Reduction of Multiple Subsystems

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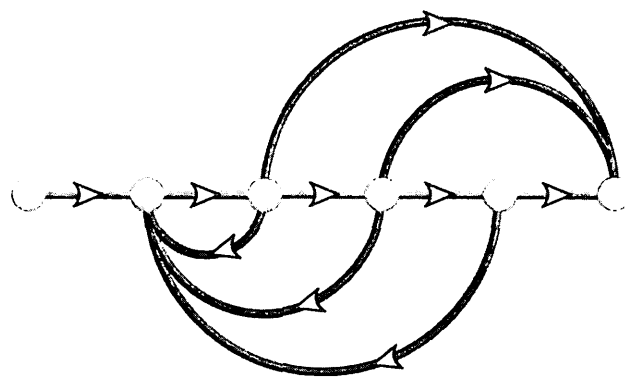


Chapter Learning Outcomes

After completing this chapter the student will be able to:

- Reduce a block diagram of multiple subsystems to a single block representing the transfer function from input to output (Sections 5.1–5.2)
- Analyze and design transient response for a system consisting of multiple subsystems (Section 5.3)
- Convert block diagrams to signal-flow diagrams (Section 5.4)
- Find the transfer function of multiple subsystems using Mason's rule (Section 5.5)
- Represent state equations as signal-flow graphs (Section 5.6) State Space SS
- Represent multiple subsystems in state space in cascade, parallel, controller canonical, and observer canonical forms (Section 5.7) State Space SS
- Perform transformations between similar systems using transformation matrices; and diagonalize a system matrix (Section 5.8) State Space SS

Case Study Learning Outcomes

You will be able to demonstrate your knowledge of the chapter objectives with case studies as follows:

- Given the antenna azimuth position control system shown on the front endpapers, you will be able to (a) find the closed-loop transfer function that represents the system from input to output; (b) find a state-space representation for the closed-loop system; (c) predict, for a simplified system model, the percent overshoot, settling time, and peak time of the closed-loop system for a step input; (d) calculate the step response for the closed-loop system; and (e) for the simplified model, design the system gain to meet a transient response requirement.

State Space
SS

- Given the block diagrams for the Unmanned Free-Swimming Submersible (UFSS) vehicle's pitch and heading control systems on the back endpapers, you will be able to represent each control system in state space.

5.1 Introduction

We have been working with individual subsystems represented by a block with its input and output. More complicated systems, however, are represented by the interconnection of many subsystems. Since the response of a single transfer function can be calculated, we want to represent multiple subsystems as a single transfer function. We can then apply the analytical techniques of the previous chapters and obtain transient response information about the entire system.

In this chapter, multiple subsystems are represented in two ways: as block diagrams and as signal-flow graphs. Although neither representation is limited to a particular analysis and design technique, block diagrams are usually used for frequency-domain analysis and design, and signal-flow graphs for state-space analysis.

Signal-flow graphs represent transfer functions as lines, and signals as small-circular nodes. Summing is implicit. To show why it is convenient to use signal-flow graphs for state-space analysis and design, consider Figure 3.10. A graphical representation of a system's transfer function is as simple as Figure 3.10(*a*). However, a graphical representation of a system in state space requires representation of each state variable, as in Figure 3.10(*b*). In that example, a single-block transfer function requires seven blocks and a summing junction to show the state variables explicitly. Thus, signal-flow graphs have advantages over block diagrams, such as Figure 3.10(*b*): They can be drawn more quickly, they are more compact, and they emphasize the state variables.

We will develop techniques to reduce each representation to a single transfer function. Block diagram algebra will be used to reduce block diagrams and Mason's rule to reduce signal-flow graphs. Again, it must be emphasized that these methods are typically used as described. As we shall see, however, either method can be used for frequency-domain or state-space analysis and design.

5.2 Block Diagrams

As you already know, a subsystem is represented as a block with an input, an output, and a transfer function. Many systems are composed of multiple subsystems, as in Figure 5.1. When multiple subsystems are interconnected, a few more schematic elements must be added to the block diagram. These new elements are *summing junctions* and *pickoff points*. All component parts of a block diagram for a linear, time-invariant system are shown in Figure 5.2. The characteristic of the summing junction shown in Figure 5.2(*c*) is that the output signal, $C(s)$, is the algebraic sum of the input signals, $R_1(s)$, $R_2(s)$, and $R_3(s)$. The figure shows three inputs, but any

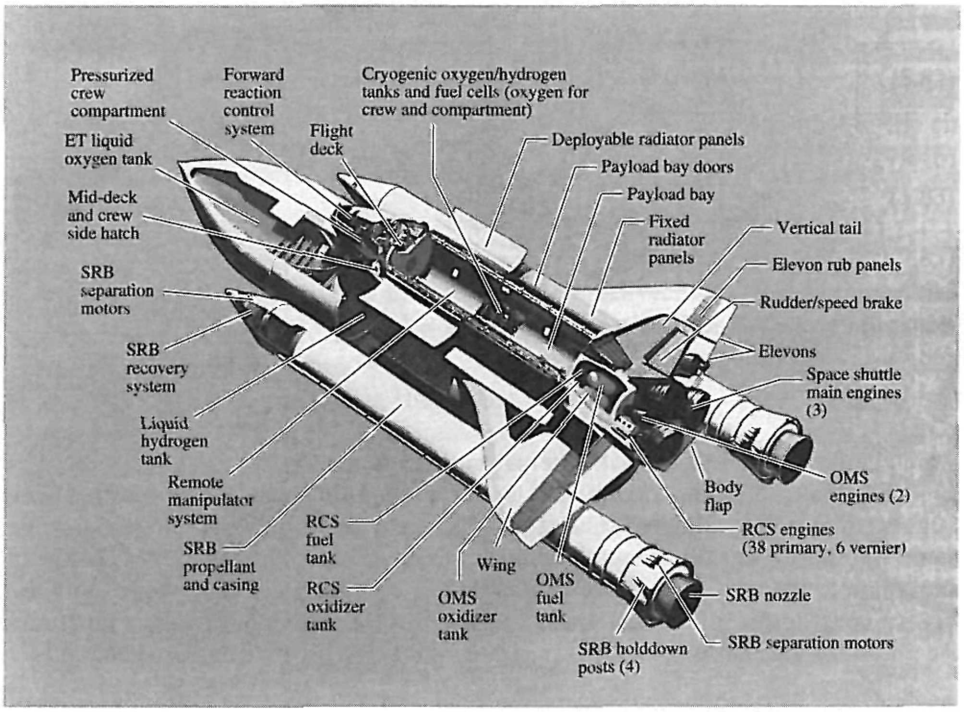


FIGURE 5.1 The space shuttle consists of multiple subsystems. Can you identify those that are control systems or parts of control systems?

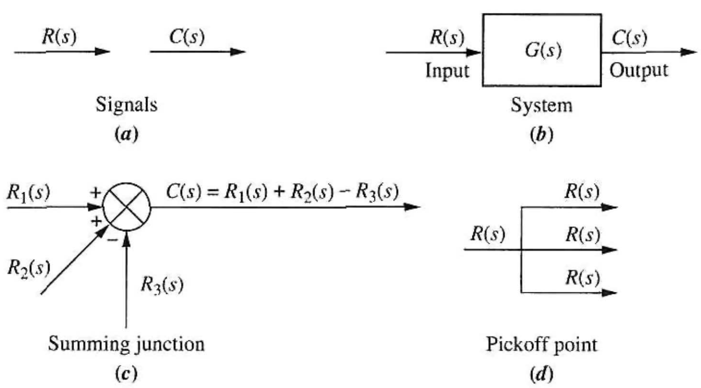


FIGURE 5.2 Components of a block diagram for a linear, time-invariant system

number can be present. A pickoff point, as shown in Figure 5.2(*d*), distributes the input signal, $R(s)$, undiminished, to several output points.

We will now examine some common topologies for interconnecting subsystems and derive the single transfer function representation for each of them. These common topologies will form the basis for reducing more complicated systems to a single block.

