Loads and Failures

1 Load Types

1.1 Limit load and ultimate load

The **limit load** is the maximum load which the aircraft may encounter at any time during its lifetime. No yielding/permanent deformation may occur at the limit load.

The **ultimate load** is the load which may occur once in the lifetime of an aircraft. All parts must be able to carry this load without failure. Permanent deformation may occur though.

1.2 Stress Concentrations

Stress concentrations occur at discontinuities in the structure, such as holes, cracks or a change in the cross-sectional shape. At such concentrations, stresses are higher. The **stress concentration factor** K is defined such that

$$\sigma_{max} = K\sigma_{average}. \tag{1.1}$$

1.3 Load Cases

A structure could be designed for just one **load case** (a way of loading it). However, some structures have multiple loads acting on them simultaneously. This is called a **combined load case**. It could also occur that multiple loads do occur, but not simultaneously. This is called a **multiple load case**.

2 Stress Criterions

2.1 Stress criterions in trusses

Members of truss structures are only subject to tensile/compressive stresses. If these stresses get too high, failure will occur. For the tensile case there is usually a maximum tensile stress $\sigma_{t_{max}}$. The **failure criterion for tension** then is

$$\sigma \leq \sigma_{t_{max}}. \tag{2.1}$$

When compression is present, things are slightly different. Sometimes a maximum compressive stress $\sigma_{c_{max}}$ is given. Sometimes **buckling** may occur. This will be discussed later.

2.2 Stress criterions in beams

In beams there are the same criterions as for truss members. However, there can also be the **maximum shear stress criterion**

$$\tau \leq \tau_{max}. \tag{2.2}$$

Besides shear stress, there can also be constraints on the bending moment present.

2.3 Stress criterions in plates

In plates stresses in multiple direction often occur. We already know how to calculate the minimum and maximum stresses σ_1 and σ_2 in a plate. These stresses must comply to the **maximum normal stress**

criterion, meaning that

$$\sigma_1 \leq \sigma_{all}, \qquad \sigma_2 \leq \sigma_{all}, \tag{2.3}$$

where σ_{all} is the maximum allowable normal stress. There is also the **maximum shear stress criterion**, meaning that

$$|\tau_{max}| \leq \tau_{all} \qquad \Leftrightarrow \qquad |\sigma_1 - \sigma_2| \leq 2\tau_{all}, \tag{2.4}$$

where τ_{max} is the maximum allowable shear stress. The **von Mises criterion** also demands that

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 \leq \sigma_{all}^2 \tag{2.5}$$

And eventually, in some cases, there is also the **maximum strain criterion**.

3 Buckling

3.1 Definition

Buckling of a structure means failure due to excessive displacements or loss of stability. Buckling occurs at the so-called **critical load**. This depends on various properties of the beam/plate itself, as how it is connected to the structure.

3.2 Buckling of beams

Let's suppose a simply supported beam with E-modulus E, length L and moment of inertia I is loaded under compression. The compressive force at which buckling will occur is

$$P_{max} = \frac{\pi^2 EI}{L^2}. \tag{3.1}$$

3.3 Buckling of plates

Let's look at a flat plate with height h, width w and thickness t, loaded in the vertical direction. Also assume all four edges of the plate are supported. The load at which failure occurs is

$$\frac{4\pi^2}{w} \frac{Et^3}{12\left(1-\nu^2\right)}. \tag{3.2}$$

Plates often have **post-buckling strength**, meaning that the plate can resist increased loads even after buckling.

3.4 Buckling of cylinder shells

Let's examine a cylinder shell with radius R, thickness t and height h. The critical load can be found by using

$$\sigma_{cr} = \frac{1}{\sqrt{3\left(1-\nu^2\right)}}\left(\frac{Et}{R}\right), \qquad \text{and} \qquad P_{cr} = 2\pi R t \sigma_{cr}. \tag{3.3}$$

This, however, is only the result of theory. In practice there are small imperfections in cylinders, which significantly reduce the strength. To compensate for this, there is the so-called **knockdown factor** γ, such that

$$\sigma_{cr_{design}} = \gamma \sigma_{cr_{theory}}. \tag{3.4}$$

The knockdown factor is a function of R/t. It decreases for thinner shells.