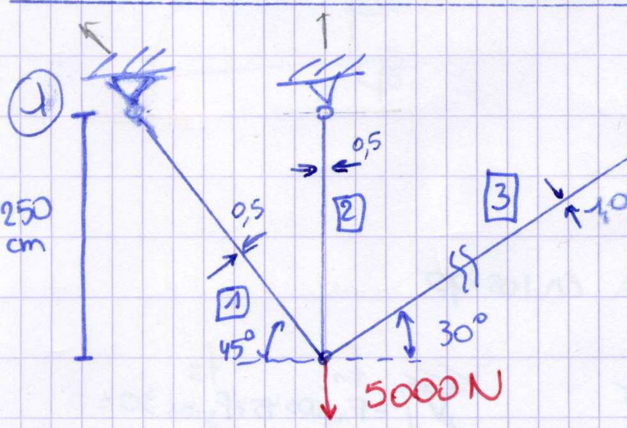
AE2-521N: A/C Stress analysis & Structural design : Extra 7.2 : dond 18okt'07

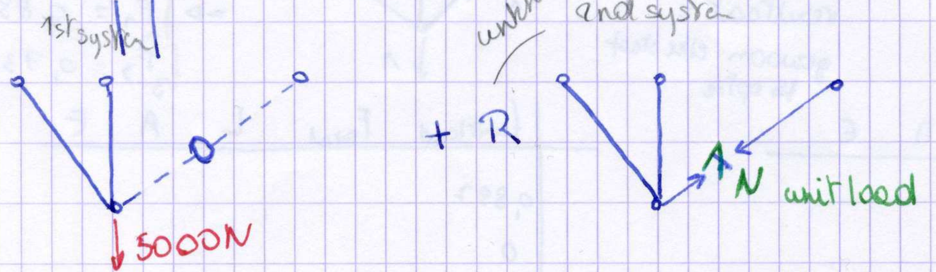
(1)

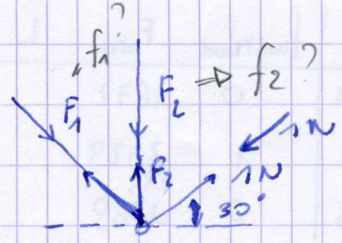


⊳ 2-force members! reaction forces in verlengde staven

\# joints : 1 ⇒ # equilib eqs = 2.1 = 2 ($\llcorner$ # joints)

\# members : 4





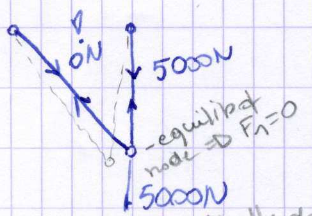
equilib:

$1 \cos 30° - F_1 \cos 45° = 0$

$F_1 = \frac{\cos 30}{\cos 45} = 1,225$ (f_1?)

$F_2 + F_1 \sin 45° + 1 . \sin 30° = 0$ ($F_2 \to f_2$, $F_1 \to f_1$)

⇒ $F_2 = -1,366$ (f_2)

=  + "

	SDS (statically determinate system) 1st system	RUL (f) (redundant unit load)	L (cm)	A (cm²)	E (N/cm²)
member 1:	0	1,225	$\frac{250}{\cos 45} \approx 353,6$	0,5	$200 . 10^5$
2:	5000	-1,366	250	0,5	$200 . 10^5$
3:	0	1	500	1	$200 . 10^5$

F : member 1: $0 + 1,225 R$

2: $5000 - 1,366 R$

3: $0 + R$

virtual work done = 0

⊳ $\Sigma \frac{F f . L}{EA}$ (F: total load, f: unit load)

$\Sigma \frac{F f L}{EA} = 0$

virtual work ⇒ $-0,1708 + 1,247 . 10^{-4} R = 0$ ⇒ $R = 1369,5 N$

$F = F_{stat\,det} + R . f$

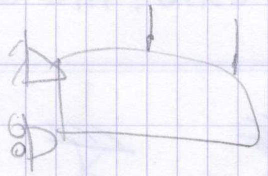
$\left(\Sigma \frac{F_{sd} f L}{EA}\right) + R \left(\Sigma \frac{f^2 L}{EA}\right)$

