# Navigational autopilot systems

In this chapter, we'll consider more advanced autopilot systems. So, the airplane is not only going to hold a certain parameter. Instead, it's going to fly on its own. Examples of such manoeuvres are following a glide slope, automatically flaring during landing, following a localizer or following a VOR beacon. We'll examine all these actions in this chapter. First, we start with the longitudinal actions. Later on, we'll consider the lateral actions.

## 1 Longitudinal navigational autopilot systems

#### 1.1 The glide slope hold mode

The **glide slope hold mode** is a system that automatically follows a glide slope. This reduces the pilot workload, and moreover it is more accurate than when the pilot follows the glide slope.

Before we're going to discuss the glide slope hold mode, we first make some assumptions. We assume that the glide slope antenna is positioned at the aircraft CG. This antenna measures the **glide slope error angle**  $\Gamma$ . (We model this sensor as  $H_{glideslope\ receiver} \approx 1$ .) The CG of the aircraft is then driven along the glide slope. To accomplish this, the airplane is kept on the glide slope using pitch attitude control. The airspeed is controlled using the autothrottle. (So we also assume that pitch attitude control and airspeed control are already present. This makes sense, as we've already discussed them in the previous chapter.)

Let's denote the **deviation** from the glide slope by d. We can find an expression for it using

$$\dot{d} = V\sin\left(\gamma + 3^{\circ}\right) \approx V\left(\gamma + 3^{\circ}\right) \frac{\pi}{180} \qquad \Rightarrow \qquad d(s) = \frac{V}{s} \frac{\pi}{180} \mathcal{L}\left(\gamma + 3^{\circ}\right). \tag{1.1}$$

(In the above equation, the  $\mathcal{L}(...)$  denotes the Laplace transform.) Of course, we also need some kind of feedback. But we can't measure d. Instead, we measure the error angle  $\Gamma$ . This angle is related to the deviation d according to

$$\Gamma \approx \sin \Gamma = \frac{d}{R} \frac{180}{\pi}.$$
(1.2)

Here, R is the **slant range**. Based on the measured error angle  $\Gamma$  (which should of course be kept at zero), we calculate a desired pitch angle  $\theta$ . We then pass this angle on to the pitch attitude control system. The desired pitch angle is calculated using a **glide slope coupler**. Its transfer function is

$$H_{coupler}(s) = K_c \left(1 + \frac{W_1}{s}\right).$$
(1.3)

In this equation,  $K_c$  is the **coupler gain**. It needs to be chosen such that we have acceptable closed loop behaviour. In fact, it is the only actual parameter that we as designers can control. Also,  $W_1$  is a **weighting constant**. It is present to cope with turbulence and such. Usually, a value of  $W_1 = 0.1$  is prescribed.

In our model, we also need to know the relation/transfer function between  $\theta$  and  $\gamma$ . This relation  $\gamma(s)/\theta(s)$  can't be obtained from the aircraft model directly. Instead, we use

$$\frac{\gamma(s)}{\theta(s)} = 1 - \frac{\alpha(s)}{\theta(s)} = 1 - \frac{\alpha(s)/\delta_e(s)}{\theta(s)/\delta_e(s)} = 1 - \frac{N_\alpha(s)}{N_\theta(s)}.$$
(1.4)

The glide slope hold mode does have a problem. When the slant range R changes, also the properties of the system change. In fact, if the gain  $K_c$  remains constant, then the closer the aircraft, the worse the performance becomes. Luckily, several solutions for this problem are available. We can apply some sort

of gain scheduling: we let  $K_c$  depend on the distance measured by the DME beacon. Or even simpler but less accurate, we let it depend on time. Finally, we can also add a lead-lag compensator to the system. If done well, this can reduce the effects of this problem significantly.

When designing an autopilot, it should be made robust. In other words, if certain parameters change, the autopilot should still work. Parameters that are subject to change in the real world are the airplane CG location, the airplane weight, the airplane speed and the presence/intensity of turbulence. The autopilot should be able to cope with these variations.

#### **1.2** Automatic flare mode

Getting the right vertical velocity on landing is difficult. The velocity shouldn't be too high. Such hard landings ( $\dot{h} \leq -6$  ft/s) are challenging for both the landing gear and the passengers. As such, they're not really acceptable. Too soft landings ( $\dot{h} \approx 0$  ft/s) are however also undesirable, as there will be floatation of the aircraft. Ideally, we have a firm landing with  $\dot{h} = -2$  to -3 ft/s.

The relationship between the normal velocity and the vertical velocity during landing is usually  $h = -V \sin 3^{\circ}$ . So, the faster an aircraft flies, the harder the touchdown. This can be a problem for airplanes with a low minimum speed, which thus have to fly fast. Therefore, such aircraft usually **flare** right before touching down: they pull up their nose. By doing this, the airplane follows the so-called **flare path**. This path starts at the height  $h_{flare}$ . It ends (by touching down) 1100 ft further than the point where the glide slope ends. (That is, where the glide slope antenna is positioned.)

