The total velocity of the helicopter is

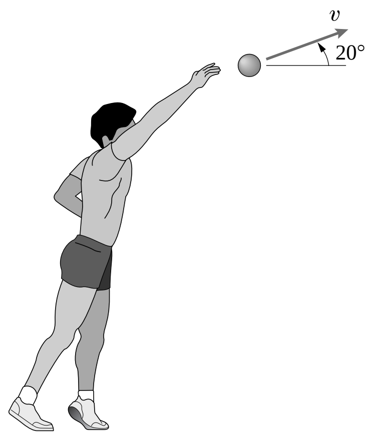
$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(4.80 \text{ m/s})^2 + (4.32 \text{ m/s})^2} = 6.46 \text{ m/s}.$$

By definition, the velocity is in the tangential direction, therefore

$$\boxed{\mathbf{v} = 6.46 \text{ m/s})\mathbf{e}_t.}$$

Problem 13.123 The athlete releases the shot with velocity $v = 16$ m/s at 20° above the horizontal.

(a) What are the velocity and acceleration of the shot in terms of normal and tangential components when it is at the highest point of its trajectory?
(b) What is the instantaneous radius of curvature of the shot's path when it is at the highest point of its trajectory?



Solution:

$a_x = 0$

$v_x = v_{x0} = 16\cos 20°,\ v_{y0} = 16\sin 20°$

$a_y = -9.81\ \text{m/s}^2$

$v_y = v_{y0} - 9.81t = 5.47 - 9.81t$

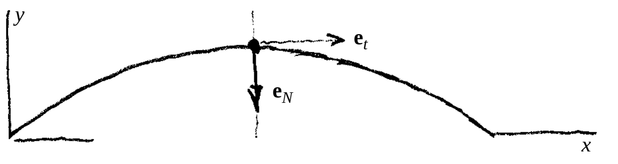
At highest point, $v_y = 0$

(a) $\underline{\mathbf{v} = 16\cos 20°\mathbf{e}_t = 15.0\mathbf{e}_t\ \text{(m/s)}}$

(b) $\underline{\mathbf{a} = 9.81\mathbf{e}_n\ (\text{m/s}^2)}$

(c) $|\mathbf{a}_N| = v^2/\rho, \quad \rho = v^2/|a_n|$

$\rho = (15)^2/9.81 = 23.0$ m



Problem 13.124 At $t = 0$, the athlete releases the shot with velocity $v = 16$ m/s.

(a) What are the velocity and acceleration of the shot in terms of normal and tangential components at $t = 0.3$ s?
(b) Use the relation $a_n = v^2/\rho$ to determine the instantaneous radius of curvature of the shot's path at $t = 0.3$ s.

Solution: From the solution to Problem 13.123,

$v_x = 15.0$ m/s $\quad v_y = 5.47 - 9.81t$ m/s

$v = \sqrt{v_x^2 + v_y^2}$

At $t = 0.3$ s, $\quad v = 15.2$ m/s

$\underline{\mathbf{v} = 15.2\mathbf{e}_t\ \text{(m/s)}}$

We have the following geometry
From the diagram

$\tan r = v_y/v_x \quad r = 9.55°$

$|\mathbf{a}_n| = 9.81\cos r = 9.67\ \text{m/s}^2$

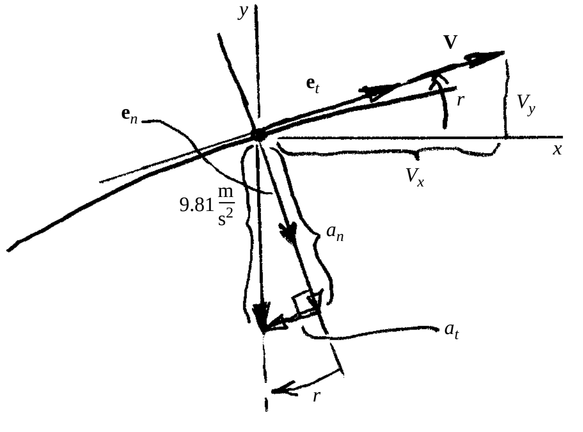
$|\mathbf{a}_t| = 9.81\sin r = 1.63\ \text{m/s}^2$

$\underline{\mathbf{a} = -1{,}63\mathbf{e}_t + 9.67\mathbf{e}_n\ (\text{m/s}^2)}$

$|a_n| = v^2/\rho$

$\rho = v^2/|a_n| = (15.2)^2/9.67$

$\underline{\rho = 24.0\ \text{m}}$



Problem 13.125 At $t = 0$, the athlete releases the shot with velocity $v = 16$ m/s. Use Eq. (13.42) to determine the instantaneous radius of curvature of the shot's path at $t = 0.3$ s.

Solution:

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

From the solution to 13.123,

$$v_y = v_{y_0} - 9.81t, \text{ hence}$$

$$y = y_0 + v_{y_0}t - 9.8/(t^2/2), \quad y_0 \equiv 0$$

Also, $v_x = v_{x_0}$

$$x = x_0 + v_{x_0} \quad t x_0 \equiv 0$$

Hence $t = x/v_{x_0}$ and

$$y = v_{y_0}(\frac{x}{v_{x_0}}) - \frac{9.81}{2}\left(\frac{x}{v_{x_0}}\right)^2$$

We now have $y(x)$

$$\frac{dy}{dx} = \frac{v_{y_0}}{v_{x_0}} - 9.81\left(\frac{x}{v_{x_0}^2}\right)$$

$$\frac{d^2y}{dx^2} = -9.81/v_{x_0}^2$$

We also know $v_0 = 16$ m/s and

$$v_{y_0} = v_0 \sin 20^\circ = 5.47 \text{ m/s}$$

$$v_{x_0} = v_0 \cos 20^\circ = 15.04 \text{ m/s}$$

At $t = 0.3$ s, $x = 4.5$ m,

$$\frac{dy}{dx} = 0.168$$

$$\frac{d^2y}{dx^2} = -0.0434$$

and $\underline{\rho = 24.0 \text{ m}}$

Problem 13.126 The cartesian coordinates of a point moving in the xy-plane are $x = 20 + 4t^2$ m, $y = 10 - t^3$ m. What is the instantaneous radius of curvature of the path of the point at $t = 3$ s?

Solution: The components of the velocity: $\mathbf{v} = 8t\mathbf{i} - (3t^2)\mathbf{j}$. At $t = 3$ seconds, the magnitude of the velocity is $|\mathbf{v}|_{t=3} = \sqrt{(8t)^2 + (-3t^2)^2} = 36.12$ m/s. The components of the acceleration are $\mathbf{a} = 8\mathbf{i} - (6t)\mathbf{j}$. The instantaneous path angle is

$$\tan\beta = \frac{v_y}{v_x} = \frac{-3t^2}{8t}.$$

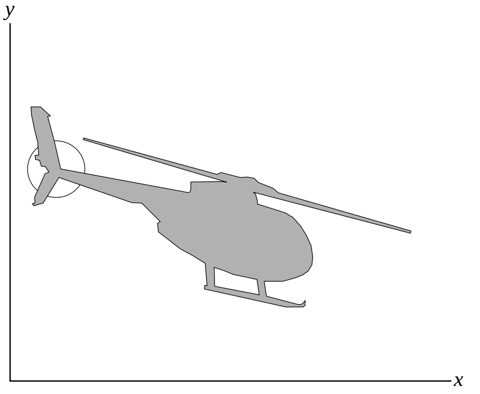
At $t = 3$ seconds, $\beta = -0.8442$ rad. The unit vector parallel to the path is $\mathbf{e}_t = \mathbf{i}\cos\beta + \mathbf{j}\sin\beta$. The unit vector normal to the path pointing toward the instantaneous radial center is

$$\mathbf{e}_n = \mathbf{i}\cos\left(\beta - \frac{\pi}{2}\right) + \mathbf{j}\sin\left(\beta - \frac{\pi}{2}\right) = \mathbf{i}\sin\beta - \mathbf{j}\cos\beta.$$

The normal acceleration is the component of acceleration in the direction of $\mathbf{e}_n$. Thus, $a_n = \mathbf{e}_n \cdot \mathbf{a}$ or $a_n = 8\sin\beta + (6t)\cos\beta$. At $t = 3$ seconds, $a_n = 5.98$ m/s^2. The radius of curvature at $t = 3$ seconds is

$$\boxed{\rho = \frac{|\mathbf{v}|^2}{a_n} = 218 \text{ m}}$$

Problem 13.127 The helicopter starts from rest at $t = 0$. The cartesian components of its acceleration are $a_x = 0.6t$ m/s^2 and $a_y = 1.8 - 0.36t$ m/s^2. Determine the tangential and normal components of its acceleration at $t = 6$ s.



Solution: The solution will follow that of Example 13.11, with the time changed to $t = 6$ s. The helicopter starts from rest $(v_x, v_y) = (0, 0)$ at $t = 0$. Assume that motion starts at the origin (0, 0). The equations for the motion in the x direction are $a_x = 0.6t$ m/s^2, $v_x = 0.3t^2$ m/s, $x = 0.1t^3$ m, and the equations for motion in the y direction are $a_y = 1.8 - 0.36t$ m/s^2, $v_y = 1.8t - 0.18t^2$ m/s, and $y = 0.9t^2 - 0.06t^3$ m. At $t = 6$ s, the variables have the values $a_x = 3.6$ m/s^2, $a_y = -0.36$ m/s^2, $v_x = 10.8$ m/s, $v_y = 4.32$ m/s, $x = 21.6$ m, and $y = 19.44$ m. The magnitude of the velocity is given by

$$|\mathbf{v}| = \sqrt{v_x^2 + v_y^2} = 11.63 \text{ m/s}.$$

The unit vector in the tangential direction is given by

$$\mathbf{e}_T = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{v_x\mathbf{i} + v_y\mathbf{j}}{|\mathbf{v}|} = 0.928\mathbf{i} + 0.371\mathbf{j}.$$

The tangential acceleration component is given by $a_T = \mathbf{a} \cdot \mathbf{e}_T = 0.928a_x + 0.371a_y = 3.21$ m/s^2.

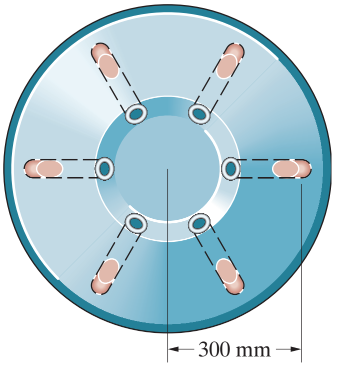
The magnitude of the acceleration is given by

$$|\mathbf{a}| = \sqrt{a_x^2 + a_y^2} = 3.62 \text{ m/s}^2.$$

The normal acceleration component is given by

$$a_N = \sqrt{|\mathbf{a}|^2 - a_T^2} = 1.67 \text{ m/s}^2$$

Problem 13.128 Suppose that when the centrifuge in Example 13.12 is turned on, its motor and control system give it an angular acceleration (in rad/s^2) $\alpha = 12 - 0.02\omega$, where ω is the centrifuge's angular velocity. Determine the tangential and normal components of the acceleration of the sample at $t = 0.2$ s.



Solution: We will first integrate to find the angular velocity at t = 0.2 s.

$$\alpha = \frac{d\omega}{dt} = ([12 \text{ rad/s}^2] - [0.02 \text{ s}^{-1}]\omega)$$

$$\int_0^\omega \frac{d\omega}{[12 \text{ rad/s}^2] - [0.02 \text{ s}^{-1}]\omega} = \int_0^{0.2 \text{ s}} dt$$

$$\frac{-1 \text{ s}}{0.02} \ln\left(\frac{[12 \text{ rad/s}^2] - [0.02 \text{ s}^{-1}]\omega}{[12 \text{ rad/s}^2]}\right) = 0.2 \text{ s}$$

$$[12 \text{ rad/s}^2] - [0.02 \text{ s}^{-1}]\omega = [12 \text{ rad/s}^2]e^{-0.004}$$

$$\omega = \frac{1 \text{ s}}{0.02}(12 \text{ rad/s}^2)(1 - e^{-0.004}) = 2.40 \text{ rad/s}$$

At this time, the angular acceleration is

$$\alpha = (12 \text{ rad/s}^2) - (0.02 \text{ s}^{-1})(2.40 \text{ rad/s}) = 11.95 \text{ rad/s}^2.$$

The components of acceleration are

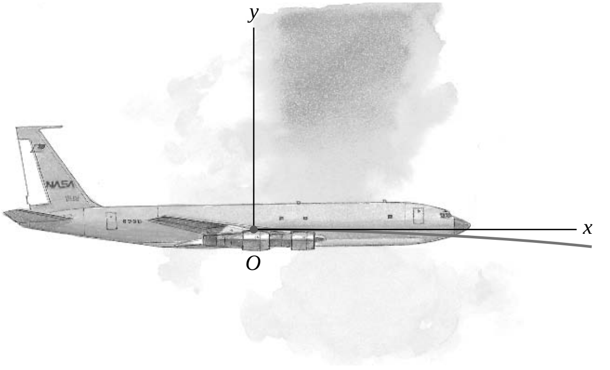
$$a_t = r\alpha = (0.3 \text{ m})(11.95 \text{ rad/s}^2) = 3.59 \text{ m/s}^2$$

$$\alpha_n = r\omega^2 = (0.3 \text{ m})(2.40 \text{ rad/s})^2 = 1.72 \text{ m/s}^2.$$

$$\boxed{\mathbf{a} = (3.59\mathbf{e}_t + 1.72\mathbf{e}_n) \text{ m/s}^2.}$$

Problem 13.129* For astronaut training, the airplane shown is to achieve "weightlessness" for a short period of time by flying along a path such that its acceleration is $a_x = 0$ and $a_y = -g$. If the velocity of the plane at O at time $t = 0$ is $\mathbf{v} = v_0\mathbf{i}$, show that the autopilot must fly the airplane so that its tangential component of the acceleration as a function of time is

$$a_t = g\frac{\left(\dfrac{gt}{v_0}\right)}{\sqrt{1+\left(\dfrac{gt}{v_0}\right)^2}}.$$



Solution: The velocity of the path is $\mathbf{v}(t) = v_0\mathbf{i} - gt\mathbf{j}$. The path angle is

$$\beta : \tan\beta = \frac{v_y}{v_x} = \frac{-gt}{v_0},$$

$$\sin\beta = \frac{-gt}{\sqrt{v_0^2 + (gt)^2}}.$$

The unit vector parallel to the velocity vector is $\mathbf{e} = \mathbf{i}\cos\beta + \mathbf{j}\sin\beta$. The acceleration vector is $\mathbf{a} = -\mathbf{j}g$. The component of the acceleration tangent to the flight path is $a_t = -g\sin\beta$., from which

$$a_t = g\frac{gt}{\sqrt{v_0^2 + (gt)^2}}.$$

Divide by v_0,

$$\boxed{a_t = g\left[1 + \left(\frac{gt}{v_0}\right)^2\right]^{-\frac{1}{2}}\left(\frac{gt}{v_0}\right)}$$

Problem 13.130* In Problem 13.129, what is the airplane's normal component of acceleration as a function of time?