Cascade Form

Figure 5.3(*a*) shows an example of cascaded subsystems. Intermediate signal values are shown at the output of each subsystem. Each signal is derived from the product of the input times the transfer function. The equivalent transfer function, $G_e(s)$, shown in Figure 5.3(*b*), is the output Laplace transform divided by the input Laplace

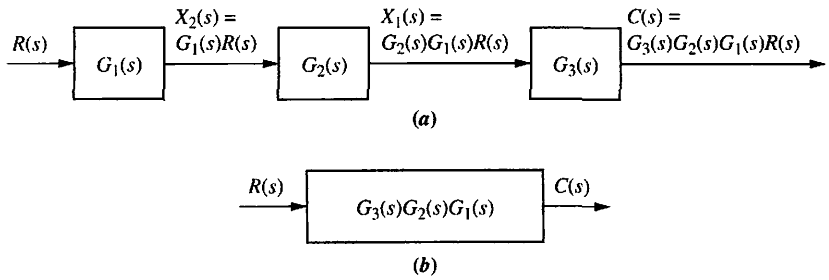


FIGURE 5.3 **a.** Cascaded subsystems; **b.** equivalent transfer function

transform from Figure 5.3(a), or

$$\boxed{G_e(s) = G_3(s)G_2(s)G_1(s)} \tag{5.1}$$

which is the product of the subsystems' transfer functions.

Equation (5.1) was derived under the assumption that interconnected subsystems do not load adjacent subsystems. That is, a subsystem's output remains the same whether or not the subsequent subsystem is connected. If there is a change in the output, the subsequent subsystem loads the previous subsystem, and the equivalent transfer function is not the product of the individual transfer functions. The network of Figure 5.4(a) demonstrates this concept. Its transfer function is

$$G_1(s) = \frac{V_1(s)}{V_i(s)} = \frac{\dfrac{1}{R_1C_1}}{s + \dfrac{1}{R_1C_1}} \tag{5.2}$$

Similarly, the network of Figure 5.4(b) has the following transfer function:

$$G_2(s) = \frac{V_2(s)}{V_1(s)} = \frac{\dfrac{1}{R_2C_2}}{s + \dfrac{1}{R_2C_2}} \tag{5.3}$$

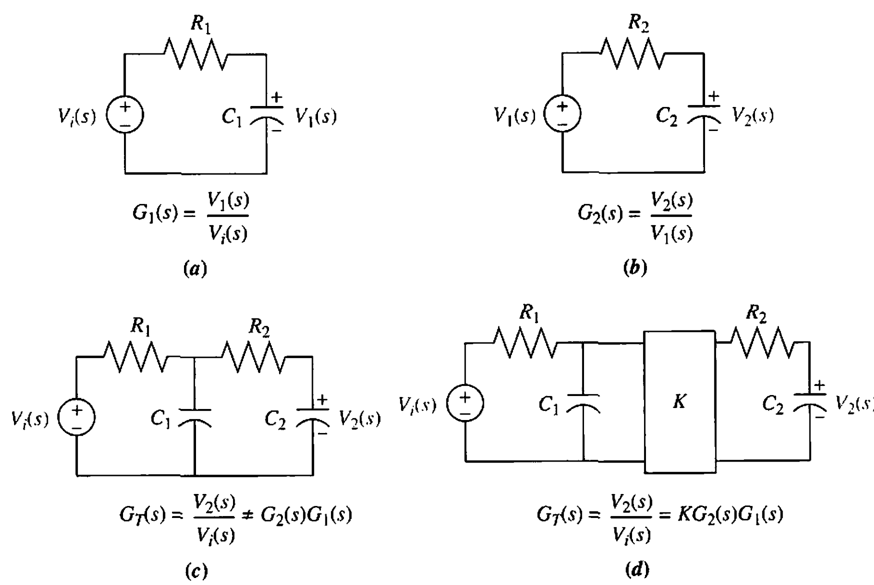


FIGURE 5.4 Loading in cascaded systems

If the networks are placed in cascade, as in Figure 5.4(c), you can verify that the transfer function found using loop or node equations is

$$G(s) = \frac{V_2(s)}{V_i(s)} = \frac{\dfrac{1}{R_1C_1R_2C_2}}{s^2 + \left(\dfrac{1}{R_1C_1} + \dfrac{1}{R_2C_2} + \dfrac{1}{R_2C_1}\right)s + \dfrac{1}{R_1C_1R_2C_2}} \tag{5.4}$$

But, using Eq. (5.1),

$$G(s) = G_2(s)G_1(s) = \frac{\dfrac{1}{R_1C_1R_2C_2}}{s^2 + \left(\dfrac{1}{R_1C_1} + \dfrac{1}{R_2C_2}\right)s + \dfrac{1}{R_1C_1R_2C_2}} \tag{5.5}$$

Equations (5.4) and (5.5) are not the same: Eq. (5.4) has one more term for the coefficient of s in the denominator and is correct.

One way to prevent loading is to use an amplifier between the two networks, as shown in Figure 5.4(d). The amplifier has a high-impedance input, so that it does not load the previous network. At the same time it has a low-impedance output, so that it looks like a pure voltage source to the subsequent network. With the amplifier included, the equivalent transfer function is the product of the transfer functions and the gain, K, of the amplifier.

Parallel Form

Figure 5.5 shows an example of parallel subsystems. Again, by writing the output of each subsystem, we can find the equivalent transfer function. Parallel subsystems have a common input and an output formed by the algebraic sum of the outputs from all of the subsystems. The equivalent transfer function, $G_e(s)$, is the output transform divided by the input transform from Figure 5.5(a), or

$$\boxed{G_e(s) = \pm G_1(s) \pm G_2(s) \pm G_3(s)} \tag{5.6}$$

which is the algebraic sum of the subsystems' transfer functions; it appears in Figure 5.5(b).

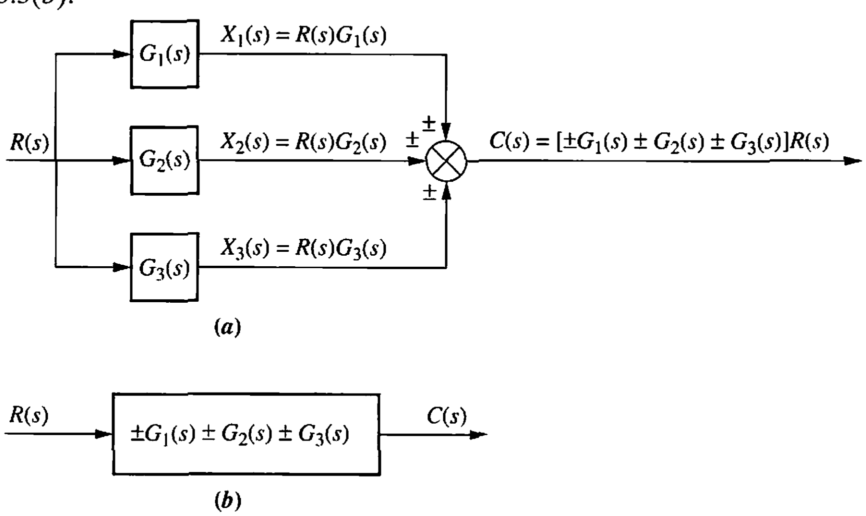


FIGURE 5.5 **a.** Parallel subsystems; **b.** equivalent transfer function

Feedback Form

The third topology is the feedback form, which will be seen repeatedly in subsequent chapters. The feedback system forms the basis for our study of control systems engineering. In Chapter 1, we defined open-loop and closed-loop systems and pointed out the advantage of closed-loop, or feedback control, systems over open-loop systems. As we move ahead, we will focus on the analysis and design of feedback systems.

