Determining your orientation

The first step to knowing where you are, is knowing how you're oriented. How can we measure our orientation?

1 Indicating your orientation

Before we can determine our orientation, we need to be able to indicate it. For this, we first need a **reference frame**. There are several reference frames around, which we can use. We will examine two, both having their origin positioned at the center of gravity (CG) of the aircraft. In the **geodetical frame of reference** $\mathscr{F}^g$, the X_g axis points to the North, the Y_g axis points to the East and the Z_g axis points downward. In the **body frame of reference** $\mathscr{F}^b$, the X_b axis is the longitudinal axis of the aircraft, Y_b points to the right of the aircraft and Z_b points downward.

To go from one reference frame to another, we use **Euler angles**. These angles indicate rotations. For example, to go from $\mathscr{F}^g$ to $\mathscr{F}^b$, we first rotate the coordinate system by the **heading angle** ψ about the Z axis. We then rotate it by the **pitch angle** θ about the Y axis. Finally, we rotate it by the **roll angle** ϕ about the X axis. These three angles indicate the orientation of the aircraft. The system measuring the orienation of the aircraft is called the **attitude and heading reference system** (AHRS).

2 Measuring orientation using gyroscopes

One way to measure orientation, is by using gyroscopes. Gyroscopes are discs that have been given a very big angular velocity about one axis. This makes them inert to rotations about the other two axes. (**Inertia** is the resistance to a change in momentum.)

2.1 Two degree of freedom gyroscopes

We can distinguish two important types of gyroscopes: **two degree of freedom gyroscopes** and **one degree of freedom gyroscopes**. First, let's examine the two degree of freedom gyroscopes, also known as **free gyros**. They can measure orientation with respect to two axes.

Within the category of free gyros, we can again distinguish two types. A **vertical gyro unit** (VGU) is set up such that its spin axis is vertical. It gives the **gyro horizon**. (You might be wondering, can't you use gravity for that? But when performing a turn, the apparent gravity doesn't point downward. That's why gyros come in handy here: they don't have this problem.) On the other hand, a **directional gyro unit** (DGU) point in a certain direction (e.g. North). DGUs are used to determine the heading of the aircraft.

So how do they work? Well, let's apply a torque T to a disc. If the disc is not spinning, then the angular position ψ of the disc will satisfy

$$T = J\ddot{\psi} \qquad \Rightarrow \qquad \psi = \frac{1}{2}\frac{T}{J}t^2. \tag{2.1}$$

In this equation, J is the moment of inertia of the disc. The above equation implies that the angular position of the disc grows quadratically. However, if the disc is spinning with an angular velocity ω_R, things are different. If we apply the torque T about one of the axes about which the disc is not spinning, we have

$$T = H\dot{\theta} \qquad \Rightarrow \qquad \theta = \frac{T}{H}t = \frac{T}{J\omega_R}t. \tag{2.2}$$

Here, H is the angular momentum of the disc. This time, the angular position of the disc evolves only linearly. And if ω_R is very big, this happens very slowly as well. Thus gyroscopes are very **rigid**: they resist any rotation about an axis other than their rotational axis. We use this to measure our own orientation.

There is one important thing to remember. Let's suppose that the gyroscope is rotating about the Z axis. If we apply a torque T about the X axis, then the gyroscope will rotate about the Y axis. Similarly, if we apply the torque about the Y axis, the gyroscope will rotate about the X axis. This phenomenon (together with the above equation) is known as **precession**.

2.2 One degree of freedom gyroscopes

One degree of freedom gyroscopes are also known as **rate gyros**: they measure angular rates. Free gyros have two gimbals: one for each axis about which they're free to rotate. With a rate gyro, one of these gimbals is constrained by some sort of spring. A damping mechanism is attached as well.

Let's again suppose that the gyro is rotating about the Z axis. When the aircraft turns, about (for example) the Y axis, the gyro is forced to rotate along. However, this will cause the gyro to, in fact, rotate about the X axis instead. The spring will now cause a moment about the Y axis, based on the rotation of the gyroscope along the X axis. In other words, the bigger the rotation of the gyro about the X axis, the bigger the force of the spring, and the faster the aircraft is rotating. The force on the spring is thus an indication of how fast the aircraft rotates.