Solution: From Problem 13.129, the velocity is $\mathbf{v}(t) = v_0\mathbf{i} - gt\mathbf{j}$. The flight path angle is β, from which

$$\cos\beta = \frac{v_0}{\sqrt{v_0^2 + (gt)^2}}.$$

The unit vector parallel to the flight path is $\mathbf{e} = \mathbf{i}\cos\beta + \mathbf{j}\sin\beta$. The unit vector normal to $\mathbf{e}$ is

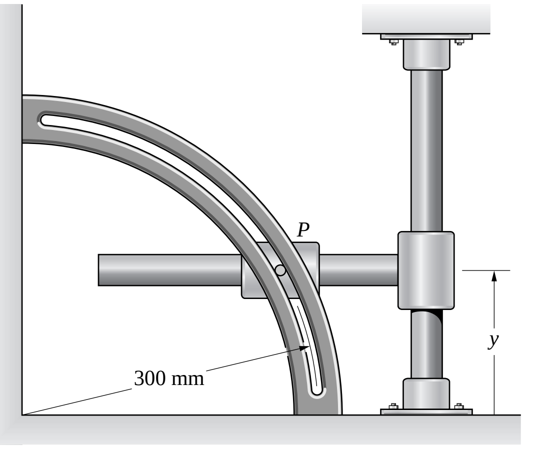
$$\mathbf{e}_n = \mathbf{i}\cos\left(\beta - \frac{\pi}{2}\right) + \mathbf{j}\sin\left(\beta - \frac{\pi}{2}\right)$$

$$= \mathbf{i}\sin\beta - \mathbf{j}\cos\beta,$$

pointing toward the instantaneous radial center of the path. The acceleration is $\mathbf{a} = -\mathbf{j}g$. The component parallel to the normal component is $a_n = g\cos\beta$, from which

$$\boxed{a_n = g\frac{v_0}{\sqrt{v_0^2 + (gt)^2}} = g\left[1 + \left(\frac{gt}{v_0}\right)^2\right]^{-\frac{1}{2}}}$$

Problem 13.131 If $y = 100$ mm, $\dfrac{dy}{dt} = 200$ mm/s, and $\dfrac{d^2y}{dt^2} = 0$, what are the velocity and acceleration of P in terms of normal and tangential components?



Solution: The equation for the circular guide is $R^2 = x^2 + y^2$, from which $x = \sqrt{R^2 - y^2} = 0.283$ m, and

$$\frac{dx}{dt} = -\left(\frac{y}{x}\right)\frac{dy}{dt} = v_x = -0.0707 \text{ m/s}.$$

The velocity of point **P** is $\mathbf{v_p} = \mathbf{i}v_x + \mathbf{j}v_y$, from which the velocity is $|\mathbf{v}| = \sqrt{v_x^2 + v_y^2} = 0.212$ m/s. The angular velocity

$$\omega = \frac{|\mathbf{v}|}{R} = 0.7071 \text{ rad/s}.$$

The angle is

$$\beta = \tan^{-1}\left(\frac{y}{x}\right) = 19.5^\circ$$

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(-\frac{y}{x}\frac{dy}{dt}\right)$$

$$= -\frac{1}{x}\left(\frac{dy}{dt}\right)^2 + \frac{y}{x^2}\left(\frac{dx}{dt}\right)\left(\frac{dy}{dt}\right) - \left(\frac{y}{x}\right)\left(\frac{d^2y}{dt^2}\right)$$

$$= -0.1591 \text{ m/s}^2$$

The unit vector tangent to the path (normal to the radius vector *for a circle*) is $\mathbf{e}_p = -\mathbf{i}\sin\beta + \mathbf{j}\cos\beta$, from which

$$\boxed{a_t = -a_x \sin\beta = 53.0 \text{ mm/s}^2}$$

since $a_y = 0$

$$\boxed{a_n = -R\omega^2 = -0.150 \text{ m/s}^2}.$$

Check: $a_n = a_x \cos\beta = -0.15$ m/s^2 *check*.

Problem 13.132* Suppose that the point P in Problem 13.131 moves upward in the slot with velocity $\mathbf{v} = 300\mathbf{e}_t$ (mm/s). When $y = 150$ mm, what are $\frac{dy}{dt}$ and $\frac{d^2y}{dt^2}$?

Solution: The position in the guide slot is $y = R\sin\theta$, from which

$$\theta = \sin^{-1}\left(\frac{y}{R}\right) = \sin^{-1}(0.5) = 30^\circ.$$

$$x = R\cos\theta = 259.8 \text{ mm}.$$

From the solution to Problem 13.131,

$$v_x = -\left(\frac{y}{x}\right)\frac{dy}{dt} = -\left(\frac{y}{x}\right)v_y.$$

The velocity is $|\mathbf{v}| = 300 = \sqrt{v_x^2 + v_y^2} = v_y\sqrt{\left(\frac{y}{x}\right)^2 + 1}$, from which

$$\boxed{v_y = 300\left(\left(\frac{y}{x}\right)^2 + 1\right)^{-\frac{1}{2}} = 259.8 \text{ mm/s}}$$

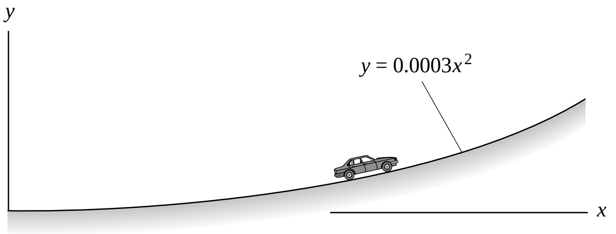
and $v_x = -150$ mm/s (Since the point is moving upward in the slot, v_y is positive.). The velocity along the path in the guide slot is assumed constant, hence $a_t = 0$. The normal acceleration is

$$a_n = \frac{|\mathbf{v}|^2}{R} = 300 \text{ mm/s}^2$$

directed toward the radius center, from which

$$\boxed{\frac{d^2y}{dt^2} = -a_n\sin\theta = -150 \text{ mm/s}^2}$$

Problem 13.133* A car travels at 100 km/h on a straight road of increasing grade whose vertical profile can be approximated by the equation shown. When $x = 400$ m, what are the tangential and normal components of the car's acceleration?



Solution: The strategy is to use the acceleration in cartesian coordinates found in the solution to Problem 13.90, find the angle with respect to the x-axis,

$$\theta = \tan^{-1}\left(\frac{dy}{dx}\right),$$

and use this angle to transform the accelerations to tangential and normal components. From the solution to Problem 13.90 the accelerations are $\mathbf{a} = -0.0993\mathbf{i} + 0.4139\mathbf{j}$ (m/s^2). The angle at

$$\theta = \tan^{-1}\left(\frac{d}{dx}Cx^2\right)_{x=400} = \tan^{-1}(6x \times 10^{-4})_{x=400} = 13.5^\circ.$$

From trigonometry (see figure) the transformation is $a_t = a_x\cos\theta + a_y\sin\theta$, $a_n = -a_x\sin\theta + a_y\cos\theta$, from which

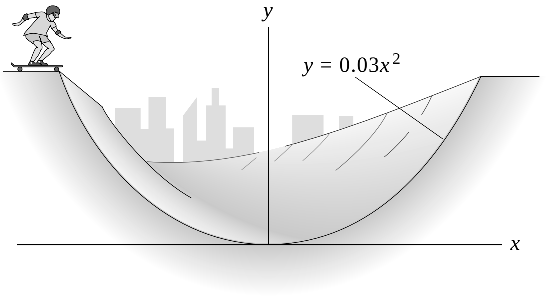
$\boxed{a_t = 0.000035\ldots = 0}$,

$\boxed{a_n = 0.4256 \text{ m/s}^2}$

Check: The velocity is constant along the path, so the tangential component of the acceleration is zero, $a_t = \frac{dv}{dt} = 0$, *check*.

Problem 13.134 A boy rides a skateboard on the concrete surface of an empty drainage canal described by the equation shown. He starts at $y = 20$ m, and the magnitude of his velocity is approximated by $v = \sqrt{2(9.81)(20 - y)}$ m/s.

(a) Use Equation (13.42) to determine the instantaneous radius of curvature of the boy's path when he reaches the bottom.
(b) What is the normal component of his acceleration when he reaches the bottom?



Solution:

(a) $y = 0.03x^2$, so

$$\frac{dy}{dx} = 0.06x \text{ and } \frac{d^2y}{dx^2} = 0.06.$$

From Eq (13.42),

$$\rho = \frac{[1 + (0.06x)^2]^{3/2}}{0.06} \text{ m.}$$

At $x = 0$, $\underline{\rho = 16.7 \text{ m}}$.

(b) The magnitude of the velocity is

$$\sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2} = v = K(20 - y)^{\frac{1}{2}} = K(20 - Cx^2)^{\frac{1}{2}},$$

where $K = 8.025$, $C = 0.03$. From $y = Cx^2$,

$$\frac{dy}{dt} = 2Cx\left(\frac{dx}{dt}\right),$$

$$\frac{d^2y}{dt^2} = 2C\left(\frac{dx}{dt}\right)^2 + 2Cx\left(\frac{d^2x}{dt^2}\right).$$

Substitute:

$$\left|\frac{dx}{dt}\right| = \frac{K(20 - Cx^2)^{\frac{1}{2}}}{(4C^2x^2 + 1)^{\frac{1}{2}}}.$$

Since the boy is moving the right,

$$\frac{dx}{dt} > 0, \quad \text{and} \quad \left|\frac{dx}{dt}\right| = \frac{dx}{dt}.$$

The acceleration is

$$\frac{d^2x}{dt^2} = \frac{-KCx}{(20 - Cx^2)^{\frac{1}{2}}(4C^2x^2 + 1)^{\frac{1}{2}}}\left(\frac{dx}{dt}\right)$$

$$- \frac{K(4C^2x)(20 - Cx^2)^{\frac{1}{2}}}{(4C^2x^2 + 1)^{\frac{3}{2}}}\left(\frac{dx}{dt}\right).$$

At the bottom of the canal the values are

$$\left(\frac{dx}{dt}\right)_{x=0} = K\sqrt{20} = 35.89 \text{ m/s.}$$

$$\left(\frac{dy}{dt}\right)_{x=0} = 0, \left(\frac{d^2x}{dt^2}\right)_{x=0} = 0,$$

$$\left(\frac{d^2y}{dt^2}\right)_{x=0} = 2C\left(\frac{dx}{dt}\right)^2\Bigg|_{x=0} = 77.28 \text{ m/s}^2.$$

The angle with respect to the x axis at the bottom of the canal is

$$\theta = \tan^{-1}\left(\frac{dy}{dx}\right)_{x=0} = 0.$$

From the solution to Problem 2.133, the tangential and normal accelerations are $a_t = a_x \cos\theta + a_y \sin\theta$, $a_n = -a_x \sin\theta + a_y \cos\theta$, from which

$\boxed{a_t = 0}$, and $\boxed{a_n = 77.28 \text{ m/s}^2}$.

Check: The velocity is constant along the path, so the tangential component of the acceleration is zero, $a_t = \frac{dv}{dt} = 0$. *check*. By inspection, the normal acceleration at the bottom of the canal is identical to the y component of the acceleration. *check*.

Problem 13.135 In Problem 13.134, what is the normal component of the boy's acceleration when he has passed the bottom and reached $y = 10$ m?

Solution: Use the results of the solutions to Problems 13.133 and 13.134. From the solution to Problem 13.134, at $y = 10$ m,

$$x = \left(\sqrt{\frac{y}{C}}\right) = 18.257 \text{ m}, \quad \text{from which}$$

$$\left(\frac{dx}{dt}\right)_{y=10} = \left(K(20 - Cx^2)^{\frac{1}{2}}(4C^2x^2 + 1)^{-\frac{1}{2}}\right)_{y=10} = 17.11 \text{ m/s}.$$

$$\left(\frac{d^2x}{dt^2}\right)_{y=10} = -K\left(\frac{dx}{dt}\right)_{y=10}\left(\frac{Cx}{(20 - Cx^2)^{\frac{1}{2}}(4C^2x^2 + 1)^{\frac{1}{2}}} + \frac{(4C^2x)(20 - Cx^2)^{\frac{1}{2}}}{(4C^2x^2 + 1)^{\frac{3}{2}}}\right)_{y=10}$$

$$= -24.78 \text{ m/s}.$$

$$\left(\frac{d^2y}{dt^2}\right)_{y=10} = \left(2C\left(\frac{dx}{dt}\right)^2 + 2Cx\left(\frac{d^2x}{dt^2}\right)\right)_{y=10} = -9.58 \text{ m/s}^2.$$

The angle is $\theta = \tan^{-1}\left(\frac{dy}{dx}\right)_{y=10} = 47.61°$.

From the solution to Problem 13.133,

$$a_t = a_x \cos\theta + a_y \sin\theta,\ a_n = -a_x \sin\theta + a_y \cos\theta,$$

from which

$$\boxed{a_t = -23.78 \text{ m/s}^2}, \boxed{a_n = 11.84 \text{ m/s}^2}$$

Problem 13.136* Using Eqs (13.41): (a) Show that the relations between the cartesian unit vectors and the unit vectors $\mathbf{e}_t$ and $\mathbf{e}_n$ are

$$\mathbf{i} = \cos\theta\,\mathbf{e}_t - \sin\theta\,\mathbf{e}_n$$

and $\quad \mathbf{j} = \sin\theta\,\mathbf{e}_t + \cos\theta\,\mathbf{e}_n$

(b) Show that

$$d\mathbf{e}_t/dt = d\theta/dt\,\mathbf{e}_n \quad \text{and} \quad d\mathbf{e}_n/dt = -d\theta/dt\,\mathbf{e}_t.$$

Solution: Equations (13.41) are $\mathbf{e}_t = \cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j}$ and $\mathbf{e}_n = -\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j}$.

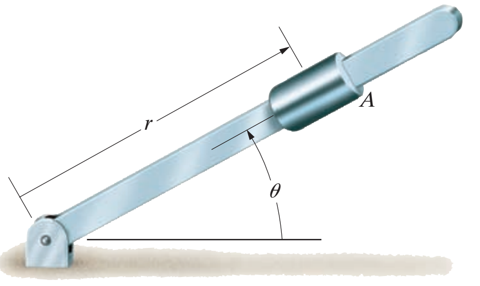
(a) Multiplying the equation for $\mathbf{e}_t$ by $\cos\theta$ and the equation for $\mathbf{e}_n$ by $(-\sin\theta)$ and adding the two equations, we get $\mathbf{i} = \cos\theta\,\mathbf{e}_t - \sin\theta\,\mathbf{e}_n$. Similarly, by multiplying the equation for $\mathbf{e}_t$ by $\sin\theta$ and the equation for $\mathbf{e}_n$ by $\cos\theta$ and adding, we get $\mathbf{j} = \sin\theta\,\mathbf{e}_t + \cos\theta\,\mathbf{e}_n$.

(b) Taking the derivative of $\mathbf{e}_t = \cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j}$, we get $\frac{d\mathbf{e}_t}{dt} = (-\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j})\frac{d\theta}{dt} = \mathbf{e}_n\frac{d\theta}{dt}$.
Similarly, taking the derivative of $\mathbf{e}_n = -\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j}$, we get $d\mathbf{e}_n/dt = -(d\theta/dt)\mathbf{e}_t$

Problem 13.137 The polar coordinates of the collar A are given as functions of time in seconds by

$$r = 1 + 0.2t^2 \text{ m and } \theta = 2t \text{ rad.}$$

What are the magnitudes of the velocity and acceleration of the collar at $t = 2$ s?