Let us derive the transfer function that represents the system from its input to its output. The typical feedback system, described in detail in Chapter 1, is shown in Figure 5.6(*a*); a simplified model is shown in Figure 5.6(*b*).[1] Directing our attention to the simplified model,

$$E(s) = R(s) \mp C(s)H(s) \tag{5.7}$$

But since $C(s) = E(s)G(s)$,

$$E(s) = \frac{C(s)}{G(s)} \tag{5.8}$$

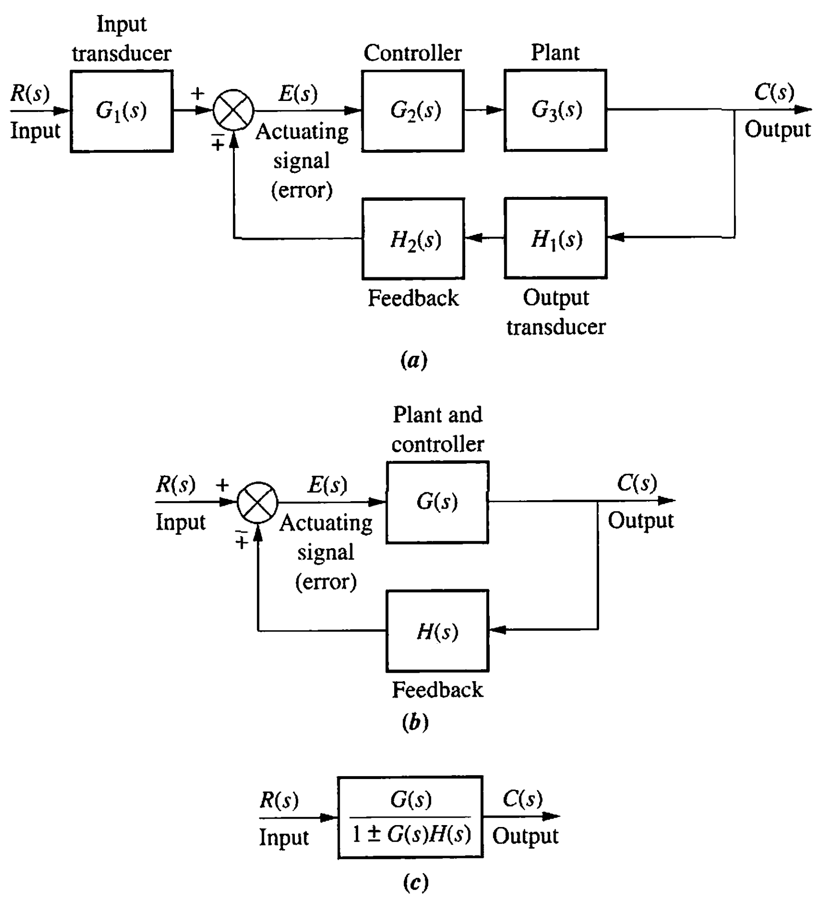


FIGURE 5.6 **a.** Feedback control system; **b.** simplified model; **c.** equivalent transfer function

[1] The system is said to have *negative feedback* if the sign at the summing junction is negative and *positive feedback* if the sign is positive.

Substituting Eq. (5.8) into Eq. (5.7) and solving for the transfer function, $C(s)/R(s) = G_e(s)$, we obtain the equivalent, or *closed-loop*, transfer function shown in Figure 5.6(*c*),

$$G_e(s) = \frac{G(s)}{1 \pm G(s)H(s)} \tag{5.9}$$

The product, $G(s)H(s)$, in Eq. (5.9) is called the *open-loop transfer function*, or *loop gain*.

So far, we have explored three different configurations for multiple subsystems. For each, we found the equivalent transfer function. Since these three forms are combined into complex arrangements in physical systems, recognizing these topologies is a prerequisite to obtaining the equivalent transfer function of a complex system. In this section, we will reduce complex systems composed of multiple subsystems to single transfer functions.

Moving Blocks to Create Familiar Forms

Before we begin to reduce block diagrams, it must be explained that the familiar forms (cascade, parallel, and feedback) are not always apparent in a block diagram. For example, in the feedback form, if there is a pickoff point after the summing junction, you cannot use the feedback formula to reduce the feedback system to a single block. That signal disappears, and there is no place to reestablish the pickoff point.

This subsection will discuss basic block moves that can be made to order to establish familiar forms when they almost exist. In particular, it will explain how to move blocks left and right past summing junctions and pickoff points.

Figure 5.7 shows equivalent block diagrams formed when transfer functions are moved left or right past a summing junction, and Figure 5.8 shows equivalent block diagrams formed when transfer functions are moved left or right past a pickoff point. In the diagrams the symbol ≡ means "equivalent to." These equivalences,

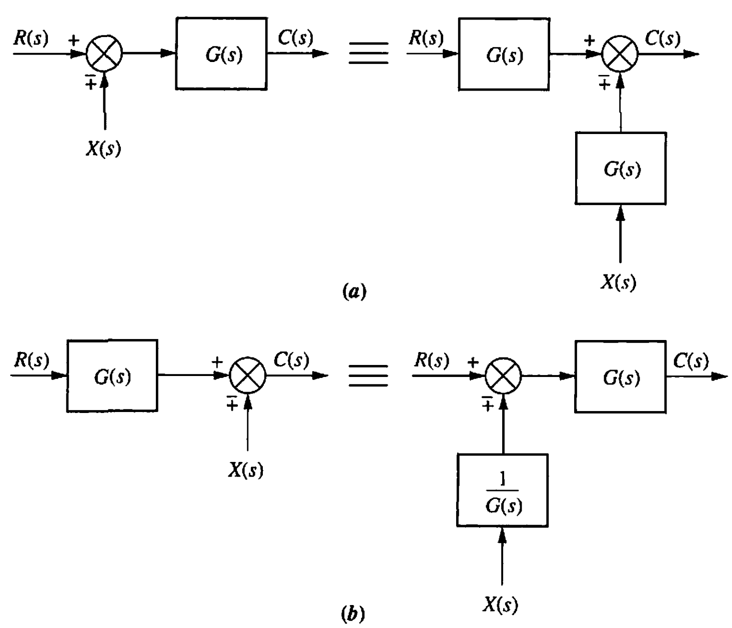


FIGURE 5.7 Block diagram algebra for summing junctions—equivalent forms for moving a block **a.** to the left past a summing junction; **b.** to the right past a summing junction

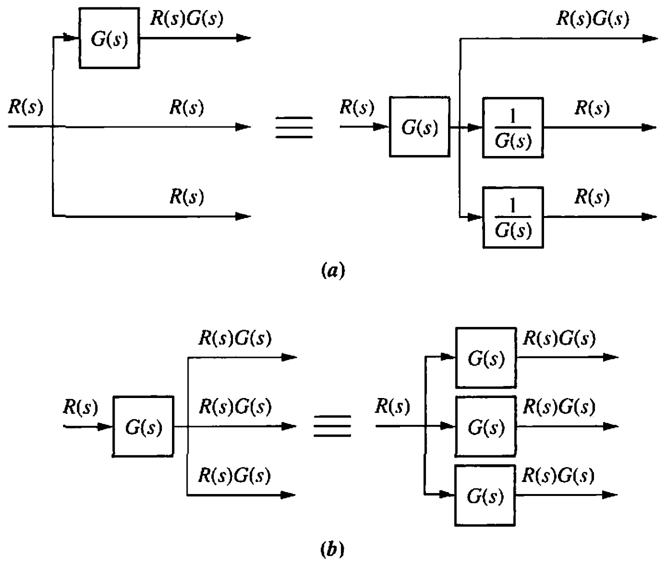


FIGURE 5.8 Block diagram algebra for pickoff points—equivalent forms for moving a block **a.** to the left past a pickoff point; **b.** to the right past a pickoff point

along with the forms studied earlier in this section, can be used to reduce a block diagram to a single transfer function. In each case of Figures 5.7 and 5.8, the equivalence can be verified by tracing the signals at the input through to the output and recognizing that the output signals are identical. For example, in Figure 5.7(a), signals $R(s)$ and $X(s)$ are multiplied by $G(s)$ before reaching the output. Hence, both block diagrams are equivalent, with $C(s) = R(s)G(s) \mp X(s)G(s)$. In Figure 5.7($b$), $R(s)$ is multiplied by $G(s)$ before reaching the output, but $X(s)$ is not. Hence, both block diagrams in Figure 5.7(b) are equivalent, with $C(s) = R(s)G(s) \mp X(s)$. For pickoff points, similar reasoning yields similar results for the block diagrams of Figure 5.8(a) and (b).