⇒ member 1: F = 1677N

2: 3129N

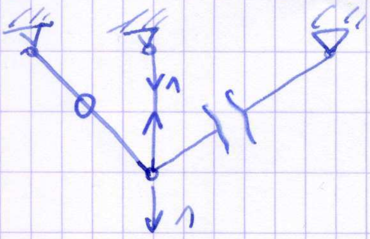
3: 1369N

	$(F_{sd} \cdot f L)/EA$	$\frac{f^2 L}{EA}$
1:		
2:		
3:		
	$\varepsilon = -0{,}1708$	$1{,}247 \cdot 10^{-4}$



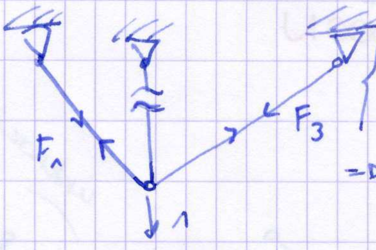
Tip: gebruik andere member as redundant en los op

Find the vertical displacement.



OF

resultaat = gewoon elke staaf is optie



$f_1 \quad f_3$

$-F_1 \cos 45 + F_3 \cos 30 = 0$

$F_1 \sin 45 + F_3 \sin 30 - 1 = 0$

$\Rightarrow f_1 F_1 = 0{,}897$

$f_3 F_3 = 0{,}732$

	$f_{unit load}$	F_{total}	L	A	E
1	0	1677			
2	1	3129			
3	0	1368			

$f_{unit load}$	F_{total}	L	A	E
0,897				
0				
0,732				

$\delta = \Sigma \frac{fFL}{EA} = 0{,}078\,cm$

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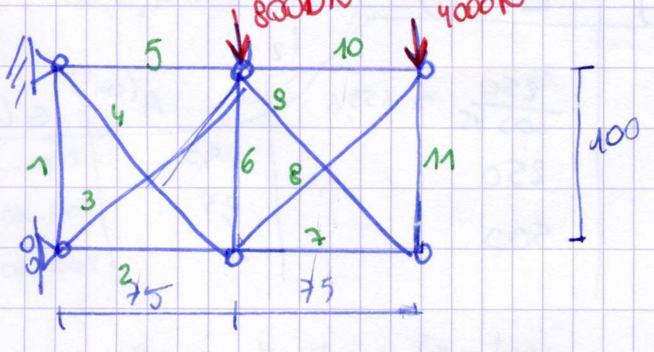
② 

11 members

6 joints

$6 \times 2 = 12$ eqs

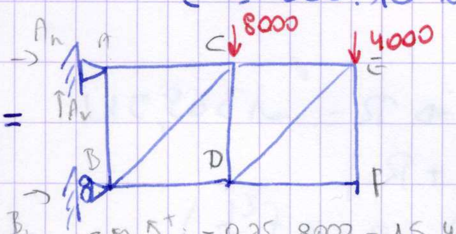
$11 + 3 = 14$ # onbek

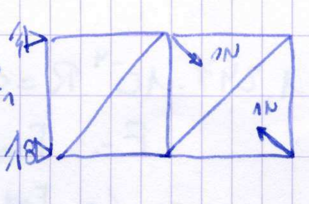
} 2 onder 2nd degr.

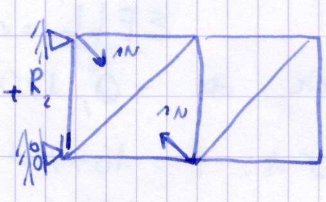
$A = 0{,}5\,cm^2$

$E = 200 \cdot 10^5\,N/cm^2$

NIET 2 staven uit zelfde "hokje" halen $\Rightarrow$ mechanisme!