Solution: We have

$$r = (1 \text{ ft}) + (0.2 \text{ m/s}^2)t^2, \quad \theta = (2 \text{ rad/s})t,$$

$$\frac{dr}{dt} = (0.4 \text{ ft/s}^2)t, \qquad \frac{d\theta}{dt} = 2 \text{ rad/s},$$

$$\frac{d^2r}{dt^2} = 0.4 \text{ m/s}^2, \qquad \frac{d^2\theta}{dt^2} = 0$$

At time $t = 2$ s, we have

$$r = 1.8 \text{ ft}, \quad \frac{dr}{dt} = 0.8 \text{ m/s}, \qquad \frac{d^2r}{dt^2} = 0.4 \text{ m/s}^2,$$

$$\theta = 8 \text{ rad}, \quad \frac{d\theta}{dt} = 2 \text{ rad/s}, \qquad \frac{d^2\theta}{dt^2} = 0$$

The components of the velocity and acceleration are

$$v_r = \frac{dr}{dt} = 0.8 \text{ m/s}, v_\theta = r\frac{d\theta}{dt} = (1.8 \text{ m})(2 \text{ rad/s}) = 3.6 \text{ m/s},$$

$$a_r = \frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = (0.4 \text{ m/s}^2) - (1.8 \text{ m})(2 \text{ rad/s}^2) = -\ 6.8 \text{ m/s}^2,$$

$$a_\theta = r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt} = 0 + 2(0.8 \text{ m/s})(2 \text{ rad/s}) = 3.2 \text{ m/s}^2.$$

The magnitudes are

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(0.8 \text{ m/s})^2 + (3.6 \text{ m/s})^2} = 3.69 \text{ m/s},$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-6.8 \text{ m/s}^2)^2 + (3.2 \text{ m/s}^2)^2} = 7.52 \text{ m/s}^2.$$

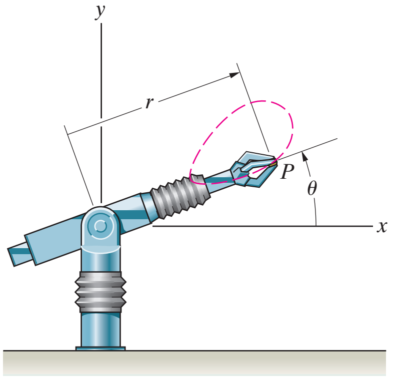
$$\boxed{v = 3.69 \text{ m/s}, a = 7.52 \text{ m/s}^2.}$$

Problem 13.138 In Active Example 13.13, suppose that the robot arm is reprogrammed so that the point P traverses the path described by

$r = 1 - 0.5 \sin 2\pi t$ m,

$\theta = 0.5 - 0.2 \cos 2\pi t$ rad.

What is the velocity of P in terms of polar coordinates at $t = 0.8$ s?



Solution: We have

$$r = \left[1 - 0.5 \sin\left(2\pi \frac{t}{s}\right)\right] \text{ m}, \qquad \theta = \left[0.5 - 0.2 \cos\left(2\pi \frac{t}{s}\right)\right] \text{rad},$$

$$\frac{dr}{dt} = -\pi \cos\left(2\pi \frac{t}{s}\right) \text{ m/s}, \qquad \frac{d\theta}{dt} = 0.4\pi \sin\left(2\pi \frac{t}{s}\right) \text{ rad/s}.$$

At time t = 0.8 s,

$$r = 1.48 \text{ m}, \frac{dr}{dt} = -0.971 \text{ m/s}, \theta = 0.438 \text{ rad}, \frac{d\theta}{dt} = -1.20 \text{ rad/s}.$$

The velocity is

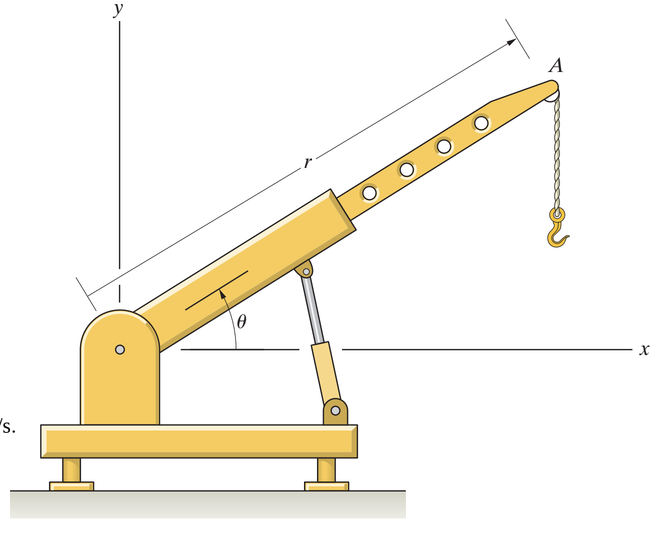
$$v_r = \frac{dr}{dt} = -0.971 \text{ m/s}, v_\theta = r\frac{d\theta}{dt} = (1.48 \text{ m})(-1.20 \text{ rad/s}) = -1.76 \text{ m/s}.$$

$$\boxed{\mathbf{v} = (-0.971\mathbf{e}_r - 1.76\mathbf{e}_\theta) \text{ m/s.}}$$

Problem 13.139 At the instant shown, $r = 3$ m and $\theta = 30^\circ$. The cartesian components of the velocity of point A are $v_x = 2$ m/s and $v_y = 8$ m/s.

(a) Determine the velocity of point A in terms of polar coordinates.

(b) What is the angular velocity $d\theta/dt$ of the crane at the instant shown?



Solution: To transform to polar coordinates we have

$$v_r = v_x \cos\theta + v_y \sin\theta = (2 \text{ m/s}) \cos 30^\circ + (8 \text{ m/s}) \sin 30^\circ = 5.73 \text{ m/s}.$$

$$v_\theta = -v_x \sin\theta + v_y \cos\theta = -(2 \text{ m/s}) \sin 30^\circ + (8 \text{ m/s}) \cos 30^\circ = 5.93 \text{ m/s}.$$

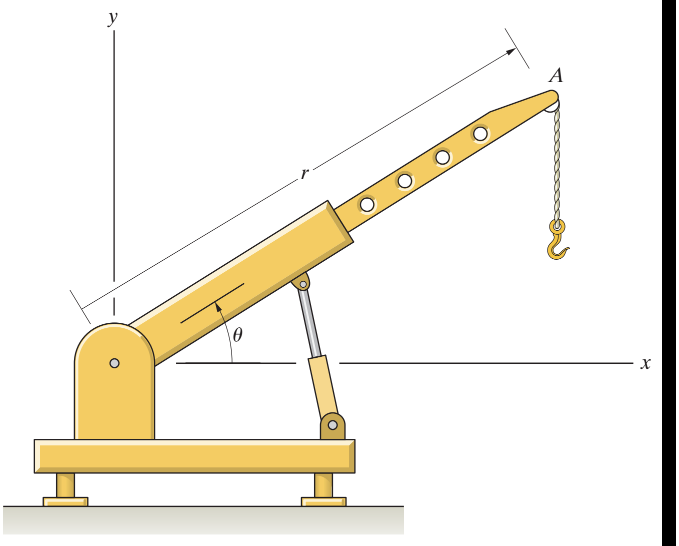
$$\boxed{\mathbf{v}_A = (5.73\mathbf{e}_r + 5.93\mathbf{e}_\theta) \text{m/s.}}$$

The angular velocity is found

$$v_\theta = r\frac{d\theta}{dt} \Rightarrow \frac{d\theta}{dt} = \frac{v_\theta}{r} = \frac{5.93 \text{ m/s}}{3\text{m}} = 1.98 \text{ rad/s}.$$

$$\boxed{\frac{d\theta}{dt} = 1.98 \text{ rad/s.}}$$

Problem 13.140 The polar coordinates of point A of the crane are given as functions of time in seconds by $r = 3 + 0.2t^2$ m and $\theta = 0.02t^2$ rad. Determine the acceleration of point A in terms of polar coordinates at $t = 3s$.



Solution: We have

$$r = (3 \text{ m}) + (0.2 \text{ m/s}^2)t^2,$$

$$\frac{dr}{dt} = (0.4 \text{ m/s}^2)t,$$

$$\frac{d^2r}{dt^2} = 0.4 \text{ m/s}^2,$$

$$\theta = (0.02)t^2 \text{ rad/s}^2.$$

$$\frac{d\theta}{dt} = (0.04)t \text{ rad/s}^2,$$

$$\frac{d^2\theta}{dt^2} = 0.04 \text{ rad/s}^2$$

At $t = 3$ s,

$$r = 4.8 \text{ m}, \quad \frac{dr}{dt} = 1.2 \text{ m/s}, \quad \frac{d^2r}{dt^2} = 0.4 \text{ m/s}^2,$$

$$\theta = 0.18 \text{ rad}, \quad \frac{d\theta}{dt} = 0.12 \text{ rad/s}, \quad \frac{d^2\theta}{dt^2} = 0.04 \text{ rad/s}^2.$$

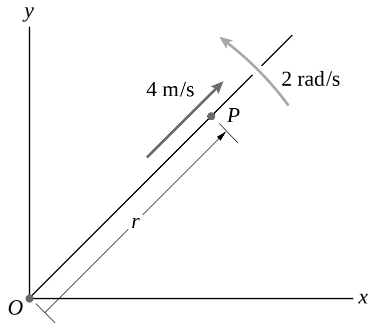
The acceleration components are

$$a_r = \frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = (0.4 \text{ m/s}^2) - (4.8 \text{ m})(0.12 \text{ rad/s})^2,$$

$$a_\theta = r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt} = (4.8 \text{ m})(0.04 \text{ rad/s}^2) + 2(1.2 \text{ m/s})(0.12 \text{ rad/s})$$

$$\boxed{\mathbf{a}_A = (0.331\mathbf{e}_r + 0.480\mathbf{e}_\theta) \text{ m/s}^2.}$$

Problem 13.141 The radial line rotates with a constant angular velocity of 2 rad/s. Point P moves along the line at a constant speed of 4 m/s. Determine the magnitude of the velocity and acceleration of P when $r = 2$ m. (See Example 13.14.)



Solution: The angular velocity of the line is

$$\frac{d\theta}{dt} = \omega = 2 \text{ rad/s},$$

from which $\dfrac{d^2\theta}{dt^2} = 0.$

The radial velocity of the point is

$$\frac{dr}{dt} = 4 \text{ m/s},$$

from which $\dfrac{d^2 r}{dt^2} = 0.$

The vector velocity is

$$\mathbf{v} = \left(\frac{dr}{dt}\right)\mathbf{e}_r + r\left(\frac{d\theta}{dt}\right)\mathbf{e}_\theta = 4\mathbf{e}_r + 4\mathbf{e}_\theta \text{ (m/s)}.$$

The magnitude is

$$\boxed{|\mathbf{v}| = \sqrt{4^2 + 4^2} = 5.66 \text{ m/s}}.$$

The acceleration is

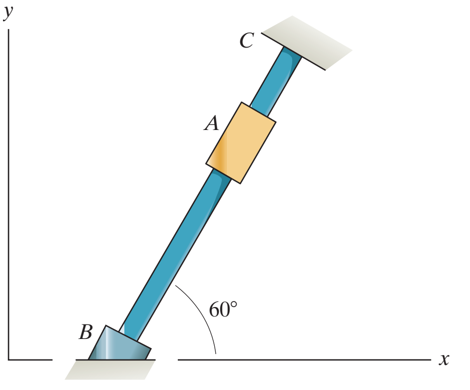
$$\mathbf{a} = [-2(4)]\mathbf{e}_r + [2(4)(2)]\mathbf{e}_\theta = -8\mathbf{e}_r + 16\mathbf{e}_\theta \text{ (m/s}^2\text{)}.$$

The magnitude is

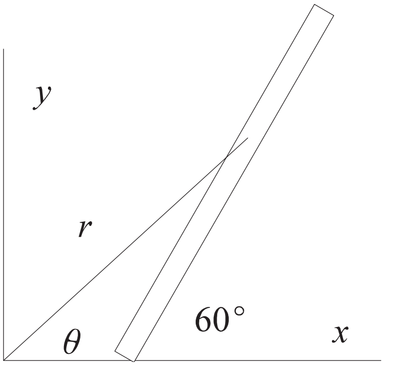
$$\boxed{|\mathbf{a}| = \sqrt{8^2 + 16^2} = 17.89 \text{ m/s}^2}$$

Problem 13.142 At the instant shown, the coordinates of the collar A are $x = 2.3$ m, $y = 1.9$ m. The collar is sliding on the bar from B toward C at a constant speed of 4 m/s.

(a) What is the velocity of the collar in terms of polar coordinates?
(b) Use the answer to part (a) to determine the angular velocity of the radial line from the origin to the collar A at the instant shown.



Solution: We will write the velocity in terms of cartesian coordinates.



$v_x = (4 \text{ m/s}) \cos 60^\circ = 2 \text{ m/s},$

$v_y = (4 \text{ m/s}) \sin 60^\circ = 3.46 \text{ m/s}.$

The angle θ is

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{1.9 \text{ m}}{2.3 \text{ m}}\right) = 39.6^\circ.$$

Now we can convert to polar coordinates

$v_r = v_x \cos\theta + v_y \sin\theta = (2 \text{ m/s}) \cos 39.6^\circ + (3.46 \text{ m/s}) \sin 39.6^\circ,$

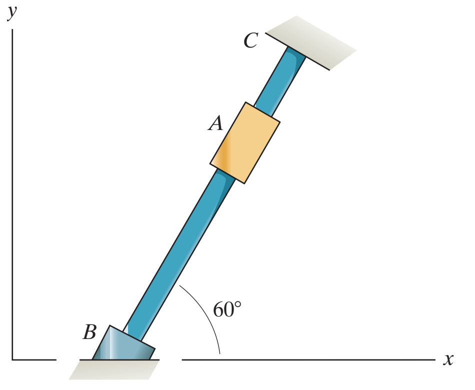
$v_\theta = -v_x \sin\theta + v_y \cos\theta = -(2 \text{ m/s}) \sin 39.6^\circ + (3.46 \text{ m/s}) \cos 39.6^\circ.$

(a) $\boxed{\mathbf{v} = (3.75\mathbf{e}_r + 1.40\mathbf{e}_\theta) \text{ m/s.}}$

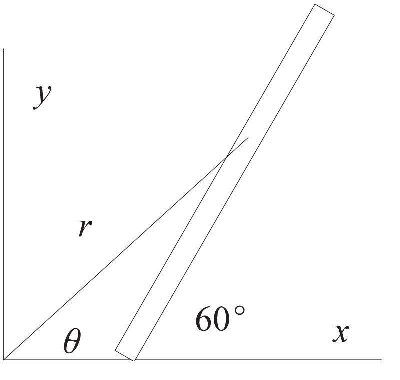
(b) $\dfrac{d\theta}{dt} = \dfrac{v_\theta}{r} = \dfrac{1.40 \text{ m/s}}{\sqrt{(2.3 \text{ m})^2 + (1.9 \text{ m})^2}} = 0.468 \text{ rad/s.}$ $\boxed{\dfrac{d\theta}{dt} = 0.468 \text{ rad/s.}}$

Problem 13.143 At the instant shown, the coordinates of the collar A are $x = 2.3$ m, $y = 1.9$ m. The collar is sliding on the bar from B toward C at a constant speed of 4 m/s.

(a) What is the acceleration of the collar in terms of polar coordinates?
(b) Use the answer to part (a) to determine the angular acceleration of the radial line from the origin to the collar A at the instant shown.



Solution: The velocity is constant, so the acceleration is zero.