Let us now put the whole story together with examples of block diagram reduction.

Example 5.1

Block Diagram Reduction via Familiar Forms

PROBLEM: Reduce the block diagram shown in Figure 5.9 to a single transfer function.

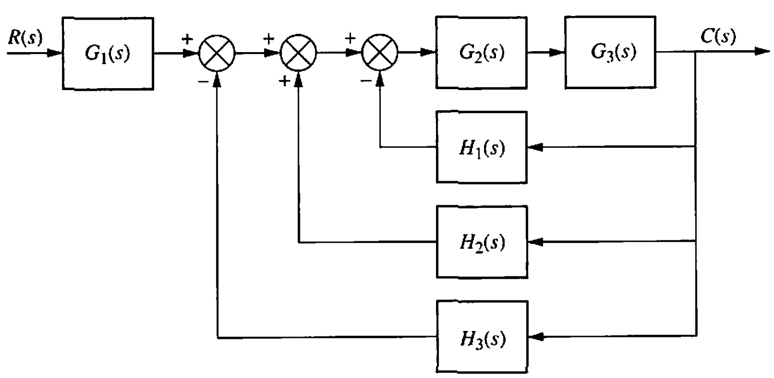


FIGURE 5.9 Block diagram for Example 5.1

SOLUTION: We solve the problem by following the steps in Figure 5.10. First, the three summing junctions can be collapsed into a single summing junction, as shown in Figure 5.10(*a*).

Second, recognize that the three feedback functions, $H_1(s)$, $H_2(s)$, and $H_3(s)$, are connected in parallel. They are fed from a common signal source, and their outputs are summed. The equivalent function is $H_1(s) - H_2(s) + H_3(s)$. Also recognize that $G_2(s)$ and $G_3(s)$ are connected in cascade. Thus, the equivalent transfer function is the product, $G_3(s)G_2(s)$. The results of these steps are shown in Figure 5.10(*b*).

Finally, the feedback system is reduced and multiplied by $G_1(s)$ to yield the equivalent transfer function shown in Figure 5.10(*c*).

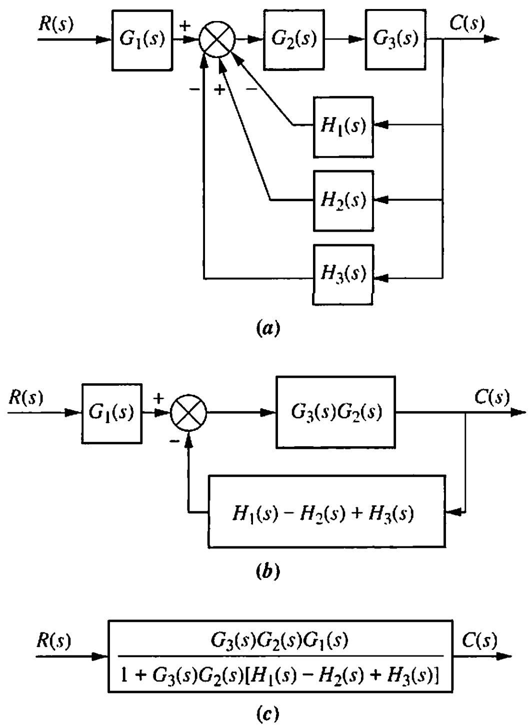


FIGURE 5.10 Steps in solving Example 5.1: **a.** Collapse summing junctions; **b.** form equivalent cascaded system in the forward path and equivalent parallel system in the feedback path; **c.** form equivalent feedback system and multiply by cascaded $G_1(s)$

Example 5.2

Block Diagram Reduction by Moving Blocks

PROBLEM: Reduce the system shown in Figure 5.11 to a single transfer function.

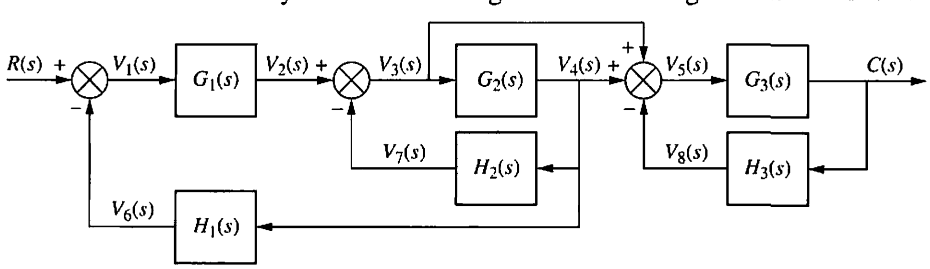


FIGURE 5.11 Block diagram for Example 5.2

SOLUTION: In this example we make use of the equivalent forms shown in Figures 5.7 and 5.8. First, move $G_2(s)$ to the left past the pickoff point to create parallel subsystems, and reduce the feedback system consisting of $G_3(s)$ and $H_3(s)$. This result is shown in Figure 5.12(*a*).

Second, reduce the parallel pair consisting of $1/G_2(s)$ and unity, and push $G_1(s)$ to the right past the summing junction, creating parallel subsystems in the feedback. These results are shown in Figure 5.12(*b*).