= ... + R_1 

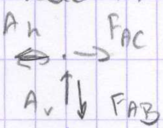
+ R_2 

$\Sigma M_B \uparrow^+ : -0{,}75 \cdot 8000 - 1{,}5 \cdot 4000 + 1 \cdot B_h = 0 \Rightarrow B_h = 12000\,N$

$\Sigma F_x \rightarrow^+ : A_h = -12000$

$\Sigma F_y \uparrow^+ : A_v = 12000$

node A:



$F_{AC} = A_h = 12000$ .. krachten berekenen

	F_{sd}	f_1	f_2
1	12000	0	-0,8
2	-3000	0	-0,6
3	-15000	0	1
4	0	0	1
5	12000	0	-0,6
6	4000	-0,8	-0,8
7	0	-0,6	0
8	-5000	1	0
9	0	1	0
10	3000	-0,6	0
11	0	-0,8	0

2 eqs needed due to 2 redundancies

$$\Sigma \frac{F f_1 L}{EA} = 0$$

$$\Sigma \frac{F f_2 L}{EA} = 0$$

$$F = F_{sd} + R_1 f_1 + R_2 f_2$$

bij meerdere (3) redundancies:

$$\Sigma \frac{F_{sd} f_1 L}{EA} + R_1 \left(\Sigma \frac{f_1^2 L}{EA}\right) + R_2 \left(\frac{\Sigma f_1 f_2 L}{EA}\right) + \left(\Sigma \frac{f_1 f_3 L}{EA}\right) R_3 = 0$$

$$\Sigma \frac{F_{sd} f_1 L}{EA} + \left(\Sigma \frac{f_1^2 L}{EA}\right) R_1 + \left(\frac{\Sigma f_1 f_2 L}{EA}\right) R_2 = 0$$

$$\Sigma \frac{F_{sd} f_2 L}{EA} + \left(\Sigma \frac{f_1 f_2 L}{EA}\right) R_1 + \left(\Sigma \frac{f_2^2 L}{EA}\right) R_2 = 0$$

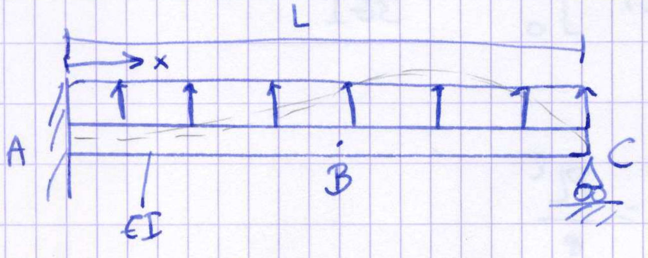
$$\begin{cases} 4{,}32 \cdot 10^{-5} \cdot R_1 + 6{,}40 \cdot 10^{-6} R_2 - 0{,}108 = 0 \\ 6{,}40 \cdot 10^{-6} R_1 + 4{,}32 \cdot 10^{-5} R_2 - 0{,}316 = 0 \end{cases}$$