(a) $\boxed{\mathbf{a}_A = 0.}$

Form Problem 13.141 we know that

$$r = \sqrt{(2.3\text{ m})^2 + (1.9\text{ m})^2} = 2.98\text{ m},$$

$$\frac{dr}{dt} = v_r = 3.75\text{ m/s},$$

$$\frac{d\theta}{dt} = 0.468\text{ rad/s}.$$

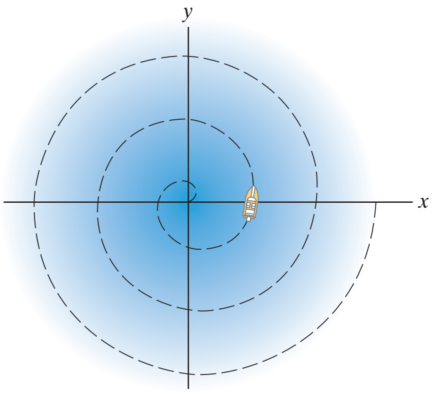
Using the θ component of the acceleration we can solve for the angular acceleration.

$$a_\theta = r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt} = 0$$

$$\frac{d^2\theta}{dt^2} = -\frac{2}{r}\frac{dr}{dt}\frac{d\theta}{dt} = -\frac{2}{2.98\text{ m}}(3.75\text{ m/s})(0.486\text{ rad/s})$$

$$\boxed{\frac{d^2\theta}{dt^2} = -1.18\text{ rad/s}^2.}$$

Problem 13.144 A boat searching for underwater archaeological sites in the Aegean Sea moves at 2.06 m/s and follows the path $r = 10\theta$ m, where θ is in radians. When $\theta = 2\pi$ rad, determine the boat's velocity (a) in terms of polar coordinates and (b) in terms of cartesian coordinates.



Solution: The velocity along the path is

$$v = 2.06 \text{ m/s.}$$

(a) The path is $r = 10\theta$. The velocity

$$v_r = \frac{dr}{dt} = \frac{d}{dt}(10\theta) = 10\frac{d\theta}{dt} \text{ m/s.}$$

The velocity along the path is related to the components by

$$v^2 = v_r^2 + v_\theta^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2 = 2.06^2.$$

At $\theta = 2\pi$, $r = 10(2\pi) = 62.8$ m. Substitute:

$$2.06^2 = \left(10\frac{d\theta}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2 = (100 + 62.8^2)\left(\frac{d\theta}{dt}\right)^2,$$

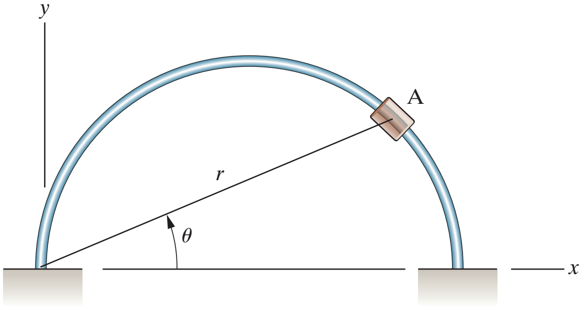
from which $\frac{d\theta}{dt} = 0.0323$ rad/s,

$$\boxed{v_r = 10\frac{d\theta}{dt} = 0.323 \text{ m/s}}, \quad \boxed{v_\theta = r\frac{d\theta}{dt} = 2.032 \text{ m/s}}$$

(b) From geometry, the cartesian components are $v_x = v_r\cos\theta + v_\theta\sin\theta$, and $v_y = v_r\sin\theta + v_\theta\cos\theta$. At $\theta = 2\pi$,

$$\boxed{v_x = v_r}, \quad \text{and} \quad \boxed{v_y = v_\theta}$$

Problem 13.145 The collar A slides on the circular bar. The radial position of A (in meters) is given as a function of θ by $r = 2\cos\theta$. At the instant shown, $\theta = 25^\circ$ and $d\theta/dt = 4$ rad/s. Determine the velocity of A in terms of polar coordinates.



Solution:

$r = 2\cos\theta$, $\dot{r} = -2\sin\theta\dot{\theta}$, $\ddot{r} = -2\sin\theta\ddot{\theta} - 2\cos\theta\dot{\theta}^2$

Using the given data we have

$\theta = 25^\circ$, $\dot{\theta} = 4$, $\ddot{\theta} = 0$

$r = 1.813$, $\dot{r} = -3.381$, $\ddot{r} = -29.00$

$$\boxed{\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta = (-3.381\mathbf{e}_r + 7.25\mathbf{e}_\theta) \text{ m/s}}$$

Problem 13.146 In Problem 13.145, $d^2\theta/dt^2 = 0$ at the instant shown. Determine the acceleration of A in terms of polar coordinates.

Solution: See Problem 13.145

$$\boxed{\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta = (-58.0\mathbf{e}_r - 27.0\mathbf{e}_\theta)\ \text{m/s}^2}$$

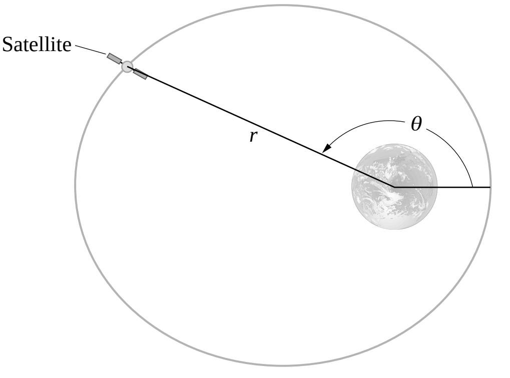
Problem 13.147 The radial coordinate of the earth satellite is related to its angular position θ by

$$r = \frac{1.91 \times 10^7}{1 + 0.5\cos\theta}\ \text{m}.$$

The product of the radial position and the transverse component of the velocity is

$$rv_\theta = 8.72 \times 10^{10}\ \text{m}^2/\text{s}.$$

What is the satellite's velocity in terms of polar coordinates when $\theta = 90^\circ$?



Solution:

At $\theta = 90^\circ$, $r = 1.91 \times 10^7\ \text{m} = p$

$$r = \frac{p}{1 + 0.5\cos\theta},$$

$$\dot{r} = \frac{(-p)(0.5)(-\sin\theta)\dot{\theta}}{(1 + 0.5\cos\theta)^2}$$

We also know that

$$rv_\theta = 8.72 \times 10^{10}\ \text{m}^2/\text{s}$$

However $v_\theta = r\dot{\theta}$, hence

$$r^2\dot{\theta} = 8.72 \times 10^{10}\ \text{m}^2/\text{s}$$

Solving for $\dot{\theta}$, we get

$$\dot{\theta} = 0.000239\ \text{rad/s}$$

and $\dot{r} = 2283$ m/s and from above

$$v_\theta = 4565\ \text{m/s}$$

$$\underline{\mathbf{v} = 2283\mathbf{e}_r + 4565\mathbf{e}_\theta\ (\text{m/s})}$$

Problem 13.148* In Problem 13.147, what is the satellite's acceleration in terms of polar coordinates when $\theta = 90^\circ$?

Solution: Set $A = 1.91 \times 10^7$ m, $B = 8.72 \times 10^{10}$ m^2/s

$$r = \frac{A}{1 + 0.5\cos\theta}, \; r v_\theta = r(r\dot{\theta}) = B$$

$$\dot{\theta} = \frac{B}{r^2} = \left(\frac{B}{A^2}\right)(1 + 0.5\cos\theta)^2$$

$$\ddot{\theta} = -\left(\frac{B}{A^2}\right)(1 + 0.5\cos\theta)\sin\theta\dot{\theta} = -\left(\frac{B^2}{A^4}\right)(1 + 0.5\cos\theta)^3\sin\theta$$

$$\dot{r} = \frac{0.5A\sin\theta}{(1 + 0.5\cos\theta)^2}\dot{\theta} = \frac{0.5B\sin\theta}{A}$$

$$\ddot{r} = \frac{0.5B\cos\theta}{A}\dot{\theta} = 0.5\left(\frac{B^2}{A^3}\right)\cos\theta(1 + 0.5\cos\theta)^2$$

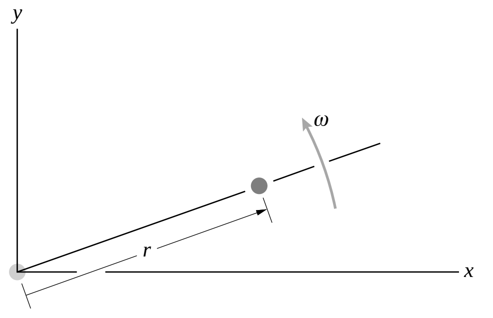
When $\theta = 90^\circ$ we have

$$r = A, \; \dot{r} = \frac{B}{2A}, \; \ddot{r} = 0, \; \dot{\theta} = \frac{B}{A^2}, \; \ddot{\theta} = -\frac{B^2}{A^4}$$

Thus

$$\boxed{\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta = (-1.091\mathbf{e}_r) \text{ m/s}^2}$$

Problem 13.149 A bead slides along a wire that rotates in the xy-plane with constant angular velocity ω_0. The radial component of the bead's acceleration is zero. The radial component of its velocity is v_0 when $r = r_0$. Determine the polar components of the bead's velocity as a function of r.



Strategy: The radial component of the bead's velocity is $v_r = \dfrac{dr}{dt}$, and the radial component of its acceleration is

$$a_r = \frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = \left(\frac{dv_r}{dt}\right) - r\omega_0^2.$$

By using the chain rule,

$$\frac{dv_r}{dt} = \frac{dv_r}{dr}\frac{dr}{dt} = \frac{dv_r}{dr}v_r.$$

you can express the radial component of the acceleration in the form $a_r = \dfrac{dv_r}{dr}v_r - r\omega_0^2$.

Solution: From the strategy:

$$a_r = 0 = v_r\frac{dv_r}{dr} - \omega_0^2 r.$$

Separate variables and integrate: $v_r dv_r = \omega_0^2 r\,dr$, from which

$$\frac{v_r^2}{2} = \omega_0^2\frac{r^2}{2} + C.$$

At $r = r_0$, $v_r = v_0$, from which

$$C = \frac{v_0^2 - \omega_0^2 r_0^2}{2}, \quad \text{and}$$

$$v_r = \sqrt{v_0^2 + \omega_0^2(r^2 - r_0^2)}.$$

The transverse component is

$$v_\theta = r\left(\frac{d\theta}{dt}\right) = r\omega_0, \text{ from which}$$

$$\boxed{\mathbf{v} = \sqrt{v_0^2 + \omega_0^2(r^2 - r_0^2)}\mathbf{e}_r + r\omega_0\mathbf{e}_\theta}$$

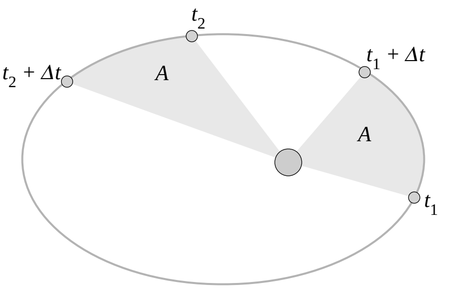
Problem 13.150 If the motion of a point in the $x-y$-plane is such that its transverse component of acceleration a_θ is zero, show that the product of its radial position and its transverse velocity is constant: $rv_\theta =$ constant.

Solution: We are given that $a_\theta = r\alpha + 2v_r\omega = 0$. Multiply the entire relationship by r. We get

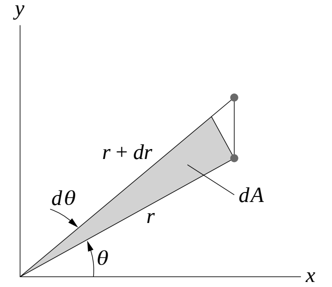
$$0 = (r^2\alpha + 2rv_r\omega) = \left(r^2\left(\frac{d\omega}{dt}\right) + 2r\left(\frac{dr}{dt}\right)_r\omega\right) = \frac{d}{dt}(r^2\omega).$$

Note that if $\dfrac{d}{dt}(r^2\omega) = 0$, then $r^2\omega =$ constant. Now note that $v_\theta = r\omega$. We have $r^2\omega = r(r\omega) = rv_\theta =$ constant. This was what we needed to prove.

Problem 13.151* From astronomical data, Kepler deduced that the line from the sun to a planet traces out equal areas in equal times (Fig. a). Show that this result follows from the fact that the transverse component a_θ of the planet's acceleration is zero. [When r changes by an amount dr and θ changes by an amount $d\theta$ (Fig. b), the resulting differential element of area is $dA = \frac{1}{2}r(r\,d\theta)$].



(a)



(b)

Solution: From the solution to Problem 13.150, $a_\theta = 0$ implies that

$$r^2\omega = r^2\frac{d\theta}{dt} = \text{constant}.$$

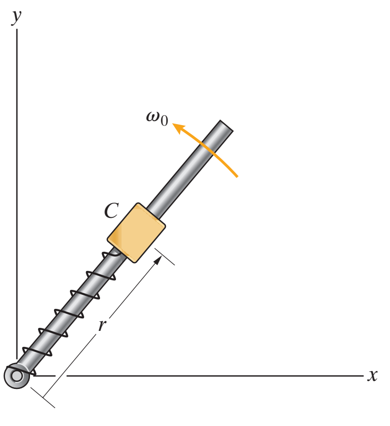
The element of area is

$$dA = \frac{1}{2}r(r\,d\theta),$$

or $$\frac{dA}{dt} = \frac{1}{2}r\left(r\frac{d\theta}{dt}\right) = \frac{1}{2}r^2\omega = \text{constant}.$$

Thus, if $\dfrac{dA}{dt}$ = constant, then equal areas are swept out in equal times.

Problem 13.152 The bar rotates in the $x-y$ plane with constant angular velocity $\omega_0 = 12$ rad/s. The radial component of acceleration of the collar C (in m/s^2) is given as a function of the radial position in meters by $a_r = -8r$. When $r = 1$ m, the radial component of velocity of C is $v_r = 2$ m/s. Determine the velocity of C in terms of polar coordinates when $r = 1.5$ m.



Strategy: Use the chain rule to write the first term in the radial component of the acceleration as

$$\frac{d^2r}{dt^2} = \frac{dv_r}{dt} = \frac{dv_r}{dr}\frac{dr}{dt} = \frac{dv_r}{dr}v_r$$

Solution: We have

$$a_r = \frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = -(8\text{ rad/s}^2)r,$$

$$\frac{d^2r}{dt^2} = \left(\left[\frac{d\theta}{dt}\right]^2 - (8\text{ rad/s}^2)\right)r = ([12\text{ rad/s}]^2 - [8\text{ rad/s}^2])r = ([136\text{ rad/s}^2)r$$

Using the supplied strategy we can solve for the radial velocity

$$\frac{d^2r}{dt^2} = v_r\frac{dv_r}{dr} = (136\text{ rad/s}^2)r$$

$$\int_{2\text{ m/s}}^{v_r} v_r\,dv_r = (136\text{ rad/s}^2)\int_{1\text{ m}}^{1.5\text{ m}} r\,dr$$

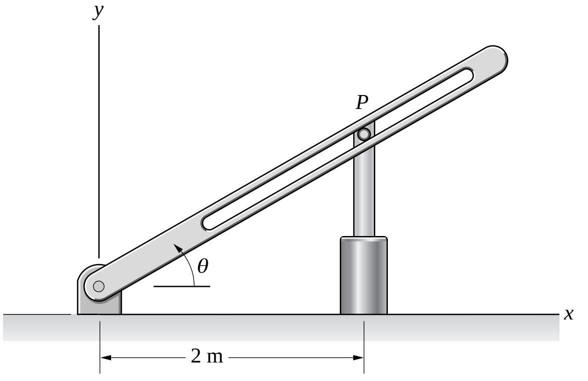
$$\frac{v_r^2}{2} - \frac{(2\text{ m/s})^2}{2} = (136\text{ rad/s}^2)\left(\frac{[1.5\text{ m}]^2}{2} - \frac{[1\text{ m}]^2}{2}\right)$$

Solving we find $v_r = 13.2$ m/s.