The airplane is kept on the flare path by the pitch attitude control system. But this system of course needs to have some input. For that, we approximate the flare path by

$$h = h_{flare} e^{-t/\tau}.$$
(1.5)

All that we need to find are the constants  $h_{flare}$  and  $\tau$ . They both depend on the time  $t_{td}$  between the start of the flare and touchdown. To see how, we first examine the horizontal distance which the airplane travels during the flare manoeuvre. This is

$$Vt_{td} = 1100 + \frac{h_{flare}}{\tan 3^{\circ}}.$$
 (1.6)

From this, we can derive the height  $h_{flare}$ . To also find the time constant  $\tau$ , we differentiate the equation for h. This gives

$$\dot{h} = -\frac{h_{flare}}{\tau} e^{-t/\tau} = -\frac{h}{\tau} \qquad \Rightarrow \qquad h_{flare} = -\dot{h}_{at\ h_{flare}}\tau. \tag{1.7}$$

Okay, we do need to know the vertical velocity  $\dot{h}_{at \ h_{flare}}$  at the flare height. But this can simply be found using  $\dot{h} = -V \sin 3^{\circ}$ . And once we know  $\tau$ , we will have the control law for our automatic flare mode system:  $\dot{h} = -h/\tau$ .

But how do we make sure that the aircraft stays at the correct altitude? Well, we know the vertical speed of the aircraft  $\dot{h}$ . (It can be measured.) We also know the desired vertical airspeed, which follows from our control law. Based on the difference, we calculate a desired pitch angle  $\theta$ . This is done using a **coupler**. Its transfer function is again given by

$$H_{coupler}(s) = K_c \left( 1 + \frac{W_1}{s} \right).$$
(1.8)

 $K_c$  is again the coupler gain and  $W_1 = 0.1$  is again the weighting constant. The desired pitch angle is then passed on to the pitch attitude control system.

In our system, we do need a model of the aircraft. How do changes in the pitch angle effect the vertical velocity? We can find the transfer function between these two parameters using

$$\dot{h} \approx V\gamma \qquad \Rightarrow \qquad \frac{\dot{h}(s)}{\theta(s)} = \frac{\gamma(s)}{\theta(s)}V.$$
 (1.9)

Earlier in this chapter we already derived an expression for  $\gamma(s)/\theta(s)$ . So we can apply that again here. For the automatic flare mode to work, an accurate altitude measurement system is required. A radar altimeter is usually sufficiently accurate. In fact, when it is used, the system is often so precise that aircraft always land on exactly the same spot. This often resulted in runway damage at that point. To prevent this, a **Monte Carlo scheme** is used. The automatic flare mode system now chooses a random point inside a certain acceptable box. It then makes sure that the airplane touches down at that point. This method effectively solves the problem.

## 2 Lateral navigational autopilot systems

#### 2.1 The localizer hold mode

During an instrument landing, pilots need to follow the ILS localizer. But the **localizer hold mode** system can perform this much more accurately. Plus, it reduces the pilot workload.

Similar to the glide slope hold mode, we first need to make some assumptions. We assume that the airplane CG follows the localizer beam centerline. Also, we assume that the **localizer error angle**  $\lambda$  is sensed by the on-board localizer receiver. The airplane is then kept on the centerline using the heading angle controller (which we assume to be present).

We again denote the deviation from the intended path by d. We now have

$$\dot{d}(s) = V \sin\left(\psi(s) - \psi_{ref}(s)\right) \approx V\left(\psi(s) - \psi_{ref}(s)\right) \qquad \Rightarrow \qquad d(s) = \frac{V}{s}\left(\psi(s) - \psi_{ref}(s)\right). \tag{2.1}$$

In this equation,  $\psi_{ref}$  is the reference heading angle. It is the heading angle which we want to have. In other words, it is the heading angle of the runway.

We also need to have some feedback. For this, we can use the localizer error angle  $\lambda$ . It is related to the deviation d according to

$$\lambda \approx \sin \lambda = \frac{d}{R} \frac{180}{\pi}.$$
(2.2)

Based on the measured error angle  $\lambda$  (which should of course be kept at zero), we calculate a desired heading angle  $\psi$ . This desired heading angle is then passed on to the heading angle controller. The desired heading angle is calculated using a **coupler**. Its transfer function is again given by

$$H_{coupler}(s) = K_c \left(1 + \frac{W_1}{s}\right).$$
(2.3)

 $K_c$  is again the coupler gain and  $W_1 = 0.1$  is again the weighting constant.

Just like the glide slope hold mode, also the localizer hold mode has a problem. When the slant range R becomes too small, dynamic instability may occur. So, again the gain  $K_c$  needs to depend on the slant range R. Or alternatively, a compensating network  $H_{compensator}(s)$  needs to be added. Luckily, the localizer doesn't have to work for slant ranges smaller than R = 1 nm. The reason for this is that the localizer antenna is at the end of the runway, while the aircraft already touches down near the start of the runway.

### 2.2 The VOR hold mode

The **VOR hold mode** tries to follow a certain VOR radial. The working principle of following the VOR radial is similar to the principle of following the ILS localizer path. This time, the VOR error angle  $\lambda$  is used as feedback, and should be kept at zero.

There are, of course, a few differences. The VOR transmitter has a bandwidth of  $360^{\circ}$ , whereas the ILS localizer only has a bandwidth of  $5^{\circ}$ . (The localizer only works when the aircraft is more or less in line with the runway.) Also, the range of possible slant ranges R is much different for the VOR. The maximum range of a VOR beacon is roughly 200 nm. Next to that, when an aircraft is flying at 6000 ft, the slant range simply can't become less than 6000 ft  $\approx 1$  nm. So, aircraft hardly ever come closer than 1 nm to a VOR beacon. The final difference between the VOR and the localizer is that, when an aircraft is above the VOR, it doesn't receive a signal. (The aircraft is in the so-called **cone of silence**.) The VOR hold mode system should be able to cope with that.