$$\Rightarrow R_1 = 1450\,N$$

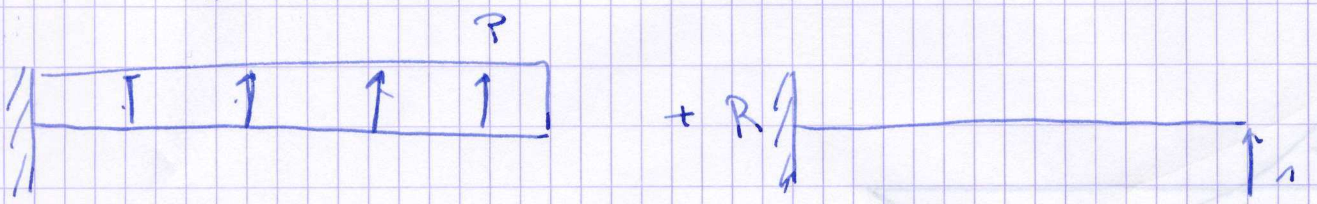
$$R_2 = 7088{,}4\,N$$

AE2 - 521N
extra 2.1

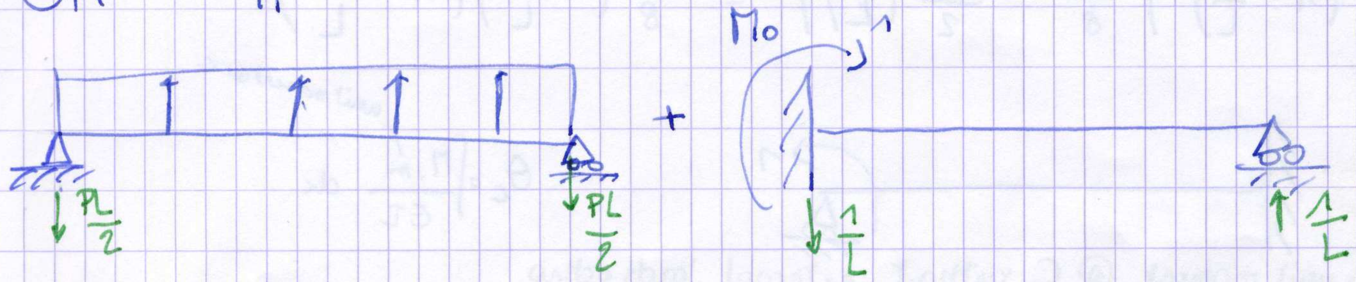
AE2-521N: A/C Stress Analysis & Structural design: Extra 2 : dond 8 nov 07



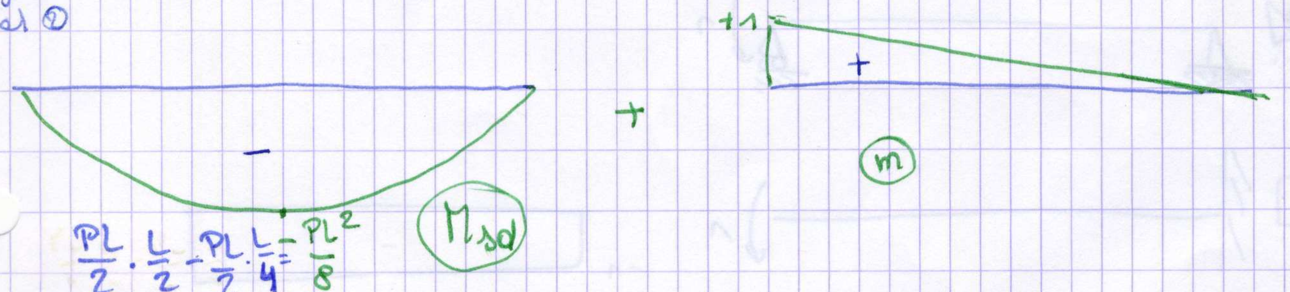
Find δ_B , θ_C



OR



M-lijnen voor geval ①



$$M = (M_{sd} + M_0 \cdot m)$$

$$\theta_A = \int \frac{Mm}{EI}\,dx = 0 \Rightarrow \left(\int_0^L \frac{M_{sd}\cdot m}{EI}\,dx\right) + M_0\left(\int_0^L \frac{m^2}{EI}\,dx\right) = 0$$

$$m = \left(1 - \frac{x}{L}\right)$$

$$\int_0^L \frac{m M_{sd}}{EI}\,dx = \frac{-PL^2}{2EI}\int_0^L \left(\frac{x}{L}\right) - 2\left(\frac{x}{L}\right)^2 + \left(\frac{x}{L}\right)^3 dx$$

$$M_{sd} = \frac{1}{2}PL^2\left(\frac{x}{L}\right)\left(1-\frac{x}{L}\right)$$

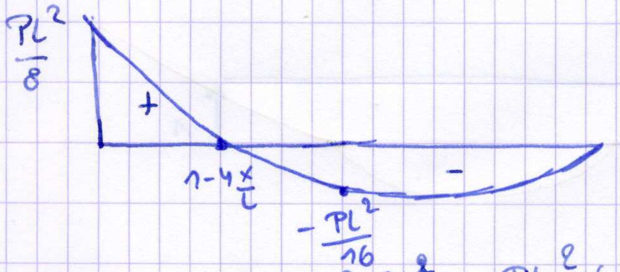
$$= \frac{-PL^3}{2EI}\left[\frac{1}{2}\left(\frac{x}{L}\right)^2 - \frac{2}{3}\left(\frac{x}{L}\right)^3 + \frac{1}{4}\left(\frac{x}{L}\right)^4\right]_0^L$$

$$= \frac{-PL^3}{2EI}\left(\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right) = \frac{-PL^3}{24EI}$$