We also have $v_\theta = r\dfrac{d\theta}{dt} = (1.5\text{ m})(12\text{ rad/s}) = 18$ m/s.

Thus $\boxed{\mathbf{v} = (13.2\mathbf{e}_r + 18\mathbf{e}_\theta)\text{ m/s.}}$

Problem 13.153 The hydraulic actuator moves the pin P upward with constant velocity $\mathbf{v} = 2\mathbf{j}$ (m/s). Determine the velocity of the pin in terms of polar coordinates and the angular velocity of the slotted bar when $\theta = 35^\circ$.



Solution:

$$\mathbf{v}_P = 2\mathbf{j} \text{ (m/s)}$$

$$\mathbf{r} = r\mathbf{e}_r$$

$$\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta$$

Also, $\mathbf{r} = 2\mathbf{i} + y\mathbf{j}$ (m)

$$\mathbf{v} = \dot{y}\mathbf{j} \text{ (m/s)} = 2\mathbf{j} \text{ (m/s)}$$

$$\dot{r} = \dot{y}\sin\theta \quad \tan\theta = \frac{y}{x}$$

$$r\dot{\theta} = \dot{y}\cos\theta$$

$$r = \sqrt{x^2 + y^2}$$

$$\theta = 35^\circ,$$

Solving, we get $y = 1.40$ m,

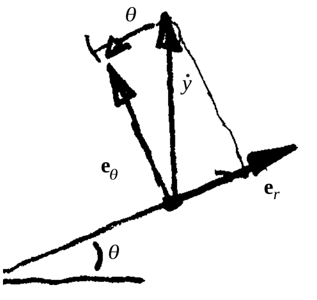
$$\dot{r} = 1.15 \text{ m/s},$$

$$r = 2.44 \text{ m},$$

$$\dot{\theta} = 0.671 \ \frac{\text{rad}}{\text{s}}$$

Hence $\mathbf{V} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta$

$$\underline{\mathbf{V} = 1.15\mathbf{e}_r + 1.64\overline{e}_\theta \text{ (m/s)}}$$



Problem 13.154 The hydraulic actuator moves the pin P upward with constant velocity $\mathbf{v} = 2\mathbf{j}$ (m/s). Determine the acceleration of the pin in terms of polar coordinates and the angular acceleration of the slotted bar when $\theta = 35^\circ$.

Solution: From Problem 13.153

$\mathbf{V} = 2\mathbf{j}$ m/s, constant

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} \equiv 0 \qquad \begin{array}{l} \dot{\theta} = \omega = 0.671 \text{ rad/s} \\ \theta = 35^\circ \end{array}$$

$$\dot{\theta} = \omega \quad \dot{y} = 2 \text{ m/s}$$

$$\ddot{\theta} = \alpha \quad \ddot{y} = 0$$

$$\tan\theta = \frac{y}{x} = \frac{y}{2}$$

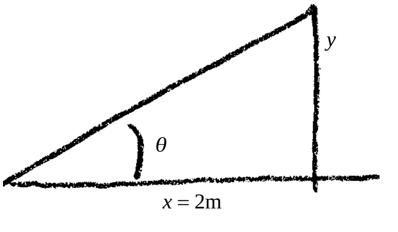
$$y = 2\tan\theta$$

$$\dot{y} = 2\sec^2\theta\dot{\theta}$$

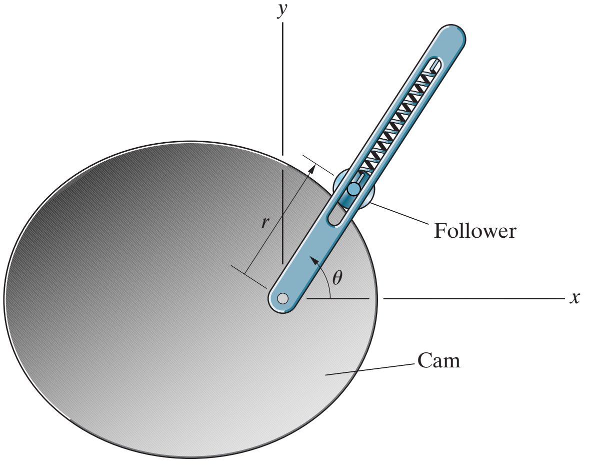
$$\ddot{y} = 2(2\sec\theta)(\sec\theta\tan\theta)\dot{\theta}^2 + 2\sec^2\theta\ddot{\theta}$$

$$\ddot{\theta} = \frac{[-2\sec\theta\tan\theta](\dot{\theta})^2}{\sec\theta}$$

$$\dot{\theta} = 0.631 \text{ rad/s}^2$$



Problem 13.155 In Example 13.15, determine the velocity of the cam follower when $\theta = 135^\circ$ (a) in terms of polar coordinates and (b) in terms of cartesian coordinates.



Solution:

(a) $\theta = 135^\circ$, $\omega = d\theta/dt = 4 rad/s$, and $\alpha = 0$.

$$r = 0.15(1 + 0.5\cos\theta)^{-1}$$

$$= 0.232 \text{ m.}$$

$$\frac{dr}{dt} = 0.075\frac{d\theta}{dt}\sin\theta(1 + 0.5\cos\theta)^{-2}$$

$$= 0.508 \text{ m/s.}$$

$$\mathbf{v} = \frac{dr}{dt}\mathbf{e}_r + r\frac{d\theta}{dt}\mathbf{e}_\theta$$

$$= 0.508\mathbf{e}_r + 0.928\mathbf{e}_\theta \text{ (m/s).}$$

(b) $v_x = v_r\cos\theta - v_\theta\sin\theta$

$$= -1.015 \text{ m/s.}$$

$$v_y = v_r\sin\theta + v_\theta\cos\theta$$

$$= -0.297 \text{ m/s.}$$

Problem 13.156* In Example 13.15, determine the acceleration of the cam follower when $\theta = 135^\circ$ (a) in terms of polar coordinates and (b) in terms of cartesian coordinates.

Solution: See the solution of Problem 13.155.

(a) $$\frac{d^2r}{dt^2} = 0.075\left(\frac{d\theta}{dt}\right)^2\cos\theta(1 + 0.5\cos\theta)^{-2}$$

$$+ 0.075\left(\frac{d\theta}{dt}\right)^2\sin^2\theta(1 + 0.5\cos\theta)^{-3}$$

$$= 0.1905 \text{ m/s}^2.$$

$$\mathbf{a} = \left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right]\mathbf{e}_r + \left[r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}\right]\mathbf{e}_\theta$$

$$= -3.52\mathbf{e}_r + 4.06\mathbf{e}_\theta \text{ (m/s}^2\text{).}$$

(b) $a_x = a_r\cos\theta - a_\theta\sin\theta$

$$= -0.381 \text{ m/s}^2$$

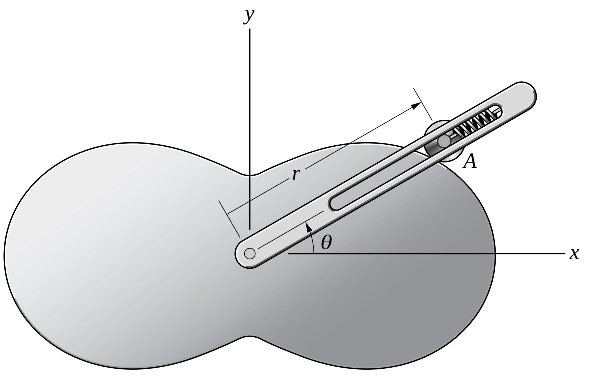
$$a_y = a_r\sin\theta + a_\theta\cos\theta$$

$$= -5.362 \text{ m/s}^2.$$

Problem 13.157 In the cam-follower mechanism, the slotted bar rotates with constant angular velocity $\omega = 10$ rad/s and the radial position of the follower A is determined by the profile of the stationary cam. The path of the follower is described by the polar equation

$$r = 1 + 0.5\cos(2\theta)\ \text{m}.$$

Determine the velocity of the cam follower when $\theta = 30^\circ$ (a) in terms of polar coordinates and (b) in terms of cartesian coordinates.



Solution:

(a) $\theta = 30^\circ$, $\omega = d\theta/dt = 10$ rad/s, and $\alpha = 0$.

$$r = 1 + 0.5\cos 2\theta$$

$$= 1.25\ \text{m}.$$

$$\frac{dr}{dt} = -\frac{d\theta}{dt}\sin 2\theta$$

$$= -8.66\ \text{m/s}.$$

$$\mathbf{v} = \frac{dr}{dt}\mathbf{e}_r + r\frac{d\theta}{dt}\mathbf{e}_\theta$$

$$= -8.66\mathbf{e}_r + 12.5\mathbf{e}_\theta\ \text{(m/s)}.$$

(b) $v_x = v_r\cos\theta - v_\theta\sin\theta$

$$= -13.75\ \text{m/s},$$

$$v_y = v_r\sin\theta + v_\theta\cos\theta$$

$$= 6.50\ \text{m/s}.$$

Problem 13.158* In Problem 13.157, determine the acceleration of the cam follower when $\theta = 30^\circ$ (a) in terms of polar coordinates and (b) in terms of cartesian coordinates.

Solution: See the solution of Problem 13.157.

(a) $$\frac{d^2r}{dt^2} = -2\theta^2\cos 2\theta$$

$$= -100\ \text{m/s}^2.$$

$$\mathbf{a} = \left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right]\mathbf{e}_r + \left[r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}\right]\mathbf{e}_\theta.$$

$$= -225\mathbf{e}_r - 173\mathbf{e}_\theta\ (\text{m/s}^2).$$

(b) $a_x = a_r\cos\theta - a_\theta\sin\theta$

$$= -108\ \text{m/s}^2,$$

$$a_y = a_r\sin\theta + a_\theta\cos\theta$$

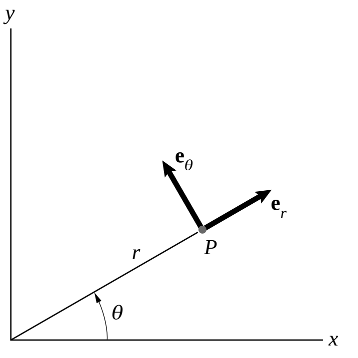
$$= -263\ \text{m/s}^2.$$

Problem 13.159 The cartesian coordinates of a point P in the $x-y$ plane are related to its polar coordinates of the point by the equations $x = r\cos\theta$ and $y = r\sin\theta$.

(a) Show that the unit vectors $\mathbf{i}$, $\mathbf{j}$ are related to the unit vectors $\mathbf{e}_r$, $\mathbf{e}_\theta$ by $\mathbf{i} = \mathbf{e}_r\cos\theta - \mathbf{e}_\theta\sin\theta$ and $\mathbf{j} = \mathbf{e}_r\sin\theta + \mathbf{e}_\theta\cos\theta$.

(b) Beginning with the expression for the position vector of P in terms of cartesian coordinates, $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$, derive Eq. (13.52) for the position vector in terms of polar coordinates.

(c) By taking the time derivative of the position vector of point P expressed in terms of cartesian coordinates, derive Eq. (13.47) for the velocity in terms of polar coordinates.



Solution:

(a) From geometry (see Figure), the radial unit vector is $\mathbf{e}_r = \mathbf{i}\cos\theta + \mathbf{j}\sin\theta$, and since the transverse unit vector is at right angles:

$$\mathbf{e}_\theta = \mathbf{i}\cos\left(\theta + \frac{\pi}{2}\right) + \mathbf{j}\sin\left(\theta + \frac{\pi}{2}\right) = -\mathbf{i}\sin\theta + \mathbf{j}\cos\theta.$$

Solve for $\mathbf{i}$ by multiplying $\mathbf{e}_r$ by $\cos\theta$, $\mathbf{e}_\theta$ by $\sin\theta$, and subtracting the resulting equations:

$$\boxed{\mathbf{i} = \mathbf{e}_r\cos\theta - \mathbf{e}_\theta\sin\theta}.$$

Solve for $\mathbf{j}$ by multiplying $\mathbf{e}_r$ by $\sin\theta$, and $\mathbf{e}_\theta$ by $\cos\theta$, and the results:

$$\boxed{\mathbf{j} = \mathbf{e}_r\sin\theta + \mathbf{e}_\theta\cos\theta}$$

(b) The position vector is $\mathbf{r} = x\mathbf{i} + y\mathbf{j} = (r\cos\theta)\mathbf{i} + (r\sin\theta)\mathbf{j} = r(\mathbf{i}\cos\theta + \mathbf{j}\sin\theta)$. Use the results of Part (a) expressing $\mathbf{i}$, $\mathbf{j}$ in terms of $\mathbf{e}_r$, $\mathbf{e}_\theta$:

$$\boxed{\begin{aligned}\mathbf{r} &= r(\mathbf{e}_r\cos^2\theta - \mathbf{e}_\theta\cos\theta\sin\theta + \mathbf{e}_r\sin^2\theta + \mathbf{e}_\theta\sin\theta\cos\theta)\\ &= r\mathbf{e}_r\end{aligned}}$$

(c) The time derivatives are:

$$\frac{d\mathbf{r}}{dt} = \mathbf{v} = \mathbf{i}\left(\frac{dr}{dt}\cos\theta - r\sin\theta\frac{d\theta}{dt}\right)$$

$$+\mathbf{j}\left(\frac{dr}{dt}\sin\theta + r\cos\theta\frac{d\theta}{dt}\right),$$

from which

$$\mathbf{v} = \frac{dr}{dt}(\mathbf{i}\cos\theta + \mathbf{j}\sin\theta) + r\frac{d\theta}{dt}(-\mathbf{i}\sin\theta + \mathbf{j}\cos\theta).$$

Substitute the results of Part (a)

$$\boxed{\mathbf{v} = \frac{dr}{dt}\mathbf{e}_r + r\frac{d\theta}{dt}\mathbf{e}_\theta = \frac{dr}{dt}\mathbf{e}_r + r\omega\mathbf{e}_\theta}$$

Problem 13.160 The airplane flies in a straight line at 643.6 km/h. The radius of its propellor is 1.524 m, and the propeller turns at 2000 rpm in the counterclockwise direction when seen from the front of the airplane. Determine the velocity and acceleration of a point on the tip of the propeller in terms of cylindrical coordinates. (Let the z-axis be oriented as shown in the figure.)

Solution: The speed is

$$v = 643.6 \times 1000/3600 = 178.8 \text{ m/s}$$

The angular velocity is

$$\omega = 2000\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 209.4 \text{ rad/s}.$$

The radial velocity at the propeller tip is zero. The transverse velocity is $v_\theta = \omega r = 319.2$ m/s. The velocity vector in cylindrical coordinates is

$$\boxed{\mathbf{v} = 319.2\mathbf{e}_\theta + 178.8\mathbf{e}_z \text{ (m/s)}}.$$

The radial acceleration is

$$a_r = -r\omega^2 = -1.524\,(209.4)^2 = -66825 \text{ m/s}^2.$$

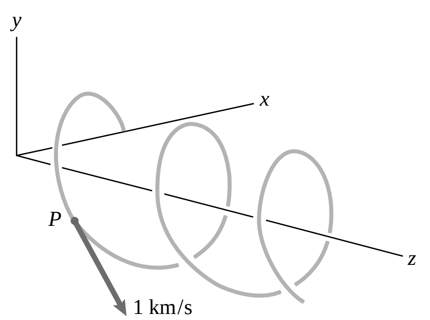
The transverse acceleration is

$$a_\theta = r\frac{d^2\theta}{dt^2} + 2\left(\frac{dr}{dt}\right)\left(\frac{d\theta}{dt}\right) = 0,$$

since the propeller rotates at a constant angular velocity. The acceleration $a_z = 0$, since the airplane travels at constant speed. Thus

$$\mathbf{a} = -66825\mathbf{e}_r \text{ (m/s}^2)$$

Problem 13.161 A charged particle P in a magnetic field moves along the spiral path described by $r = 1$ m, $\theta = 2z$ rad, where z is in meters. The particle moves along the path in the direction shown with constant speed $|\mathbf{v}| = 1$ km/s. What is the velocity of the particle in terms of cylindrical coordinates?