There is also a **rate integrating gyro** (RIG). This is a device that integrates the angular rate. It therefore gives an angular position as output. RIGs do have one downside. Because they integrate an angular velocity, errors are integrated as well. These errors accumulate over time. RIGs therefore lose precision after a couple of hours.

2.3 Drift and transport wander

Gyroscopes have several limitations. One of these is **drift**: when the spin axis of the gyroscope starts to rotate. Part of this is caused by **real drift**, due to imperfections in the gyroscope. On the other hand, there is also **apparent drift**. When the Earth rotates, the gyroscope won't rotate along. So, if we are on the equator, then after 6 hours, we will have an apparent drift of 90°. But, if we are on one of the poles, we won't have any apparent drift. Next to drift, there is also **transport wander**. This occurs when we carry a gyroscope over the Earth. (It is similar to apparent drift.)

For both apparent drift and transport wander, we can derive an equation. Let's suppose that we have a Northward velocity $V_N = V \cos\psi$ and an Eastward velocity $V_E = V \sin\psi$. Now, the error rates ω_{X_g}, ω_{Y_g} and ω_{Z_g} about the X_g, Y_g and Z_g axes, respectively, are

$$\omega_{X_g} = \omega_e \cos\Phi + \frac{V_E}{R_e + h}, \qquad \omega_{Y_g} = -\frac{V_N}{R_e + h} \qquad \text{and} \qquad \omega_{Z_g} = -\omega_e \sin\Phi + \frac{V_E}{R_e + h}\tan\Phi. \tag{2.3}$$

In this equation, ω_e is the rotational velocity of Earth. The terms with ω_e indicate the apparent drift, while the terms with V_E and V_N indicate the transport wander.

2.4 Optical gyroscopes

Next to mechanical gyroscopes (with spinning disks), there are also **optical gyroscopes**. Light is used to measure rotational rates. This results in a higher accuracy, and (since there are no moving parts) less mainenance and more reliability.

We'll briefly explain how it works. In a so-called **ring laser gyro** (RLG), two laser beams are sent along a ring, one clockwise and one counterclockwise. They are being reflected by mirrors on the sides.

Normally, both light beams require the same amount of time $T = 2\pi R/c$ to complete their journey. But when the system rotates with an angular velocity $\dot{\theta}$, one light beam needs to travel an extra distance $R\dot{\theta}T$, whereas the other one needs to travel the same distance less. So, the difference in distance travelled is

$$\Delta L = 2R\dot{\theta}T = \frac{4\pi R^2\dot{\theta}}{c}. \tag{2.4}$$

The consequence of this is a shift in the frequency of the two signals. In fact, the difference in frequency Δf is given by

$$\frac{\Delta f}{f} = \frac{\Delta L}{L} = \frac{2R\dot{\theta}}{c} \qquad \Rightarrow \qquad \Delta f = \frac{4\pi R^2 f}{cL}\dot{\theta} = \frac{4\pi R^2}{\lambda L}\dot{\theta} = K\dot{\theta}. \tag{2.5}$$

The frequency difference Δf is thus proportional to the rotational rate $\dot{\theta}$. Also, the constant K is known as the **gyro scale factor** and λ is the wave length of the light signal.

Optical gyroscopes have one small downside. This is the **lock-in phenomenon**: at rotational rates $\dot{\theta}$ below the **lock-in rate**, the two laser beams show coupling effects. (This lock-in rate is in the order of $0.01°/s$ to $0.1°/s$.) This results in a frequency difference $\Delta f = 0$ when $\dot{\theta}$ is near zero. (We thus have a **dead zone**.) Of course this is undesirable. To solve it, we simply twist the optical gyroscope back and forth with a frequency of $\omega \approx 100Hz$. This is called **dithering**. Due to this, $\dot{\theta}$ is nowhere near zero. The lock-in effect will thus not play a role anymore, making the gyroscope accurate.

3 Using Earth's magnetic field

3.1 Earth's magnetic field, and its imperfections

Let's examine Earth's magnetic field. This field can be represented by a bar magnet. The North pole of this magnet (being the Magnetic North MN) lies at Earth's South pole. Similarly, the South pole of the magnet (the Magnetic South MS) lies at Earth's North pole.