NOTE: to find displacement: DON'T apply unit load on inderstruc
WEL on determ struc

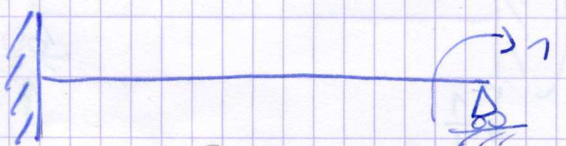
$$\int_0^L \frac{(1-x)^2}{EI}\,dx = \frac{L}{EI}\left[\left(\frac{x}{L}\right) - \left(\frac{x}{L}\right)^2 + \frac{1}{3}\left(\frac{x}{L}\right)^3\right]_0^L = \frac{L}{3EI}$$

$$-\frac{PL^3}{24EI} + M_0 \frac{L}{3EI} = 0 \quad \Rightarrow \quad M_0 = \frac{PL^2}{8}$$



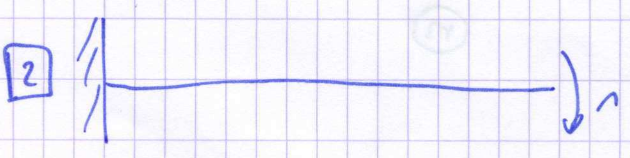
$$M = \left(1-\frac{x}{L}\right)\left\{\frac{PL^2}{8} - \frac{PL^2}{2}\left(\frac{x}{L}\right)\right\} = \frac{PL^2}{8}\left(1-\frac{x}{L}\right)\left(1-4\frac{x}{L}\right)$$

NEXT: $\theta_C = ?$



$$\theta_C = \int \frac{M \cdot m}{EI}\,dx$$ (m: unit moment @ C)

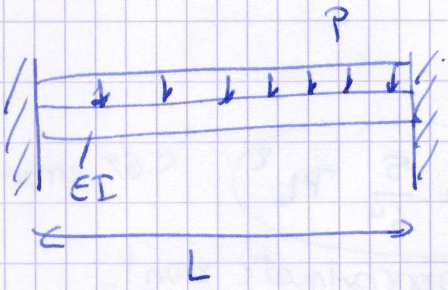
↳ apply unit moment @ C without external loads acting



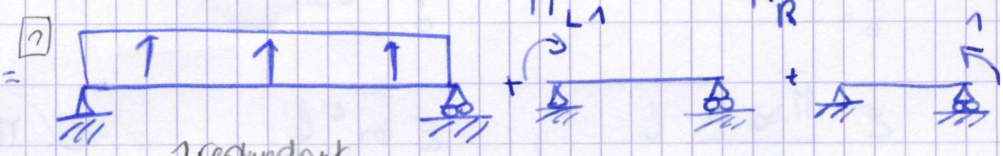
$$\theta_C = \int_0^L \frac{-PL^2}{8EI}\left[1 - 5\left(\frac{x}{L}\right) + 4\left(\frac{x}{L}\right)^2\right] dx = \frac{-PL^3}{8EI}\left[\left(\frac{x}{L}\right) - \frac{5}{2}\left(\frac{x}{L}\right)^2 + \frac{4}{3}\left(\frac{x}{L}\right)^3\right]_0^L = \frac{PL^3}{48EI}$$

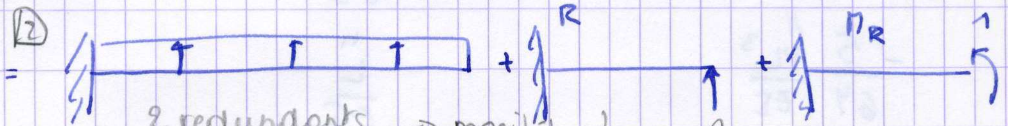
Q: to find displacement @ B? ⇒ apply unit load