Solution: The radial velocity is zero, since the path has a constant radius. The magnitude of the velocity is

$$v = \sqrt{r^2\left(\frac{d\theta}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} = 1000 \text{ m/s}.$$

The angular velocity is $\dfrac{d\theta}{dt} = 2\dfrac{dz}{dt}$.

Substitute: $$v = \sqrt{r^2\left(\frac{d\theta}{dt}\right)^2 + \frac{1}{4}\left(\frac{d\theta}{dt}\right)^2}$$

$$= \left(\frac{d\theta}{dt}\right)\sqrt{r^2 + \frac{1}{4}} = \sqrt{1.25},$$

from which $$\frac{d\theta}{dt} = \frac{1000}{\sqrt{1.25}} = 894.4 \text{ rad/s},$$

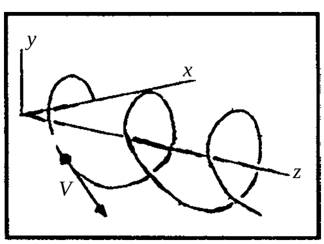
from which the transverse velocity is

$$v_\theta = r\left(\frac{d\theta}{dt}\right) = 894.4 \text{ m/s}.$$

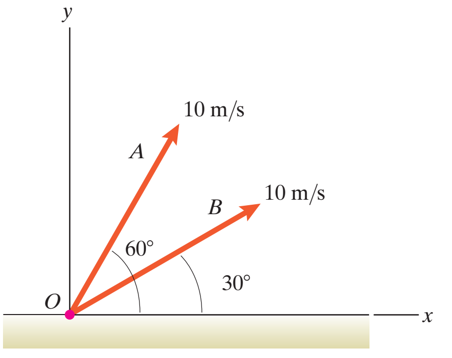
The velocity along the cylindrical axis is

$$\frac{dz}{dt} = \frac{1}{2}\left(\frac{d\theta}{dt}\right) = 447.2 \text{ m/s}.$$

The velocity vector: $\boxed{\mathbf{v} = 894.4\mathbf{e}_\theta + 447.2\mathbf{e}_z}$



Problem 13.162 At $t = 0$, two projectiles A and B are simultaneously launched from O with the initial velocities and elevation angles shown. Determine the velocity of projectile A relative to projectile B (a) at $t = 0.5$ s and (b) at $t = 1$ s.



Solution:

$$\mathbf{v}_A = -(9.81\ \text{m/s}^2\mathbf{j})t + (10\ \text{m/s})(\cos 60^\circ\mathbf{i} + \sin 60^\circ j)$$

$$\mathbf{v}_B = -(9.81\ \text{m/s}^2\mathbf{j})t + (10\ \text{m/s})(\cos 30^\circ\mathbf{i} + \sin 30^\circ j)$$

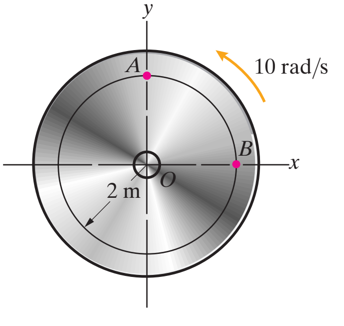
$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B = (10\ \text{m/s})(-0.366\mathbf{i} + 0.366\mathbf{j})$$

$$\mathbf{v}_{A/B} = (-3.66\mathbf{i} + 3.66\mathbf{j})\ \text{m/s}$$

Since $\mathbf{v}_{A/B}$ doesn't depend on time, the answer is the same for both times

$$\boxed{\mathbf{v}_{A/B} = (-3.66\mathbf{i} + 3.66\mathbf{j})\ \text{m/s}}$$

Problem 13.163 Relative to the earth-fixed coordinate system, the disk rotates about the fixed point O at 10 rad/s. What is the velocity of point A relative to point B at the instant shown?



Solution:

$$\mathbf{v}_A = -(10\ \text{rad/s})(2\ \text{m})\mathbf{i} = -(20\ \text{m/s})\mathbf{i}$$

$$\mathbf{v}_B = (10\ \text{rad/s})(2\ \text{m})\mathbf{j} = (20\ \text{m/s})\mathbf{j}$$

$$\boxed{\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B = (-20\mathbf{i} - 20\mathbf{j})\ \text{m/s}}$$

Problem 13.164 Relative to the earth-fixed coordinate system, the disk rotates about the fixed point O with a constant angular velocity of 10 rad/s. What is the acceleration of point A relative to point B at the instant shown?

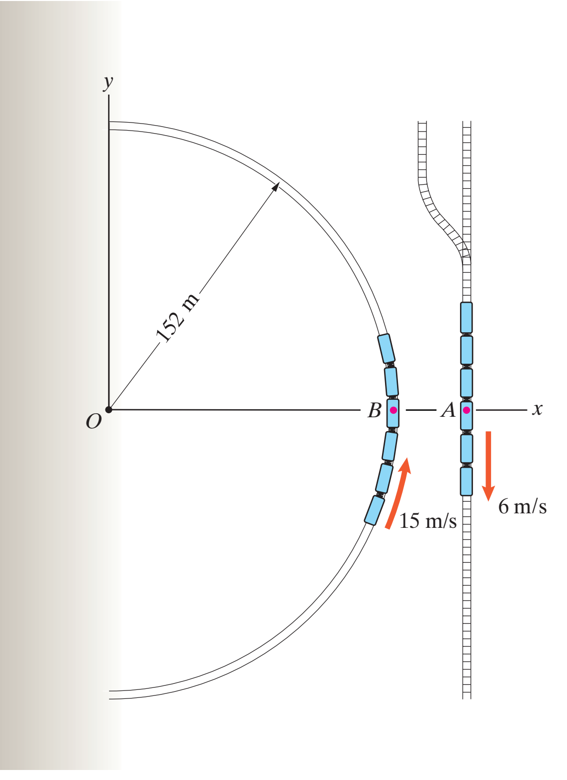
Solution:

$$\mathbf{a}_A = -(10\ \text{rad/s})^2(2\ \text{m})\mathbf{j} = -(200\ \text{m/s}^2)\mathbf{j}$$

$$\mathbf{a}_B = -(10\ \text{rad/s})^2(2\ \text{m})\mathbf{i} = -(200\ \text{m/s}^2)\mathbf{i}$$

$$\boxed{\mathbf{a}_{A/B} = \mathbf{a}_A - \mathbf{a}_B = (200\mathbf{i} - 200\mathbf{j})\ \text{m/s}^2}$$

Problem 13.165 The train on the circular track is traveling at 15 m/s. The train on the straight track is traveling at 6 m/s. In terms of the earth-fixed coordinate system shown, what is the velocity of passenger A relative to passenger B?



Solution:

$\mathbf{v}_A = (-6\mathbf{j})$ m/s, $\mathbf{v}_B = (15\mathbf{j})$ m/s

$$\boxed{\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B = (-21\mathbf{j})\ \text{m/s}}$$

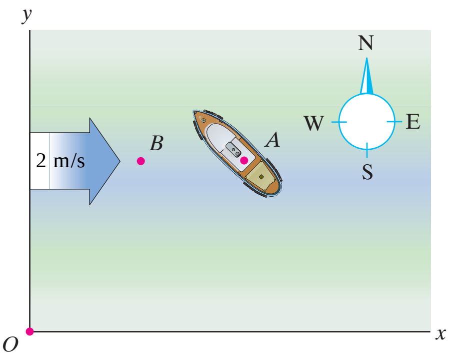
Problem 13.166 The train on the circular track is traveling at a constant speed of 15 m/s. The train on the straight track is traveling at 6 m/s and is increasing its speed at 0.6 m/s^2. In terms of the earth-fixed coordinate system shown, what is the acceleration of passenger A relative to passenger B?

Solution:

$$\mathbf{a}_A = (-0.6\mathbf{j})\ \text{m/s}^2,\ \mathbf{a}_B = -\frac{(15\ \text{m/s})^2}{152\ \text{m}}\mathbf{i} = (-1.48)\ \text{m/s}^2$$

$$\boxed{\mathbf{a}_{A/B} = \mathbf{a}_A - \mathbf{a}_B = (1.48\mathbf{i} - 0.6\mathbf{j})\ \text{m/s}^2}$$

Problem 13.167 In Active Example 13.16, suppose that the velocity of the current increases to 3 m/s flowing east. If the helmsman wants to travel northwest relative to the earth, what direction must he point the ship? What is the resulting magnitude of the ships' velocity relative to the earth?



Solution: The ship is moving at 5 m/s relative to the water. Use relative velocity concepts

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$v_A(-\cos 45^\circ \mathbf{i} + \sin 45^\circ \mathbf{j}) = (3 \text{ m/s})\mathbf{i} + (5 \text{ m/s})(-\cos\theta \mathbf{i} + \sin\theta \mathbf{j})$$

Breaking into components we have

$$-v_A \cos 45^\circ = (3 \text{ m/s}) - (5 \text{ m/s}) \cos\theta,$$

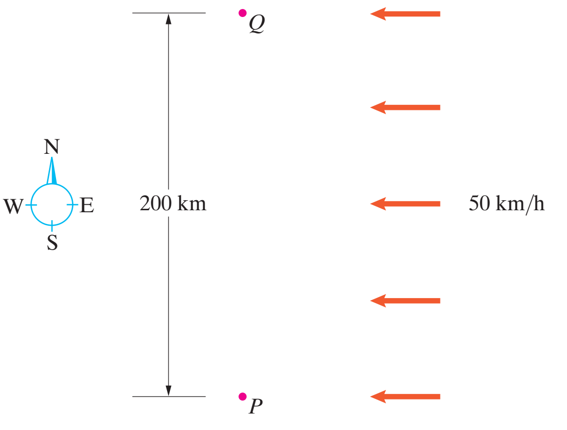
$$v_A \sin 45^\circ = (5 \text{ m/s}) \sin\theta.$$

Solving these equations we find

$$\theta = 19.9^\circ, \quad v_A = 2.41 \text{ m/s}.$$

$$\boxed{70.1^\circ \text{ west of north}, \quad 2.41 \text{ m/s}.}$$

Problem 13.168 A private pilot wishes to fly from a city P to a city Q that is 200 km directly north of city P. The airplane will fly with an airspeed of 290 km/h. At the altitude at which the airplane will be flying, there is an east wind (that is, the wind's direction is west) with a speed of 50 km/h. What direction should the pilot point the airplane to fly directly from city P to city Q? How long will the trip take?



Solution: Assume an angle θ, measured ccw from the east.

$$\mathbf{V}_{Plane/Ground} = \mathbf{V}_{Plane/Air} + \mathbf{V}_{Air/Ground}$$

$$\mathbf{V}_{Plane/Air} = (290 \text{ km/h})(\cos\theta \mathbf{i} + \sin\theta \mathbf{j})$$

$$\mathbf{V}_{Air/Ground} = -(50 \text{ km/h})\mathbf{i}$$

$$\mathbf{V}_{Plane/Ground} = [(290\cos\theta - 50)\mathbf{i} + (290\sin\theta)\mathbf{j}] \text{ km/h}$$

We want the airplane to travel due north therefore

$$290\cos\theta - 50 = 0 \Rightarrow \theta = \cos^{-1}\left(\frac{50}{290}\right) = 80.07^\circ$$

Thus the heading is

$$\boxed{90^\circ - 80.07^\circ = 9.93^\circ \text{ east of north}}$$

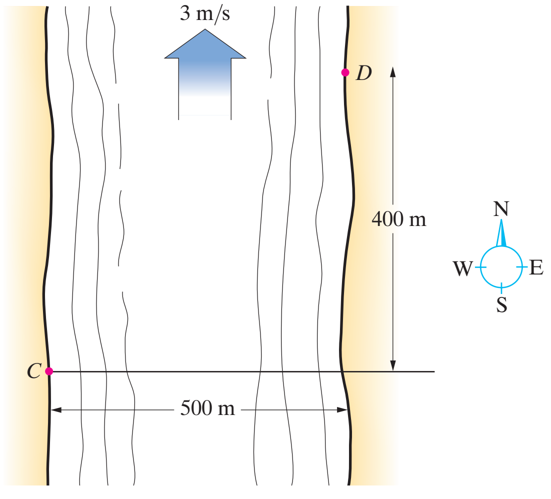
The ground speed is now

$$v = (290 \text{ km/h}) \sin(80.1^\circ) = 285.6 \text{ km/h}$$

The time is

$$\boxed{t = \frac{d}{v} = \frac{200 \text{ km}}{285.6 \text{ km/h}} = 0.700 \text{ h} = 42.0 \text{ min}}$$

Problem 13.169 The river flows north at 3 m/s. (Assume that the current is uniform.) If you want to travel in a straight line from point C to point D in a boat that moves at a constant speed of 10 m/s relative to the water, in what direction should you point the boat? How long does it take to make the crossing?



Solution: Assume an angle θ, measured ccw from the east.

$$\mathbf{V}_{Boat/Ground} = \mathbf{V}_{Boat/Water} + \mathbf{V}_{Water/Ground}$$

$$\mathbf{V}_{Boat/Water} = (10 \text{ m/s})(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})$$

$$\mathbf{V}_{Water/Ground} = (3 \text{ m/s})\,j$$

$$\mathbf{V}_{Boat/Ground} = [(10\cos\theta)\mathbf{i} + (3 + 10\sin\theta)\mathbf{j}] \text{ m/s}$$

We want the boat to travel at an angle $\tan\phi = \dfrac{400}{500}$

Therefore

$$\frac{3 + 10\sin\theta}{10\cos\theta} = \frac{400}{500} \Rightarrow \theta = 25.11^\circ$$

Thus the heading is

$$\boxed{25.11^\circ \text{ north of east}}$$

The ground speed is now

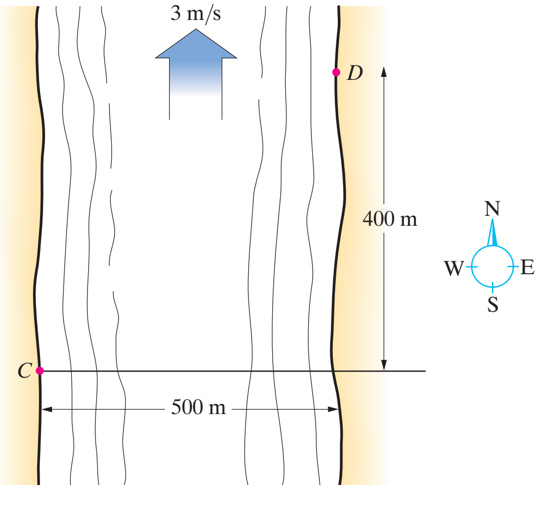
$$v = \sqrt{(10\cos\theta)^2 + (3 + 10\sin\theta)^2} = 11.60 \text{ m/s}$$

The time is

$$\boxed{t = \frac{d}{v} = \frac{\sqrt{500^2 + 400^2} \text{ m}}{11.60 \text{ m/s}} = 55.2 \text{ s}}$$

Problem 13.170 The river flows north at 3 m/s (Assume that the current is uniform.) What minimum speed must a boat have relative to the water in order to travel in a straight line form point C to point D? How long does it take to make the crossing?