Navigating by means of magnetic compasses is not very precise. This is because the magnetic poles aren't positioned at the exact geographical poles of our planet. In fact, they're slightly off. For this reason, the **magnetic variation** (or **declination**) is defined as the (horizontal) angle between the local magnetic meridian and the geographic meridian. The magnetic variation depends on where you are on our planet. It also changes over time. However, maps are available where the magnetic variation is given quite accurately. So, we can more or less compensate for the magnetic variation.

A magnetic compass aligns itself with magnetic field lines. However, these field lines aren't always horizontal. The **magnetic dip** (or **inclination**) is the (vertical) angle between the magnetic field lines and the local horizontal. So, at the magnetic equator, the inclination is zero, while at the magnetic poles, it is 90°. There are tricks to get rid of the effects of inclination. However, when manoeuvring the aircraft, these tricks will again cause errors. They are therefore not very reliable.

Finally, the **deviation** is defined as the angle between the true magnetic direction, and the direction indicated by the compass. There can be several causes for these deviations. First of all, the compass may be lagging during turns, due to friction. Also, magnetic fields caused by other aircraft systems can cause compass deviations.

3.2 Combining a magnetic compass with a gyroscope

We have seen that a compass doesn't work very well when the aircraft is turning, due to lagging. But during a long straight flight, they work fine. On the other hand, gyroscopes work well in turns. But during long straight flights, they will have drift. If we combine a magnetic compass with a DGU, we can get rid of these disadvantages.

So how does this work? During turns, our compass is inaccurate, so we use our gryo. On the other hand, during straight flight, we simply use the magnetic compass to keep our DGU accurate. This sounds like a great idea, but in practice, this means that you have to reset your DGU roughly every 15 minutes. This is quite cumbersome. It would be better if we could adjust the gyroscope electronically. But normal magnetic compasses don't give an electronic signal. However, magnetometers do. Let's examine these.

3.3 The magnetometer

A **magnetometer** uses electric signals to detect a magnetic field. Its working is quite complex, but we'll examine it nevertheless.

The main component of the magnetometer is a set of two easily magnetized spoke legs. The Earth's magnetic field runs through these spoke legs, thus magnetizing them. The amount of magnetization depends on the flux $B_1 = H \cos\theta$. Here, θ is the orientation of the coil and H is the magnetic field strength.

Now, let's put a **primary coil** around the two spoke legs. If we put an alternating current on this coil, the spoke legs will get an additional magnetization, with opposite polarity. This magnetization will result in another flux B_2. But, whereas B_1 was a **static flux**, B_2 is an **alternating flux**.

Let's take the two spoke legs, and put another coil (the **pick-off coil**) around them. Since the spokes are magnetized, an electic current will run through this pick-off coil. Due to the opposite polarity, the alternating flux of these two spokes will cancel out at the pick-off coil. We remain with a rather useless constant signal. This method thus doesn't work yet.

We therefore need to apply another trick. And this trick is based on **saturation**: we can't magnetize the two spoke legs indefinitely. After a certain amount of magnetic force H, we simply can't magnetize the spoke legs any further. (The flux density B will remain constant, even if we increase H.) We have reached the **saturation point**. We now make sure that, by applying our alternating current, we exactly reach this saturation point.

Let's ask ourselves, what happens if we also add the magnetic flux B_1 due to Earth's magnetic field to this? This will then imply that the magnetization of the spoke legs will lose its sinusoid shape. On one side of the graph, the flux has reached its maximum. The tips of the sinusoid are thus 'cut off'. The resulting voltage at the pick-off coil will then contain several jumps. The amplitude of these **voltage jumps** is an indication of the angle with respect to the magnetic field lines.

Now we can get an indication of the angle with respect to the magnetic field lines. But this still leaves some ambiguity. For example, no distinction can be made between an angle of $0°$ and $180°$. For this reason, we usually use 3 spokes, being at $120°$ angles with respect to each other. By combining the signals from the three spokes, the exact direction with respect to Earth's magnetic field can be found.

By using a magnetometer, the downsides of magnetic compasses are mostly gone. Only during steep turns can there still be problems. For this reason, we often combine a magnetometer with a directional gyro. This will result in a **Magnetic Heading Reference System** (MHRS) called a **GYROSYN compass**.