PROBLEM 2

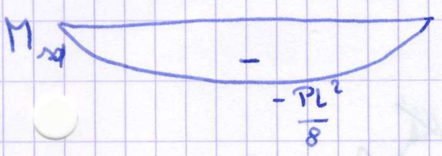


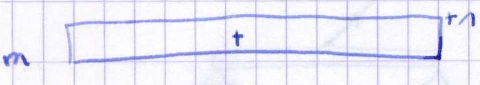
assumption: no horizontal moment since no hori loads → pure bending





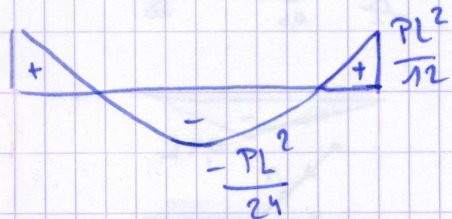
using approach [1]:



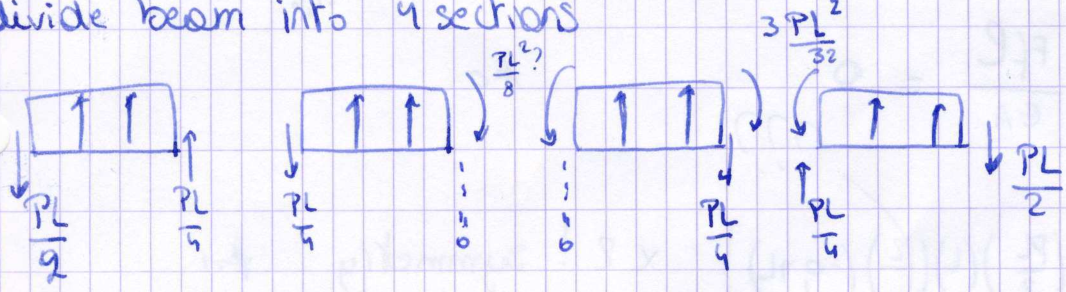


$$\int_0^L \frac{M_{sd}\, m}{EI}\, dx + M_0 \int_0^L \frac{m^2}{EI}\, dx = 0$$

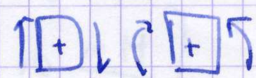
$$\frac{-PL^3}{12EI} + M_0 \frac{L}{EI} = 0 \Rightarrow M_0 = \frac{PL^2}{12}$$

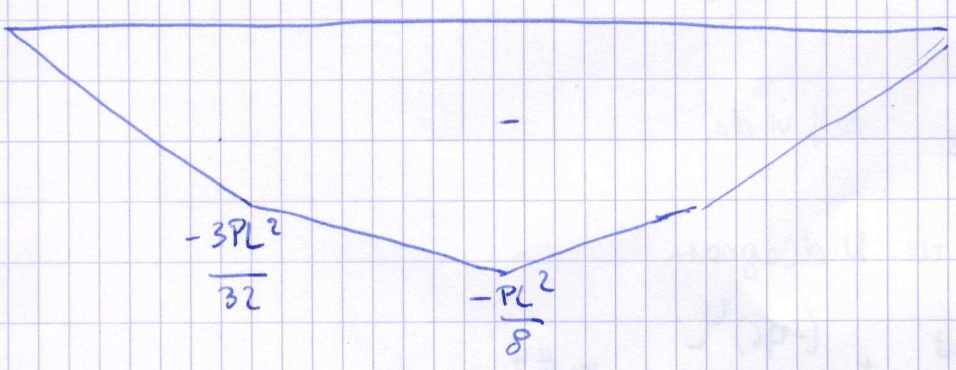


divide beam into 4 sections



remember





$$\int_0^L \frac{M_{sd} \cdot m}{EI}\, dx \quad / \quad \int_0^L \frac{m^2}{EI}\, dx$$