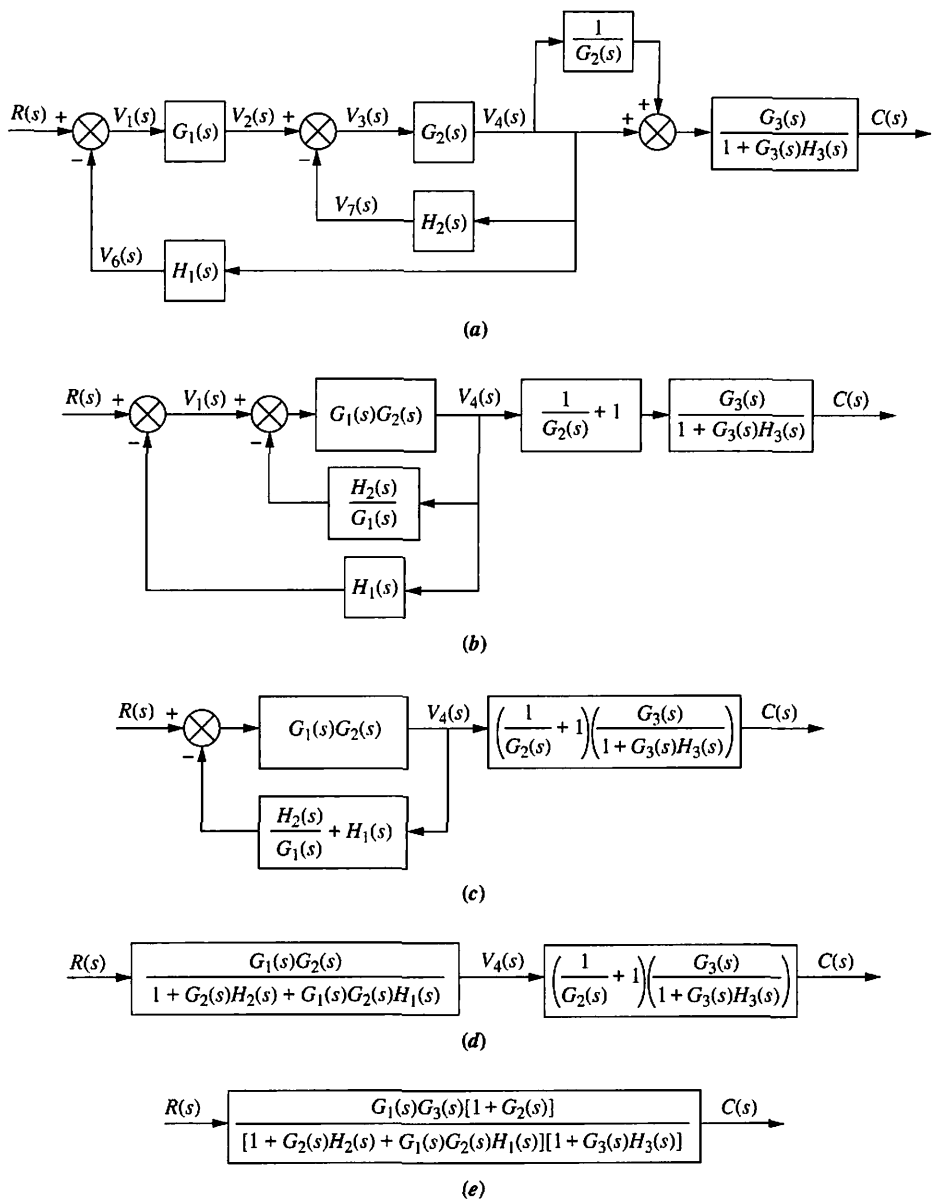


FIGURE 5.12 Steps in the block diagram reduction for Example 5.2

Third, collapse the summing junctions, add the two feedback elements together, and combine the last two cascaded blocks. Figure 5.12(*c*) shows these results.

Fourth, use the feedback formula to obtain Figure 5.12(*d*).

Finally, multiply the two cascaded blocks and obtain the final result, shown in Figure 5.12(*e*).

MATLAB
ML

Students who are using MATLAB should now run ch5p1 in Appendix B to perform block diagram reduction.

Skill-Assessment Exercise 5.1

PROBLEM: Find the equivalent transfer function, $T(s) = C(s)/R(s)$, for the system shown in Figure 5.13.

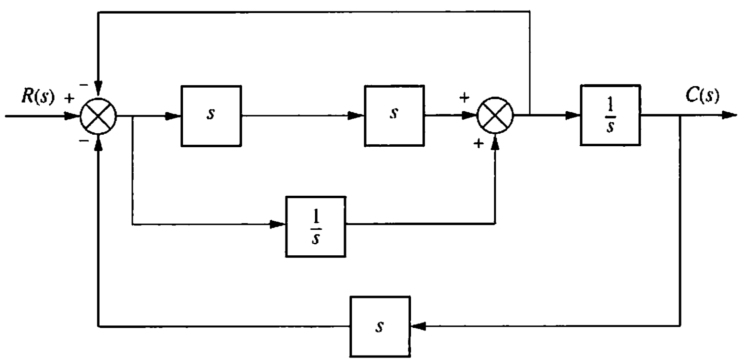


FIGURE 5.13 Block diagram for Skill-Assessment Exercise 5.1

ANSWER:

$$T(s) = \frac{s^3 + 1}{2s^4 + s^2 + 2s}$$

The complete solution is at www.wiley.com/college/nise.

TryIt 5.1

Use the following MATLAB and Control System Toolbox statements to find the closed-loop transfer function of the system in Example 5.2 if all $G_i(s) = 1/(s + 1)$ and all $H_i(s) = 1/s$.

```
G1=tf(1,[1 1]);
G2=G1;G3=G1;
H1=tf(1,[1 0]);
H2=H1;H3=H1;
System=append...
 (G1,G2,G3,H1,H2,H3);
input=1;output=3;
Q=[1  -4   0   0   0
   2   1  -5   0   0
   3   2   1  -5  -6
   4   2   0   0   0
   5   2   0   0   0
   6   3   0   0   0];
T=connect(System,...
 Q,input,output);
T=tf(T);T=minreal(T)
```

In this section, we examined the equivalence of several block diagram configurations containing signals, systems, summing junctions, and pickoff points. These configurations were the cascade, parallel, and feedback forms. During block diagram reduction, we attempt to produce these easily recognized forms and then reduce the block diagram to a single transfer function. In the next section, we will examine some applications of block diagram reduction.

5.3 Analysis and Design of Feedback Systems

An immediate application of the principles of Section 5.2 is the analysis and design of feedback systems that reduce to second-order systems. Percent overshoot, settling time, peak time, and rise time can then be found from the equivalent transfer function.

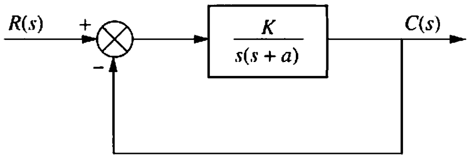


FIGURE 5.14 Second-order feedback control system

Consider the system shown in Figure 5.14, which can model a control system such as the antenna azimuth position control system. For example, the transfer function, $K/s(s+a)$, can model the amplifiers, motor, load, and gears. From Eq. (5.9), the closed-loop transfer function, $T(s)$, for this system is

$$T(s) = \frac{K}{s^2 + as + K} \tag{5.10}$$

where K models the amplifier gain, that is, the ratio of the output voltage to the input voltage. As K varies, the poles move through the three ranges of operation of a second-order system: overdamped, critically damped, and underdamped. For example, for K between 0 and $a^2/4$, the poles of the system are real and are located at

$$s_{1,2} = -\frac{a}{2} \pm \frac{\sqrt{a^2 - 4K}}{2} \tag{5.11}$$

As K increases, the poles move along the real axis, and the system remains overdamped until $K = a^2/4$. At that gain, or amplification, both poles are real and equal, and the system is critically damped.

For gains above $a^2/4$, the system is underdamped, with complex poles located at

$$s_{1,2} = -\frac{a}{2} \pm j\frac{\sqrt{4K - a^2}}{2} \tag{5.12}$$

Now as K increases, the real part remains constant and the imaginary part increases. Thus, the peak time decreases and the percent overshoot increases, while the settling time remains constant.

Let us look at two examples that apply the concepts to feedback control systems. In the first example, we determine a system's transient response. In the second example, we design the gain to meet a transient response requirement.

Example 5.3

Finding Transient Response

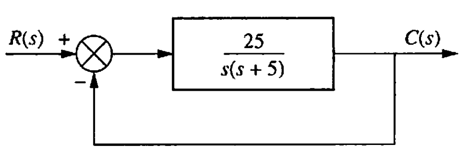


FIGURE 5.15 Feedback system for Example 5.3

PROBLEM: For the system shown in Figure 5.15, find the peak time, percent overshoot, and settling time.

SOLUTION: The closed-loop transfer function found from Eq. (5.9) is

$$T(s) = \frac{25}{s^2 + 5s + 25} \tag{5.13}$$

From Eq. (4.18),

$$\omega_n = \sqrt{25} = 5 \tag{5.14}$$

From Eq. (4.21),

$$2\zeta\omega_n = 5 \tag{5.15}$$

Substituting Eq. (5.14) into (5.15) and solving for ζ yields

$$\zeta = 0.5 \tag{5.16}$$

Using the values for ζ and ω_n along with Eqs (4.34), (4.38), and (4.42), we find respectively,

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.726 \text{ second} \tag{5.17}$$

$$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100 = 16.303 \tag{5.18}$$

$$T_s = \frac{4}{\zeta\omega_n} = 1.6 \text{ seconds} \tag{5.19}$$

MATLAB

ML

Students who are using MATLAB should now run ch5p2 in Appendix B. You will learn how to perform block diagram reduction followed by an evaluation of the closed-loop system's transient response by finding, T_p, $\%OS$, and T_s. Finally, you will learn how to use MATLAB to generate a closed-loop step response. This exercise uses MATLAB to do Example 5.3.

Simulink

SL

MATLAB's Simulink provides an alternative method of simulating feedback systems to obtain the time response. Students who are performing the MATLAB exercises and want to explore the added capability of MATLAB's Simulink should now consult Appendix C. Example C.3 includes a discussion about, and an example of, the use of Simulink to simulate feedback systems with nonlinearities.