Strategy: Draw a vector diagram showing the relationships of the velocity of the river relative to the earth, the velocity of the boat relative to the river, and the velocity of the boat relative to the earth. See which direction of the velocity of the boat relative to the river causes it magnitude to be a minimum.



Solution: The minimum velocity occurs when the velocity of the boat relative to the water is 90° from the velocity of the boat relative to the earth.

$$\mathbf{v}_B = \mathbf{v}_W + \mathbf{v}_{B/W}.$$

The angle of travel is

$$\theta = \tan^{-1}\left(\frac{400 \text{ m}}{500 \text{ m}}\right) = 38.7^\circ.$$

Using the triangle that is drawn we have

$$v_{B/W} = (3 \text{ m/s})\cos 38.7^\circ = 2.34 \text{ m/s},$$

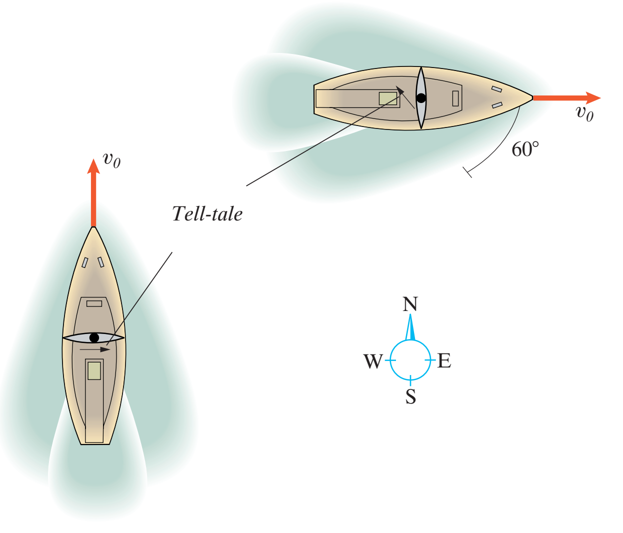
$$v_B = (3 \text{ m/s})\sin 38.7^\circ = 1.87 \text{ m/s}.$$

The time is given by

$$t = \frac{\sqrt{(400 \text{ m})^2 + (500 \text{ m})^2}}{v_B} = 342 \text{ s}.$$

$$\boxed{v_{B/W} = 2.34 \text{ m/s}. \; t = 342 \text{ s}.}$$

Problem 13.171 Relative to the earth, the sailboat sails north with speed $v_0 = 6$ m/s and then sails east at the same speed. The tell-tale indicates the direction of the wind *relative to the boat*. Determine the direction and magnitude of the wind's velocity (in m/s) relative to the earth.



Solution:

$$\mathbf{v}_{wind/ground} = \mathbf{v}_{wind/boat} + \mathbf{v}_{boat/ground}$$

In position one we have

$$\mathbf{v}_{wind/ground} = v_{wind/boat1}\,\mathbf{i} + (6\text{ m/s})\mathbf{j}$$

In position two we have

$$\mathbf{v}_{wind/ground} = v_{wind/boat2}(-\cos 60^\circ\mathbf{i} + \sin 60^\circ\mathbf{j}) + (6\text{ m/s})\mathbf{i}$$

Since the wind has not changed these two expressions must be the same. Therefore

$$\left.\begin{aligned} v_{wind/boat1} &= -v_{wind/boat2}\cos 60^\circ + 6\text{ m/s} \\ 6\text{ m/s} &= v_{wind/boat2}\sin 60^\circ \end{aligned}\right\}$$

$$\Rightarrow \left\{\begin{aligned} v_{wind/boat1} &= 2.536\text{ m/s} \\ v_{wind/boat2} &= 6.928\text{ m/s} \end{aligned}\right.$$

Using either position one or position two we have

$$\boxed{\begin{aligned} \mathbf{v}_{wind/ground} &= (2.536\mathbf{i} + 6\mathbf{j})\text{ m/s} \\ v_{wind/ground} &= \sqrt{(2.536)^2 + (6)^2}\text{ m/s} = 6.51\text{ m/s} \\ \text{direction} &= \tan^{-1}\left(\frac{2.536}{6}\right) = 22.91^\circ\text{east of north} \end{aligned}}$$

Problem 13.172 Suppose you throw a ball straight up at 10 m/s and release it at 2 m above the ground. (a) What maximum height above the ground does the ball reach? (b) How long after release it does the ball hit the ground? (c) What is the magnitude of its velocity just before it hits the ground?

Solution: The equations of motion for the ball are

$$a_y = -g = -9.81\text{ m/s}^2,$$

$$v_y = v_{y0} - gt = 10 - 9.81t\text{ (m/s)}, \text{ and}$$

$$y = y_0 + v_{y0}t - gt^2/2 = 2 + 10t - 9.81t^2/2\text{ (m)}.$$

(a) The maximum height occurs when the velocity is zero. Call this time $t = t_1$. It can be obtained by setting velocity to zero, i.e., $v_y = 0 = 10 - 9.81t_1$ (m/s). Solving, we get $t_1 = 1.02$ s. Substituting this time into the y equation, we get a maximum height of $y_{MAX} = 7.10$ m.

(b) The ball hits the ground when $y = 0$ m. To find out when this occurs, we set $y = 0$ m into the y equation and solve for the time(s) when this occurs. There will be two times, one positive and one negative. Only the positive time has meaning for us. Let this time be $t = t_2$. The equation for t_2 is $y = 0 = 2 + 10t_2 - 9.81t_2{}^2/2$ (m). Solving, we get $t_2 = 2.22$ s.

(c) The velocity at impact is determined by substituting $t_2 = 2.22$ s into the equation for v_y. Doing this, we find that at impact, $v_y = -11.8$ m/s

Problem 13.173 Suppose that you must determine the duration of the yellow light at a highway intersection. Assume that cars will be approaching the intersection traveling as fast as 104.6 km/h, that the drivers' reaction times are as long as 0.5 s, and that cars can safely achieve a deceleration of at least $0.4g$.

(a) How long must the light remain yellow to allow drivers to come to a stop safely before the light turns red?
(b) What is the minimum distance cars must be from the intersection when the light turns yellow to come to a stop safely at the intersection?

Solution: The speed-time equation from initial speed to stop is given by integrating the equation $\dfrac{d^2s}{dt^2} = -0.4g$. From which

$$\frac{ds}{dt} = -0.4gt + V_0, \text{ and } s(t) = -0.2gt^2 + V_0 t,$$

where V_0 is the initial speed and the distance is referenced from the point where the brakes are applied. The initial speed is:

$$V_0 = 104.6 \times 1000/3600 = 29.05 \text{ m/s}.$$

(a) The time required to come to a full stop

$$\frac{ds(t_0)}{dt} = 0 \text{ is } t_0 = \frac{V_0}{0.4g} = \frac{29.05}{(0.4)(9.81)} = 7.40 \text{ s}.$$

The driver's reaction time increases this by 0.5 second, hence the total time to stop after observing the yellow light is $T = t_0 + 0.5 = 7.90$ s

(b) The distance traveled after brake application is traveled from brake application to full stop is given by

$$s(t)_0 = -0.2gt_0^2 + V_0 t_0, \text{ from which } s(t_0) = 107.6 \text{ m}.$$

The distance traveled during the reaction time is

$$d = V_0(0.5) = 29.05(0.5) = 14.53 \text{ m},$$

from which the total distance is

$$d_t = 29.05 + 14.53 = 43.58 \text{ m}$$

Problem 13.174 The acceleration of a point moving along a straight line is $a = 4t + 2$ m/s^2. When $t = 2$ s, its position is $s = 36$ m, and when $t = 4$ seconds, its position is $s = 90$ meters. What is its velocity when $t = 4$ s?

Solution: The position-time equation is given by integrating

$$\frac{d^2s}{dt^2} = 4t + 2, \text{ from which } \frac{ds}{dt} = 2t^2 + 2t + V_0, \text{ and}$$

$$s(t) = \left(\frac{2}{3}\right)t^3 + t^2 + V_0 t + d_0,$$

where V_0, d_0 are the initial velocity and position. From the problem conditions:

$$s(2) = \left(\frac{2}{3}\right)2^3 + (2^2) + V_0(2) + d_0 = 36,$$

from which

$$(1) \quad 2V_0 + d_0 = \left(\frac{80}{3}\right).s(4) = \left(\frac{2}{3}\right)4^3 + (4^2) + V_0(4) + d_0 = 90,$$

from which

$$(2) \quad 4V_0 + d_0 = \left(\frac{94}{3}\right).$$

Subtract (1) from (2) to obtain

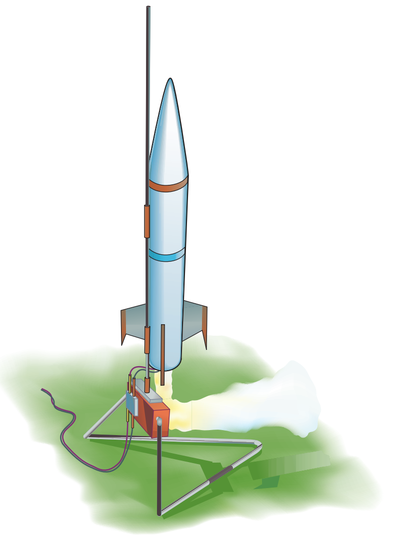
$$V_0 = \left(\frac{94 - 80}{6}\right) = 2.33 \text{ m/s}.$$

The velocity at $t = 4$ seconds is

$$\left[\frac{ds(t)}{dt}\right]_{t=4} = [2t^2 + 2t + V_0]_{t=4} = 32 + 8 + 2.33 = 42.33 \text{ m/s}$$

Problem 13.175 A model rocket takes off straight up. Its acceleration during the 2 s its motor burns is 25 m/s^2. Neglect aerodynamic drag, and determine

(a) the maximum velocity of the rocket during the flight and
(b) the maximum altitude the rocket reaches.



Solution: The strategy is to solve the equations of motion for the two phases of the flight: during burn $0 \le t \le 2$ s seconds, and after burnout: $t > 2$ s.

Phase 1: The acceleration is:

$$\frac{d^2s}{dt^2} = 25, \text{ from which } \frac{ds}{dt} = 25t, \text{ and } s(t) = 12.5t^2,$$

since the initial velocity and position are zero. The velocity at burnout is $V_{\text{burnout}} = (25)(2) = 50$ m/s. The altitude at burnout is $h_{\text{burnout}} = (12.5)(4) = 50$ m.

Phase 2. The acceleration is:

$$\frac{d^2s}{dt^2} = -g, \text{ from which } \frac{ds}{dt} = -g(t-2) + V_{\text{burnout}}(t \ge 2), \text{ and}$$

$$s(t) = -g(t-2)^2/2 + V_{\text{burnout}}(t-2) + h_{\text{burnout}}, \ (t \ge 2).$$

The velocity during phase 1 is constantly increasing because of the rocket's positive acceleration. Maximum occurs at burnout because after burnout, the rocket has negative acceleration and velocity constantly decreases until it reaches zero at maximum altitude. The velocity from maximum altitude to impact must be constantly increasing since the rocket is falling straight down under the action of gravity. Thus the maximum velocity during phase 2 occurs when the rocket impacts the ground. The issue of maximum velocity becomes this: is the velocity at burnout greater or less than the velocity at ground impact? The time of flight is given by $0 = -g(t_{\text{flight}} - 2)^2/2 + V_{\text{burnout}}(t_{\text{flight}} - 2) + h_{\text{burnout}}$, from which, in canonical form:

$$(t_{\text{flight}} - 2)^2 + 2b(t_{\text{flight}} - 2) + c = 0,$$

where $b = -(V_{\text{burnout}}/g)$ and $c = -(2h_{\text{burnout}}/g)$.

The solution $(t_{\text{flight}} - 2) = -b \pm \sqrt{b^2 - c} = 11.11, = -0.92$ s. Since the negative time is not allowed, the time of flight is $t_{\text{flight}} = 13.11$ s.

The velocity at impact is

$$V_{\text{impact}} = -g(t_{\text{flight}} - 2) + V_{\text{burnout}} = -59 \text{ m/s}$$

which is <u>higher in magnitude</u> than the velocity at burnout. The time of maximum altitude is given by

$$\frac{ds}{dt} = 0 = -g(t_{\max alt} - 2) + V_{\text{burnout}}, \quad \text{from which}$$

$$t_{\max alt} - 2 = \frac{V_{\text{burnout}}}{g} = 5.1 \text{ s}, \quad \text{from which}$$

$$t_{\max alt} = 7.1 \text{ s}.$$

The maximum altitude is

$$h_{\max} = -\frac{g}{2}(t_{\max alt} - 2)^2 + V_{\text{burnout}}(t_{\max alt} - 2) + h_{\text{burnout}} = 177.42 \text{ m}$$

Problem 13.176 In Problem 13.175, if the rocket's parachute fails to open, what is the total time of flight from takeoff until the rocket hits the ground?

Solution: The solution to Problem 13.175 was (serendipitously) posed in a manner to yield the time of flight as a peripheral answer. The time of flight is given there as $t_{\text{flight}} = 13.11$ s

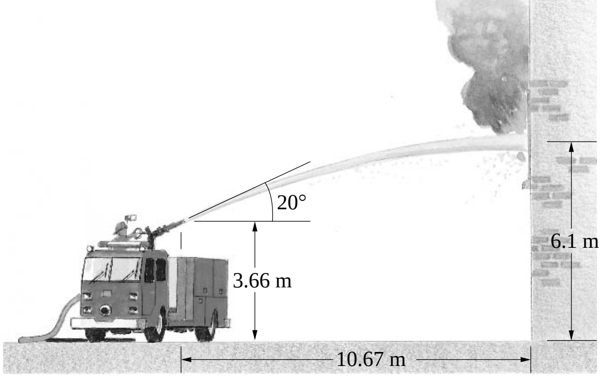
Problem 13.177 The acceleration of a point moving along a straight line is $a = -cv^3$, where c is a constant. If the velocity of the point is v_0, what distance does the point move before its velocity decreases to $\dfrac{v_0}{2}$?

Solution: The acceleration is $\dfrac{dv}{dt} = -cv^3$. Using the chain rule, $\dfrac{dv}{dt} = \dfrac{dv}{ds}\dfrac{ds}{dt} = v\dfrac{dv}{ds} = -cv^3$. Separating variables and integrating: $\dfrac{dv}{v^2} = -cds$, from which $-\dfrac{1}{v} = -cs + C$. At $s = 0$, $v = v_0$, from which $-\dfrac{1}{v} = -cs - \dfrac{1}{v_0}$, and $v = \dfrac{v_0}{1 + v_0 cs}$. Invert: $v_0 cs = \dfrac{v_0}{v} - 1$.

When $v = \dfrac{v_0}{2}$, $\boxed{s = \left(\dfrac{1}{cv_0}\right)}$

Problem 13.178 Water leaves the nozzle at 20° above the horizontal and strikes the wall at the point indicated. What was the velocity of the water as it left the nozzle?

Strategy: Determine the motion of the water by treating each particle of water as a projectile.