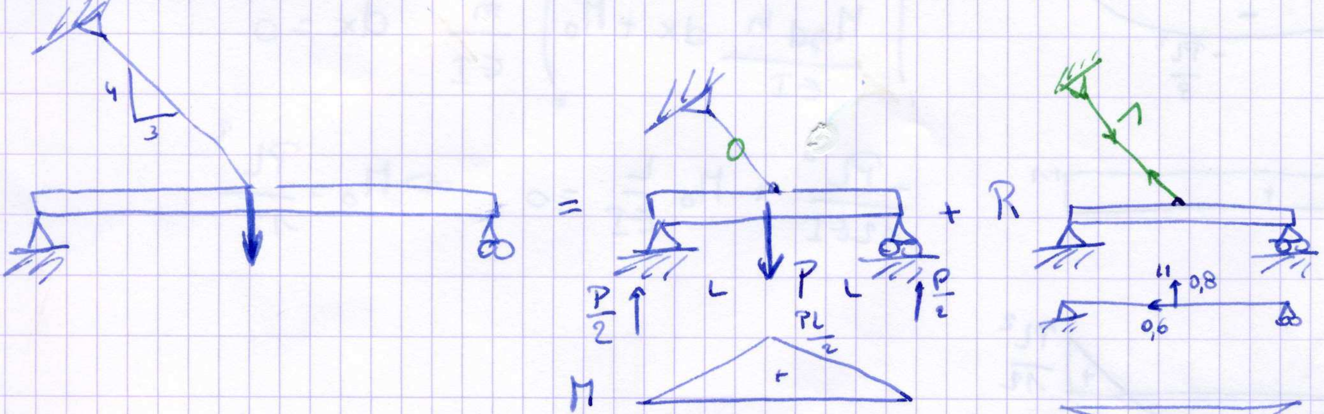
$$\Sigma \frac{\bar{M}_{sd}\, \bar{m}}{EI}\, l \quad / \quad \Sigma \frac{\bar{m}^2 l}{EI} \Rightarrow M_o = \frac{5}{64} PL^2 \approx 6\% \text{ error}$$

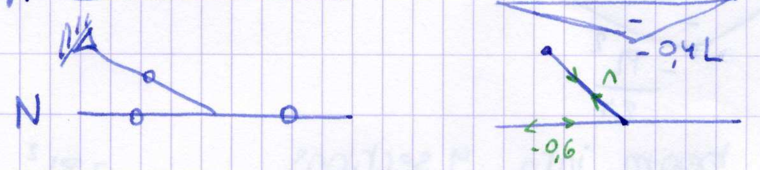
$$= -\frac{5}{64} \frac{PL^3}{EI} \qquad = \frac{L}{EI}$$

approximate want!

exact $= \frac{PL^2}{12}$

PROBLEM 3





bij puurbending $\Sigma = 0$

bij truss $\int = 0$

alg vgl

$$\int_0^L \frac{M\, m}{EI}\, dx + \Sigma \frac{F f l}{EA} = 0 \qquad \#0$$

???

displ

$$\int_0^{2L} \frac{M_{sd}\, m}{EI} = \frac{1}{EI}\left\{ \frac{1}{2}\left(\frac{PL}{2}\right)(L)\left(\frac{2}{3}\right)(-0{,}4L)\right\} \times 2 \text{ ! Symmetry} \qquad \#1$$

$$\int_0^{2L} \frac{m^2}{EI}\, dx = \qquad \#_2 \quad \text{zelf vinden}$$

$$\Sigma \frac{F_{sd}\, f l}{EA} = 0 \qquad \#_3 \text{ zie N-diagram}$$

$$\Sigma \frac{f^2 L}{EA} = \frac{(1^2)\, l_{strut}}{E_{strut} \cdot A_{strut}} + \frac{(-0{,}6)^2 L}{E_{beam} A_{beam}} = \#4$$

displacement: apply unit load

→ find moment due to this

rotation: apply unit moment → $\int_0^L \frac{M m}{EI}\, dx$

ALGEMEEN

ssing #3:

$$\left(\int_0^L \frac{m_{sd} m}{EI} \, dx + \Sigma \frac{F_{sd} f l}{EA} \right) + R \left(\int_0^L \frac{m^2}{EI} \, dx + \Sigma \frac{f^2 l}{EA} \right) = 0$$

$$(\#1 + \#3) + R(\#2 + \#4) = 0$$