Example 5.4

Gain Design for Transient Response

PROBLEM: Design the value of gain. K, for the feedback control system of Figure 5.16 so that the system will respond with a 10% overshoot.

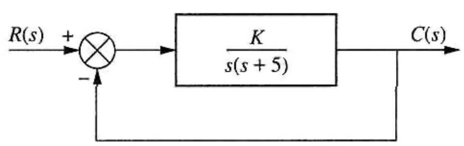


FIGURE 5.16 Feedback system for Example 5.4

SOLUTION: The closed-loop transfer function of the system is

$$T(s) = \frac{K}{s^2 + 5s + K} \tag{5.20}$$

From Eq. (5.20),

$$2\zeta\omega_n = 5 \tag{5.21}$$

and

$$\omega_n = \sqrt{K} \tag{5.22}$$

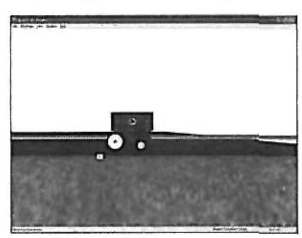
Thus,

$$\zeta = \frac{5}{2\sqrt{K}} \tag{5.23}$$

Since percent overshoot is a function only of ζ, Eq. (5.23) shows that the percent overshoot is a function of K.

Virtual Experiment 5.1
Position Control
Gain Design

Put theory into practice designing the position control gain for the Quanser Linear Position Servo and simulating its closed-loop response in LabVIEW. This concept is used, for instance, to control a rover exploring the terrain of a planet.



Virtual experiment are found on WileyPlus.

A 10% overshoot implies that $\zeta = 0.591$. Substituting this value for the damping ratio into Eq. (5.23) and solving for K yields

$$K = 17.9 \tag{5.24}$$

Although we are able to design for percent overshoot in this problem, we could not have selected settling time as a design criterion because, regardless of the value of K, the real parts, -2.5, of the poles of Eq. (5.20) remain the same.

Skill-Assessment Exercise 5.2

WileyPLUS WPCS Control Solutions

PROBLEM: For a unity feedback control system with a forward-path transfer function $G(s) = \dfrac{16}{s(s+a)}$, design the value of a to yield a closed-loop step response that has 5% overshoot.

ANSWER:

$$a = 5.52$$

The complete solution is at www.wiley.com/college/nise.

TryIt 5.2

Use the following MATLAB and Control System Toolbox statements to find ζ, ω_n, $\%OS$, T_s, T_p, and T_r for the closed-loop unity feedback system described in Skill-Assessment Exercise 5.2. Start with $a = 2$ and try some other values. A step response for the closed-loop system will also be produced.

```
a=2;
numg=16;
deng=poly([0 -a]);
G=tf(numg,deng);
T=feedback(G,1);
[numt,dent]=...
 tfdata(T,'v');
wn=sqrt(dent(3))
z=dent(2)/(2*wn)
Ts=4/(z*wn)
Tp=pi/(wn*...
 sqrt(1-z^2))
pos=exp(-z*pi...
 /sqrt(1-z^2))*100
Tr=(1.76*z^3-...
 0.417*z^2+1.039*...
 z+1)/wn
step(T)
```

5.4 Signal-Flow Graphs

Signal-flow graphs are an alternative to block diagrams. Unlike block diagrams, which consist of blocks, signals, summing junctions, and pickoff points, a signal-flow graph consists only of *branches*, which represent systems, and *nodes*, which represent signals. These elements are shown in Figure 5.17(a) and (b), respectively. A system is represented by a line with an arrow showing the direction of signal flow through the

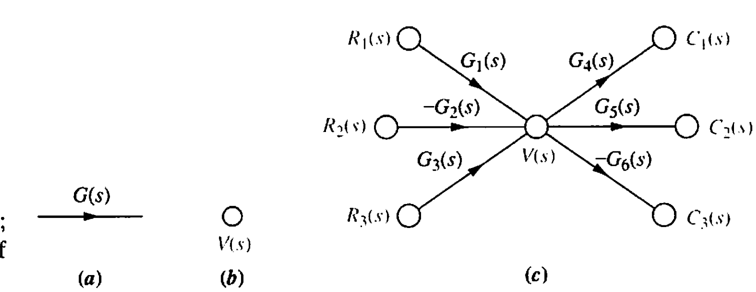


FIGURE 5.17 Signal-flow graph components: **a.** system; **b.** signal; **c.** interconnection of systems and signals

system. Adjacent to the line we write the transfer function. A signal is a node with the signal's name written adjacent to the node.

Figure 5.17(c) shows the interconnection of the systems and the signals. Each signal is the sum of signals flowing into it. For example, the signal $V(s) = R_1(s)G_1(s) - R_2(s)G_2(s) + R_3(s)G_3(s)$. The signal $C_2(s) = V(s)G_5(s) = R_1(s)G_1(s)G_5(s) - R_2(s)G_2(s)G_5(s) + R_3(s)G_3(s)G_5(s)$. The signal $C_3(s) = -V(s)G_6(s) = -R_1(s)G_1(s)G_6(s) + R_2(s)G_2(s)G_6(s) - R_3(s)G_3(s)G_6(s)$. Notice that in summing negative signals we associate the negative sign with the system and not with a summing junction, as in the case of block diagrams.

To show the parallel between block diagrams and signal-flow graphs, we will take some of the block diagram forms from Section 5.2 and convert them to signal-flow graphs in Example 5.5. In each case, we will first convert the signals to nodes and then interconnect the nodes with system branches. In Example 5.6, we will convert an intricate block diagram to a signal-flow graph.

Example 5.5

Converting Common Block Diagrams to Signal-Flow Graphs

PROBLEM: Convert the cascaded, parallel, and feedback forms of the block diagrams shown in Figures 5.3(a), 5.5(a), and 5.6(b), respectively, into signal-flow graphs.

SOLUTION: In each case, we start by drawing the signal nodes for that system. Next we interconnect the signal nodes with system branches. The signal nodes for the cascaded, parallel, and feedback forms are shown in Figure 5.18(a), (c), and (e), respectively. The interconnection of the nodes with branches that represent the subsystems is shown in Figure 5.18(b), (d), and (f) for the cascaded, parallel, and feedback forms, respectively.

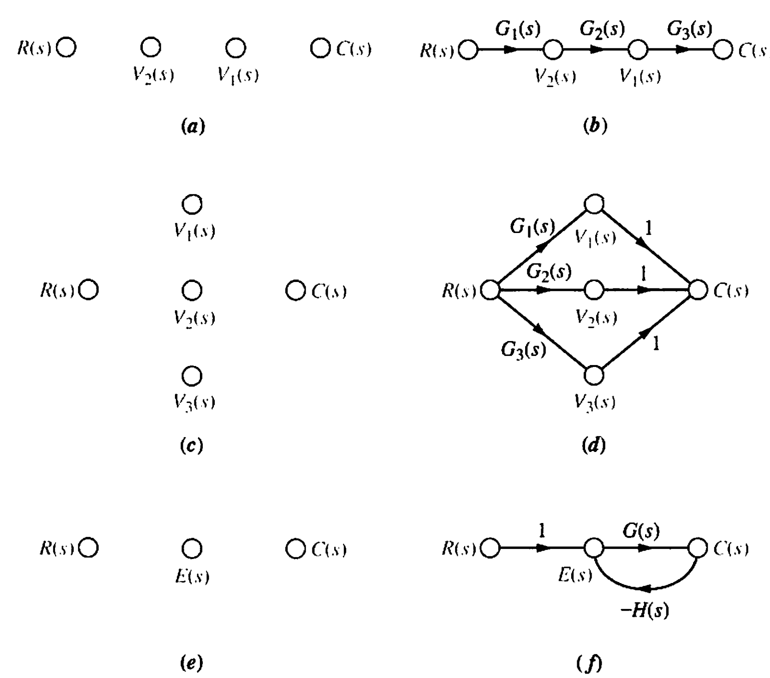


FIGURE 5.18 Building signal-flow graphs: **a.** cascaded system nodes (from Figure 5.3(a)); **b.** cascaded system signal-flow graph; **c.** parallel system nodes (from Figure 5.5(a)); **d.** parallel system signal-flow graph; **e.** feedback system nodes (from Figure 5.6(b)); **f.** feedback system signal-flow graph

Example 5.6

Converting a Block Diagram to a Signal-Flow Graph

PROBLEM: Convert the block diagram of Figure 5.11 to a signal-flow graph.

