Aircraft SA & SD Formula Overview

Trusses

Statically determinate trusses

$$\delta_j = \sum_{i=1}^{n} \frac{F_i f_i L_i}{A_i E_i} \tag{1}$$

Statically indeterminate trusses

$$\delta_{AB}^{ext} = \sum_{i=1}^{m} \frac{F_i^{ext} f_i L_i}{A_i E_i} \tag{2}$$

$$\delta_{AB}^{int} = \sum_{i=1}^{m} \frac{F_i^{int} f_i L_i}{A_i E_i} = F_j \sum_{i=1}^{m} \frac{f_i^2 L_i}{A_i E_i} \tag{3}$$

$$\delta_{AB}^{ext} + \delta_{AB}^{int} = 0 \tag{4}$$

$$F_i^{act} = F_i^{ext} + F_i^{int} = F_i^{ext} + F_j f_i \tag{5}$$

Actuation

$$\varepsilon = \varepsilon_T + \varepsilon_M = \alpha\,\Delta T + \frac{P}{AE} \tag{6}$$

$$\delta_{AB}^{ext} = \sum_{i=1}^{m} f_i L_i \left(\alpha\,\Delta T + \frac{F_i}{A_i E_i}\right) \tag{7}$$

Beams

Basic beam equations

$$\sigma = \sigma_N + \sigma_M = \frac{F}{A} + \frac{My}{I} \tag{8}$$

$$\delta = \sum_{i=1}^{m} \frac{M_i m_i L_i}{E_i I_i} \tag{9}$$

$$\delta_{A_i} = \delta_{A_i}^{ext} + R_{A_1} f_{A_i A_1} + \ldots + R_{A_n} f_{A_i A_n} \tag{10}$$

Beams of multiple materials

$$A^* = \int_A \frac{E}{E_{ref}} dA \tag{11}$$

$$\bar{x}^* = \frac{1}{A^*} \int_A x \frac{E}{E_{ref}} dA \tag{12}$$

$$I^* = \int_A (y - \bar{y}^*)^2 \frac{E}{E_{ref}} dA \tag{13}$$

$$\sigma_N(x,y) = \frac{P}{A^*} \frac{E(x,y)}{E_{ref}} \tag{14}$$

$$\sigma_M(x,y) = \frac{M\,(y - \bar{y}^*)}{I^*} \frac{E(x,y)}{E_{ref}} \tag{15}$$

Plates

Biaxial stress state

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} = \frac{R_x}{ELt} - \nu\frac{P}{Ewt} \tag{16}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} = \frac{P}{Ewt} - \nu\frac{R_x}{ELt} \tag{17}$$

$$\sigma_x = \frac{R_x}{Lt} = \nu\frac{P}{wt} = \nu\sigma_y \tag{18}$$

$$\sigma_y = \frac{P}{wt} \tag{19}$$

Mohr's circle

$$\sigma_{av} = \frac{\sigma_x + \sigma_y}{2} \tag{20}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{21}$$

$$\tau_{max} = R \tag{22}$$

$$\sigma_{min} = \sigma_{av} - R \tag{23}$$

$$\sigma_{max} = \sigma_{av} + R \tag{24}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \tag{25}$$

Other

Sensitivity

$$\frac{\partial\sigma}{\partial A} = \frac{\sigma(A_1) - \sigma(A_0)}{A_1 - A_0} = \frac{\sigma_1 - \sigma_0}{A_1 - A_0} \tag{26}$$

Stress concentrations

$$\sigma_{max} = K\sigma_{average} \tag{27}$$

Failure criteria

$$\sigma_1 \le \sigma_{all}, \qquad \sigma_2 \le \sigma_{all} \tag{28}$$

$$|\tau_{max}| \le \tau_{all} \quad \Leftrightarrow \quad |\sigma_1 - \sigma_2| \le 2\tau_{all} \tag{29}$$

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 \le \sigma_{all}^2 \tag{30}$$