Solution: Denote $\theta = 20°$. The path is obtained by integrating the equations:

$$\frac{dv_y}{dt} = -g \text{ and } \frac{dv_x}{dt} = 0, \quad \text{from which}$$

$$\frac{dy}{dt} = -gt + V_n \sin\theta, \; \frac{dx}{dt} = V_n \cos\theta.$$

$$y = -\frac{g}{2}t^2 + (V_n \sin\theta)t + y_0.$$

$$x = (V_n \cos\theta)t + x_0.$$

Choose the origin at the nozzle so that $y_0 = 0$, and $x_0 = 0$. When the stream is $y(t_{\text{impact}}) = 6.1 - 3.66 = 2.44$ m, the time is

$$0 = -\frac{g}{2}(t_{\text{impact}})^2 + (V_n \sin\theta)t_{\text{impact}} - 2.44.$$

At this same time the horizontal distance is

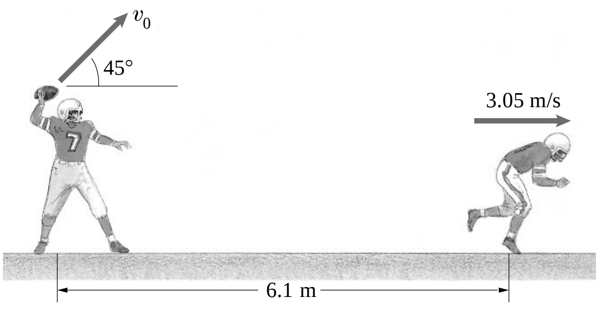
$$x(t_{\text{impact}}) = 10.67 = (V_n \cos\theta)t_{\text{impact}}, \text{ from which } t_{\text{impact}} = \frac{10.67}{V_n \cos\theta}.$$

Substitute:

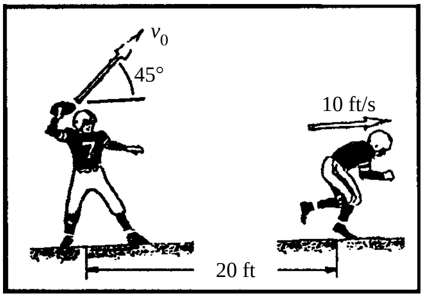
$$0 = -\frac{g}{2}\left(\frac{10.67}{V_n \cos\theta}\right)^2 + 10.67 \tan\theta - 2.44,$$

from which $\boxed{V_n = \left(\dfrac{10.67}{\cos\theta}\right)\sqrt{\dfrac{g}{2(10.67\tan\theta - 2.44)}} = 20.9 \text{ m/s}}$

Problem 13.179 In practice, a quarterback throws the football with a velocity v_0 at 45° above the horizontal. At the same instant, a receiver standing 6.1 m in front of him starts running straight down field at 3.05 m/s and catches the ball. Assume that the ball is thrown and caught at the same height above the ground. What is the velocity v_0?



Solution: Denote $\theta = 45^\circ$. The path is determined by integrating the equations;



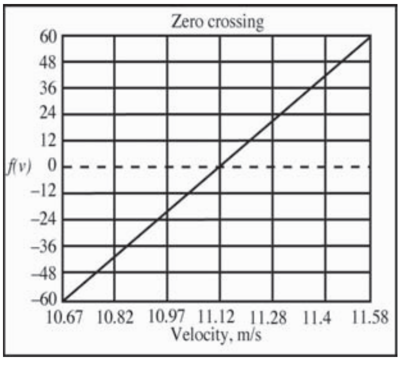
$$\frac{d^2y}{dt^2} = -g, \frac{d^2x}{dt^2} = 0, \quad \text{from which}$$

$$\frac{dy}{dt} = -gt + v_0 \sin\theta, \frac{dx}{dt} = v_0 \cos\theta.$$

$$y = -\frac{g}{2}t^2 + (v_0 \sin\theta)t,$$

$$x = (v_0 \cos\theta)t,$$

where the origin is taken at the point where the ball leaves the quarterback's hand.



When the ball reaches the receiver's hands,

$$y = 0, \text{ from which } t_{\text{flight}} = \sqrt{\frac{2v_0 \sin\theta}{g}}.$$

At this time the distance down field is the distance to the receiver:

$$x = 3.05\, t_{\text{flight}} + 6.1. \text{ But also}$$

$$x = (v_0 \cos\theta)t_{\text{flight}}, \quad \text{from which}$$

$$t_{\text{flight}} = \frac{6.1}{(v_0 \cos\theta - 3.05)}.$$

Substitute:

$$\frac{6.1}{(v_0 \cos\theta - 3.05)} = \sqrt{\frac{2v_0 \sin\theta}{g}}, \quad \text{from which}$$

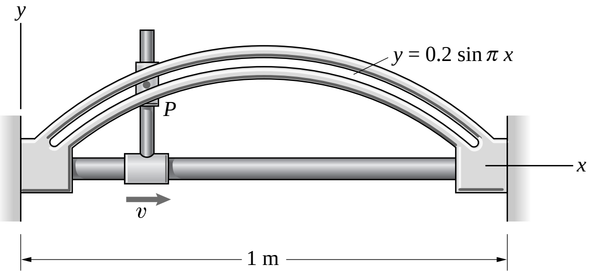
$$37.21 \text{ g } = 2v_0 \sin\theta(v_0 \cos\theta - 3.05)^2.$$

The function

$$f(v_0) = 2v_0 \sin\theta(v_0 \cos\theta - 3.05)^2 - 37.21 \text{ g}$$

was graphed to find the zero crossing, and the result refined by iteration: $\boxed{v_0 = 11.12 \text{ m/s}}$. *Check*: The time of flight is $t = 1.27$ s and the distance down field that the quarterback throws the ball is $d = 3.87 + 6.1 = 10$ m, which seem reasonable for a short, "lob" pass. *check*.

Problem 13.180 The constant velocity $v = 2$ m/s. What are the magnitudes of the velocity and acceleration of point P when $x = 0.25$ m?



Solution: Let $x = 2t$ m/s. Then $x = 0.25$ m at $t = 0.125$ s. We know that $v_x = 2$ m/s and $a_x = 0$.

From

$$y = 0.2\sin(2\pi t), \quad \text{we obtain}$$

$$\frac{dy}{dt} = 0.4\pi\cos(2\pi t) \quad \text{and}$$

$$\frac{d^2y}{dt^2} = -0.8\pi^2\sin(2\pi t).$$

At $t = 0.125$ s,

$$y = 0.141 \text{ m and}$$

$$\frac{dy}{dt} = v_y = 0.889 \text{ m/s and}$$

$$\frac{d^2y}{dt^2} = a_y = -5.58 \text{ m/s}.$$

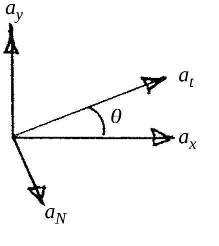
Therefore

$$|\mathbf{v}| = \sqrt{v_x^2 + v_y^2} = 2.19 \text{ m/s},$$

$$|\mathbf{a}| = \sqrt{a_x^2 + a_y^2} = 5.58 \text{ m/s}^2.$$

Problem 13.181 The constant velocity $v = 2$ m/s. What is the acceleration of point P in terms of normal and tangential components when $x = 0.25$ m?

Solution: See the solution of Problem 13.180. The angle θ between the x axis and the path is



$$\theta = \arctan\left(\frac{v_y}{v_x}\right) = \arctan\left(\frac{0.889}{2}\right) = 24.0^\circ. \quad \text{Then}$$

$$a_t = a_x\cos\theta + a_y\sin\theta = 0 + (-5.58)\sin 24.0^\circ = -2.27 \text{ m/s}^2,$$

$$a_N = a_x\sin\theta - a_y\cos\theta = 0 - (-5.58)\cos 24.0^\circ = 5.10 \text{ m/s}^2.$$

The instantaneous radius is

$$\rho = \frac{v_x^2 + v_y^2}{a_N} = \frac{(2)^2 + (0.889)^2}{5.10} = 0.939 \text{ m}.$$

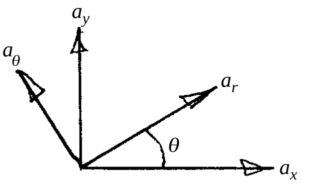
Problem 13.182 The constant velocity $v = 2$ m/s. What is the acceleration of point P in terms of polar coordinates when $x = 0.25$ m?

Solution: See the solution of Problem 13.192. The polar angle θ is

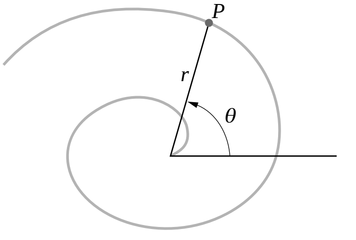
$$\theta = \arctan\left(\frac{y}{x}\right) = \arctan\left(\frac{0.141}{0.25}\right) = 29.5^\circ. \quad \text{Then}$$

$$a_r = a_x \cos\theta + a_y \sin\theta = 0 + (-5.58) \sin 29.5^\circ = -2.75 \text{ m/s}^2,$$

$$a_\theta = -a_x \sin\theta + a_y \cos\theta = 0 + (-5.58) \cos 29.5^\circ = -4.86 \text{ m/s}^2.$$



Problem 13.183 A point P moves along the spiral path $r = (0.1)\theta$ m, where θ is in radians. The angular position $\theta = 2t$ rad, where t is in seconds, and $r = 0$ at $t = 0$. Determine the magnitudes of the velocity and acceleration of P at $t = 1$ s.



Solution: The path $r = 0.2t$ m, $\theta = 2t$ rad. The velocity components are

$$v_r = \frac{dr}{dt} = 0.2 \text{ m/s}, \; v_\theta = r\frac{d\theta}{dt} = (0.2t)2 = 0.4t.$$

At $t = 1$ seconds the magnitude of the velocity is

$$\boxed{|\mathbf{v}| = \sqrt{v_r^2 + v_\theta^2} = \sqrt{0.2^2 + 0.4^2} = 0.447 \text{ m/s}}$$

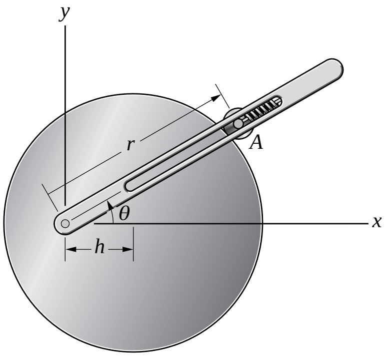
The acceleration components are:

$$a_r = \frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = -(0.2t)(2^2) \text{ m/s}^2,$$

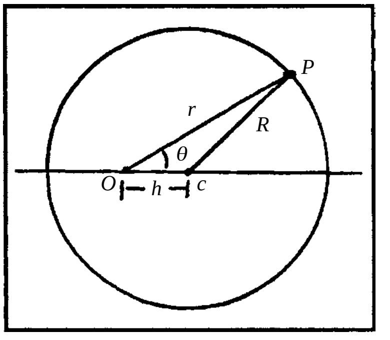
$$a_\theta = r\left(\frac{d^2\theta}{dt^2}\right) + 2\left(\frac{dr}{dt}\right)\left(\frac{d\theta}{dt}\right) = 2(0.2)(2) = 0.8 \text{ m/s}^2.$$

The magnitude of the acceleration is $\boxed{|\mathbf{a}| = \sqrt{a_r^2 + a_\theta^2} = 1.13 \text{ m/s}^2}$

Problem 13.184 In the cam-follower mechanism, the slotted bar rotates with a constant angular velocity $\omega = 12$ rad/s, and the radial position of the follower A is determined by the profile of the stationary cam. The slotted bar is pinned a distance $h = 0.2$ m to the left of the center of the circular cam. The follower moves in a circular path of 0.42 m radius. Determine the velocity of the follower when $\theta = 40°$ (a) in terms of polar coordinates, and (b) in terms of cartesian coordinates.



Solution:



(a) The first step is to get an equation for the path of the follower in terms of the angle θ. This can be most easily done by referring to the diagram at the right. Using the law of cosines, we can write $R^2 = h^2 + r^2 - 2hr\cos\theta$. This can be rewritten as $r^2 - 2hr\cos\theta + (h^2 - R^2) = 0$. We need to find the components of the velocity. These are $v_r = \dot{r}$ and $v_\theta = r\dot{\theta}$. We can differentiate the relation derived from the law of cosines to get $\dot{r}$. Carrying out this differentiation, we get $2r\dot{r} - 2h\dot{r}\cos\theta + 2hr\dot{\theta}\sin\theta = 0$. Solving for $\dot{r}$, we get

$$\dot{r} = \frac{hr\dot{\theta}\sin\theta}{(h\cos\theta - r)}.$$

Recalling that $\omega = \dot{\theta}$ and substituting in the numerical values, i.e., $R = 0.42$ m, $h = 0.2$ m, $\omega = 12$ rad/s, and $\theta = 40°$, we get $r = 0.553$ m, $v_r = -2.13$ m/s, and $v_\theta = 6.64$ m/s

(b) The transformation to cartesian coordinates can be derived from $\mathbf{e_r} = \cos\theta\mathbf{i} + \sin\theta\mathbf{j}$, and $\mathbf{e}_\theta = -\sin\theta\mathbf{i} + \cos\theta\mathbf{j}$. Substituting these into $\mathbf{v} = v_r\mathbf{e_r} + v_\theta\mathbf{e}_\theta$, we get $\mathbf{v} = (v_r\cos\theta - v_\theta\sin\theta)\mathbf{i} + (v_r\sin\theta + v_\theta\cos\theta)\mathbf{j}$. Substituting in the numbers, $\mathbf{v} = -5.90\mathbf{i} + 3.71\mathbf{j}$ (m/s)

Problem 13.185* In Problem 13.184, determine the acceleration of the follower when $\theta = 40°$ (a) in terms of polar coordinates and (b) in terms of cartesian coordinates.

Solution:

(a) Information from the solution to Problem 13.184 will be used in this solution. In order to determine the components of the acceleration in polar coordinates, we need to be able to determine all of the variables in the right hand sides of $a_r = \ddot{r} - r\dot{\theta}^2$ and that $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$. We already know everything except $\ddot{r}$ and $\ddot{\theta}$. Since ω is constant, $\ddot{\theta} = \dot{\omega} = 0$. We need only to find the value for r and the value for $\ddot{r}$ at $\theta = 40°$. Substituting into the original equation for r, we find that $r = 0.553$ m at this position on the cam. To find $\ddot{r}$, we start with $\dot{r} = v_r$. Taking a derivative, we start with $r\dot{r} - h\dot{r}\cos\theta + hr\dot{\theta}\sin\theta = 0$ from Problem 13.184 (we divided through by 2). Taking a derivative with respect to time, we get

$$\ddot{r} = \frac{\dot{r}^2 + 2hr\dot{\theta}\sin\theta + hr\dot{\theta}^2\cos\theta + hr\ddot{\theta}\sin\theta}{(h\cos\theta - r)}.$$

Evaluating, we get $\ddot{r} = -46.17$ m/s^2. Substituting this into the equation for a_r and evaluating a_n, we get $a_r = -125.81$ m/s^2 and $a_\theta = -51.2$ m/s^2

(b) The transformation of cartesian coordinates can be derived from $\mathbf{e_r} = \cos\theta\mathbf{i} + \sin\theta\mathbf{j}$, and $\mathbf{e}_\theta = \sin\theta\mathbf{i} + \cos\theta\mathbf{j}$. Substituting these into $\mathbf{a} = a_r\mathbf{e_r} + a_e\mathbf{e}_\theta$, we get $\mathbf{a} = a_r\mathbf{e_r} + a_\theta\mathbf{e}_\theta$, we get $\mathbf{a} = (a_r\cos\theta - a_\theta\sin\theta)\mathbf{i} + (a_r\sin\theta + a_\theta\cos\theta)\mathbf{j}$. Substituting in the numbers, we get $\mathbf{a} = -63.46\ \mathbf{i} - 120.1\ \mathbf{j}$(m/s^2).