SOLUTION: Begin by drawing the signal nodes, as shown in Figure 5.19(*a*). Next, interconnect the nodes, showing the direction of signal flow and identifying each transfer function. The result is shown in Figure 5.19(*b*). Notice that the negative signs at the summing junctions of the block diagram are represented by the negative transfer functions of the signal-flow graph. Finally, if desired, simplify the signal-flow graph to the one shown in Figure 5.19(*c*) by eliminating signals that have a single flow in and a single flow out, such as $V_2(s)$, $V_6(s)$, $V_7(s)$, and $V_8(s)$.

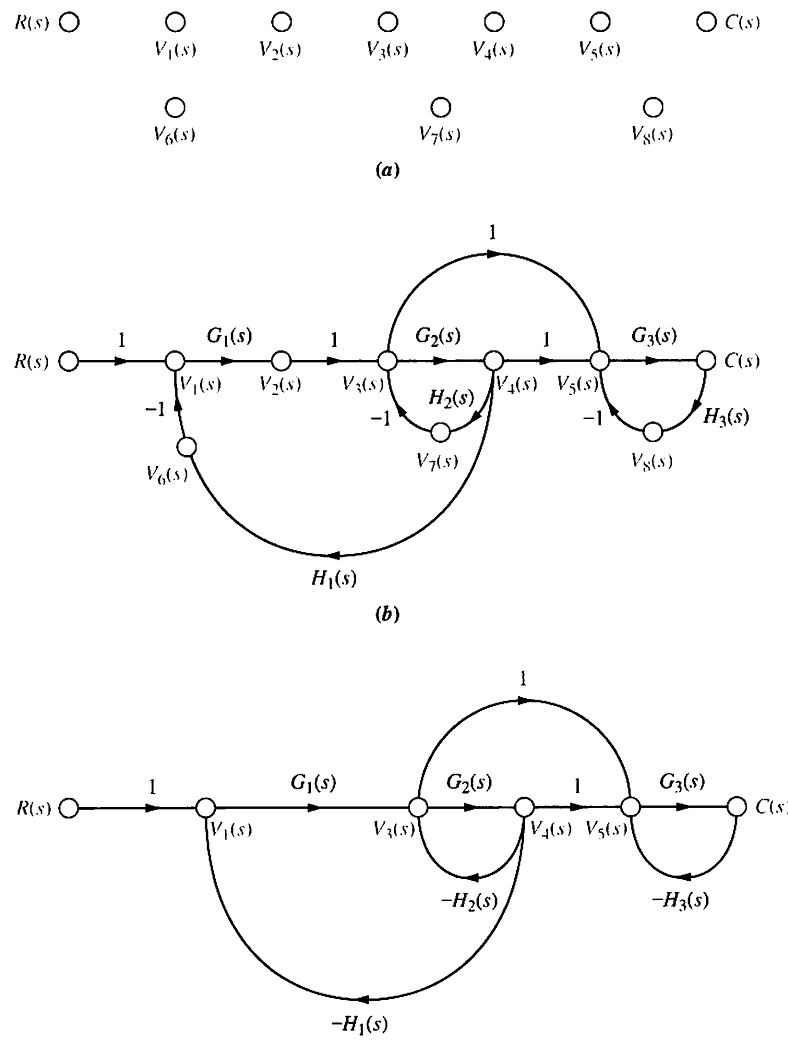


FIGURE 5.19 Signal-flow graph development: **a.** signal nodes; **b.** signal-flow graph; **c.** simplified signal-flow graph

Skill-Assessment Exercise 5.3

PROBLEM: Convert the block diagram of Figure 5.13 to a signal-flow graph.

ANSWER: The complete solution is at www.wiley.com/college/nise.

5.5 Mason's Rule

Earlier in this chapter, we discussed how to reduce block diagrams to single transfer functions. Now we are ready to discuss a technique for reducing signal-flow graphs to single transfer functions that relate the output of a system to its input.

The block diagram reduction technique we studied in Section 5.2 requires successive application of fundamental relationships in order to arrive at the system transfer function. On the other hand, Mason's rule for reducing a signal-flow graph to a single transfer function requires the application of one formula. The formula was derived by S. J. Mason when he related the signal-flow graph to the simultaneous equations that can be written from the graph (*Mason, 1953*).

In general, it can be complicated to implement the formula without making mistakes. Specifically, the existence of what we will later call nontouching loops increases the complexity of the formula. However, many systems do not have non-touching loops. For these systems, you may find Mason's rule easier to use than block diagram reduction.

Mason's formula has several components that must be evaluated. First, we must be sure that the definitions of the components are well understood. Then we must exert care in evaluating the components. To that end, we discuss some basic definitions applicable to signal-flow graphs; then we state Mason's rule and do an example.

Definitions

Loop gain. The product of branch gains found by traversing a path that starts at a node and ends at the same node, following the direction of the signal flow, without passing through any other node more than once. For examples of loop gains, see Figure 5.20. There are four loop gains:

1. $G_2(s)H_1(s)$ (5.25a)
2. $G_4(s)H_2(s)$ (5.25b)
3. $G_4(s)G_5(s)H_3(s)$ (5.25c)
4. $G_4(s)G_6(s)H_3(s)$ (5.25d)

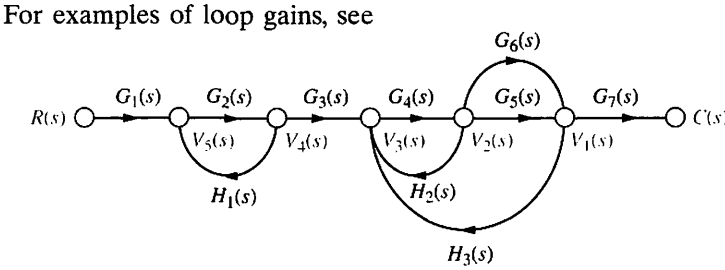


FIGURE 5.20 Signal-flow graph for demonstrating Mason's rule

Forward-path gain. The product of gains found by traversing a path from the input node to the output node of the signal-flow graph in the direction of signal flow. Examples of forward-path gains are also shown in Figure 5.20. There are two forward-path gains:

1. $G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)G_7(s)$ (5.26a)
2. $G_1(s)G_2(s)G_3(s)G_4(s)G_6(s)G_7(s)$ (5.26b)

Nontouching loops. Loops that do not have any nodes in common. In Figure 5.20, loop $G_2(s)H_1(s)$ does not touch loops $G_4(s)H_2(s)$, $G_4(s)G_5(s)H_3(s)$, and $G_4(s)G_6(s)H_3(s)$.

Nontouching-loop gain. The product of loop gains from nontouching loops taken two, three, four, or more at a time. In Figure 5.20 the product of loop gain $G_2(s)H_1(s)$ and loop gain $G_4(s)H_2(s)$ is a nontouching-loop gain taken two at a time. In summary, all three of the nontouching-loop gains taken two at a time are

1. $[G_2(s)H_1(s)][G_4(s)H_2(s)]$ (5.27a)
2. $[G_2(s)H_1(s)][G_4(s)G_5(s)H_3(s)]$ (5.27b)
3. $[G_2(s)H_1(s)][G_4(s)G_6(s)H_3(s)]$ (5.27c)

The product of loop gains $[G_4(s)G_5(s)H_3(s)][G_4(s)G_6(s)H_3(s)]$ is not a nontouching-loop gain since these two loops have nodes in common. In our example there are no nontouching-loop gains taken three at a time since three nontouching loops do not exist in the example.

