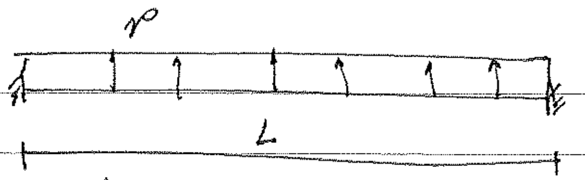
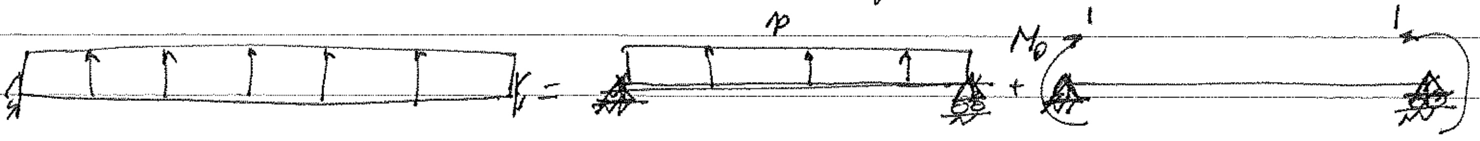
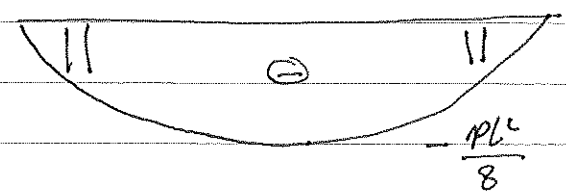


Problem I:

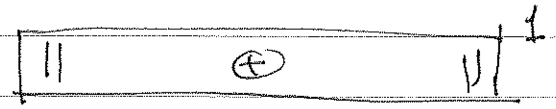
In this problem a beam made of one material and having a constant cross-section is subject to uniform loading.



The bending moment of the statically determinate beam is



The bending moment of the unit load is

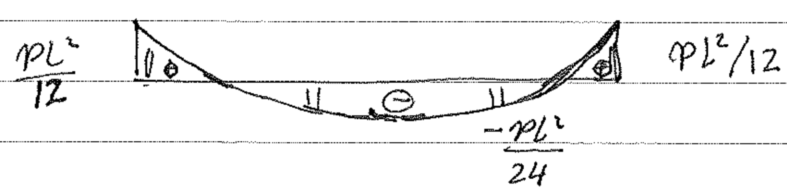


$$\int_0^L \frac{M_{sd}\, m}{EI}\, dx = \frac{1}{EI}\left(-\frac{pL^2}{8}\right) \times L \times \frac{2}{3} = -\frac{pL^3}{12EI}$$

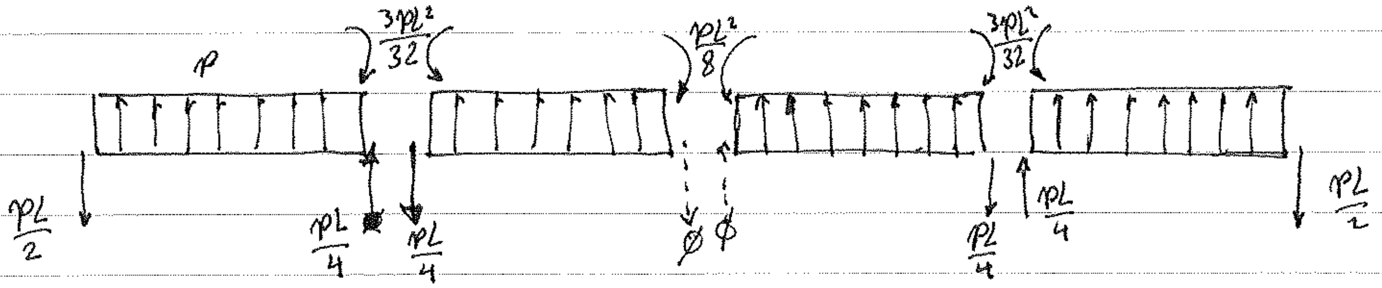
$$\int_0^L \frac{m^2}{EI}\, dx = \frac{L}{EI}$$

$$\Rightarrow \quad \frac{L}{EI} M_0 + -\frac{pL^3}{12EI} = 0 \quad \Rightarrow \quad M_0 = \frac{pL^2}{12}$$

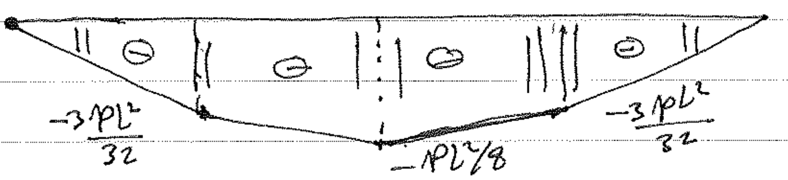
The final bending moment $M = M_{sd} + M_0\, m$



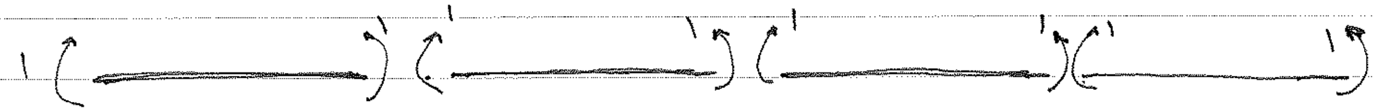
We can solve the problem by discretization into (say) 4 elements



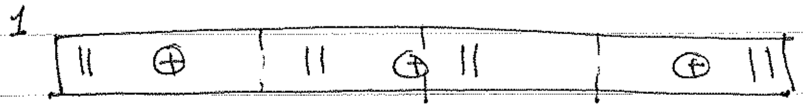
Bending moment of the statically determinate structure.



Bending moment of the unit load system.



Bending moment of the unit load



$$\int_0^L \frac{M_{sd}\, m}{EI}\, dx = \left\{ \frac{\left[\frac{1}{2}\left(\frac{-3pL^2}{32} + 0\right)\right][1]\left(\frac{L}{4}\right)}{EI} + \frac{\left[\frac{1}{2}\left(\frac{-3pL^2}{32} - \frac{pL^2}{8}\right)\right][1]\left(\frac{L}{4}\right)}{EI} \right\} \times 2$$

$$= -\frac{5}{64}\frac{pL^3}{EI}$$

$$\int_0^L \frac{m^2}{EI}\, dx = \frac{L}{EI}$$

$$\Longrightarrow \quad M_0 = -\frac{5}{64}\, pL^2 \quad (6\% \text{ off})$$