We are now ready to state Mason's rule.

Mason's Rule

The transfer function, $C(s)/R(s)$, of a system represented by a signal-flow graph is

$$G(s) = \frac{C(s)}{R(s)} = \frac{\sum_k T_k \Delta_k}{\Delta} \tag{5.28}$$

where

k = number of forward paths
T_k = the kth forward-path gain
$\Delta = 1 - \Sigma$ loop gains $+ \Sigma$ nontouching-loop gains taken two at a time $- \Sigma$ nontouching-loop gains taken three at a time $+ \Sigma$ nontouching-loop gains taken four at a time $- \ldots$
$\Delta_k = \Delta - \Sigma$ loop gain terms in Δ that touch the kth forward path. In other words, Δ_k is formed by eliminating from Δ those loop gains that touch the kth forward path.

Notice the alternating signs for the components of Δ. The following example will help clarify Mason's rule.

Example 5.7

Transfer Function via Mason's Rule

PROBLEM: Find the transfer function, $C(s)/R(s)$, for the signal-flow graph in Figure 5.21.

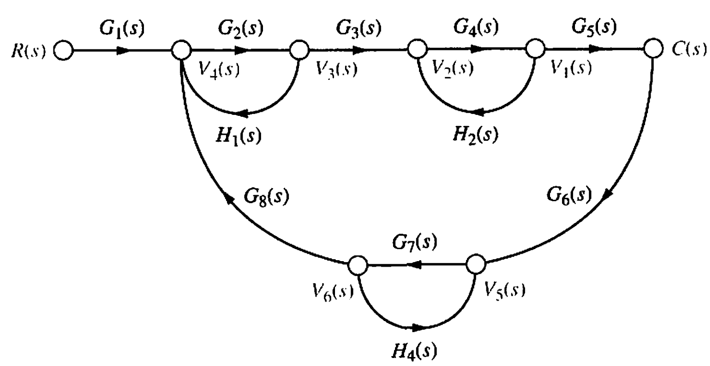


FIGURE 5.21 Signal-flow graph for Example 5.7

SOLUTION: First, identify the *forward-path gains.* In this example there is only one:

$$G_1(s)G_2(s)G_3(s)G_4(s)G_5(s) \tag{5.29}$$

Second, identify the *loop gains.* There are four, as follows:

1. $G_2(s)H_1(s)$ (5.30a)
2. $G_4(s)H_2(s)$ (5.30b)
3. $G_7(s)H_4(s)$ (5.30c)
4. $G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)$ (5.30d)

Third, identify the *nontouching loops taken two at a time.* From Eqs. (5.30) and Figure 5.21, we can see that loop 1 does not touch loop 2, loop 1 does not touch loop 3, and loop 2 does not touch loop 3. Notice that loops 1, 2, and 3 all touch loop 4. Thus, the combinations of nontouching loops taken two at a time are as follows:

$$\text{Loop 1 and loop 2}: \; G_2(s)H_1(s)G_4(s)H_2(s) \tag{5.31a}$$

$$\text{Loop 1 and loop 3}: \; G_2(s)H_1(s)G_7(s)H_4(s) \tag{5.31b}$$

$$\text{Loop 2 and loop 3}: \; G_4(s)H_2(s)G_7(s)H_4(s) \tag{5.31c}$$

Finally, the *nontouching loops taken three at a time* are as follows:

$$\text{Loops 1, 2, and 3}: \; G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s) \tag{5.32}$$

Now, from Eq. (5.28) and its definitions, we form Δ and Δ_k. Hence,

$$\begin{aligned}\Delta = 1 \; &-[G_2(s)H_1(s) + G_4(s)H_2(s) + G_7(s)H_4(s) \\ &\qquad + G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)] \\ &+[G_2(s)H_1(s)G_4(s)H_2(s) + G_2(s)H_1(s)G_7(s)H_4(s) \\ &\qquad + G_4(s)H_2(s)G_7(s)H_4(s)] \\ &-[G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)]\end{aligned} \tag{5.33}$$

We form Δ_k by eliminating from Δ the loop gains that touch the kth forward path:

$$\Delta_1 = 1 - G_7(s)H_4(s) \tag{5.34}$$

Expressions (5.29), (5.33), and (5.34) are now substituted into Eq. (5.28), yielding the transfer function:

$$G(s) = \frac{T_1\Delta_1}{\Delta} = \frac{[G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)][1 - G_7(s)H_4(s)]}{\Delta} \tag{5.35}$$

Since there is only one forward path, $G(s)$ consists of only one term, rather than a sum of terms, each coming from a forward path.

Skill-Assessment Exercise 5.4

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Control Solutions

PROBLEM: Use Mason's rule to find the transfer function of the signal-flow diagram shown in Figure 5.19(*c*). Notice that this is the same system used in Example 5.2 to find the transfer function via block diagram reduction.

ANSWER:

$$T(s) = \frac{G_1(s)G_3(s)[1 + G_2(s)]}{[1 + G_2(s)H_2(s) + G_1(s)G_2(s)H_1(s)][1 + G_3(s)H_3(s)]}$$

The complete solution is at www.wiley.com/college/nise.

5.6 Signal-Flow Graphs of State Equations

State Space
SS

In this section, we draw signal-flow graphs from state equations. At first this process will help us visualize state variables. Later we will draw signal-flow graphs and then write alternate representations of a system in state space.

Consider the following state and output equations:

$$\dot{x}_1 = 2x_1 - 5x_2 + 3x_3 + 2r \quad (5.36a)$$

$$\dot{x}_2 = -6x_1 - 2x_2 + 2x_3 + 5r \quad (5.36b)$$

$$\dot{x}_3 = x_1 - 3x_2 - 4x_3 + 7r \quad (5.36c)$$

$$y = -4x_1 + 6x_2 + 9x_3 \quad (5.36d)$$

First, identify three nodes to be the three state variables, x_1, x_2, and x_3; also identify three nodes, placed to the left of each respective state variable, to be the derivatives of the state variables, as in Figure 5.22(*a*). Also identify a node as the input, r, and another node as the output, y.

Next interconnect the state variables and their derivatives with the defining integration, $1/s$, as shown in Figure 5.22(*b*). Then using Eqs. (5.36), feed to each node the indicated signals. For example, from Eq. (5.36*a*), $\dot{x}_1$ receives $2x_1 - 5x_2 + 3x_3 + 2r$, as shown in Figure 5.22(*c*). Similarly, $\dot{x}_2$ receives $-6x_1 - 2x_2 + 2x_3 + 5r$, as shown in Figure 5.22(*d*), and $\dot{x}_3$ receives $x_1 - 3x_2 - 4x_3 + 7r$, as shown in Figure 5.22(*e*). Finally, using Eq. (5.36*d*), the output, y, receives $-4x_1 + 6x_2 + 9x_3$, as shown in Figure 5.19(*f*), the final phase-variable representation, where the state variables are the outputs of the integrators.

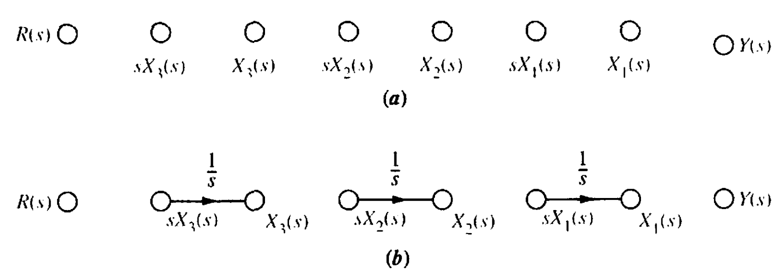


FIGURE 5.22 Stages of development of a signal-flow graph for the system of Eqs. (5.36): **a.** Place nodes; **b.** interconnect state variables and derivatives; (*figure continues*)

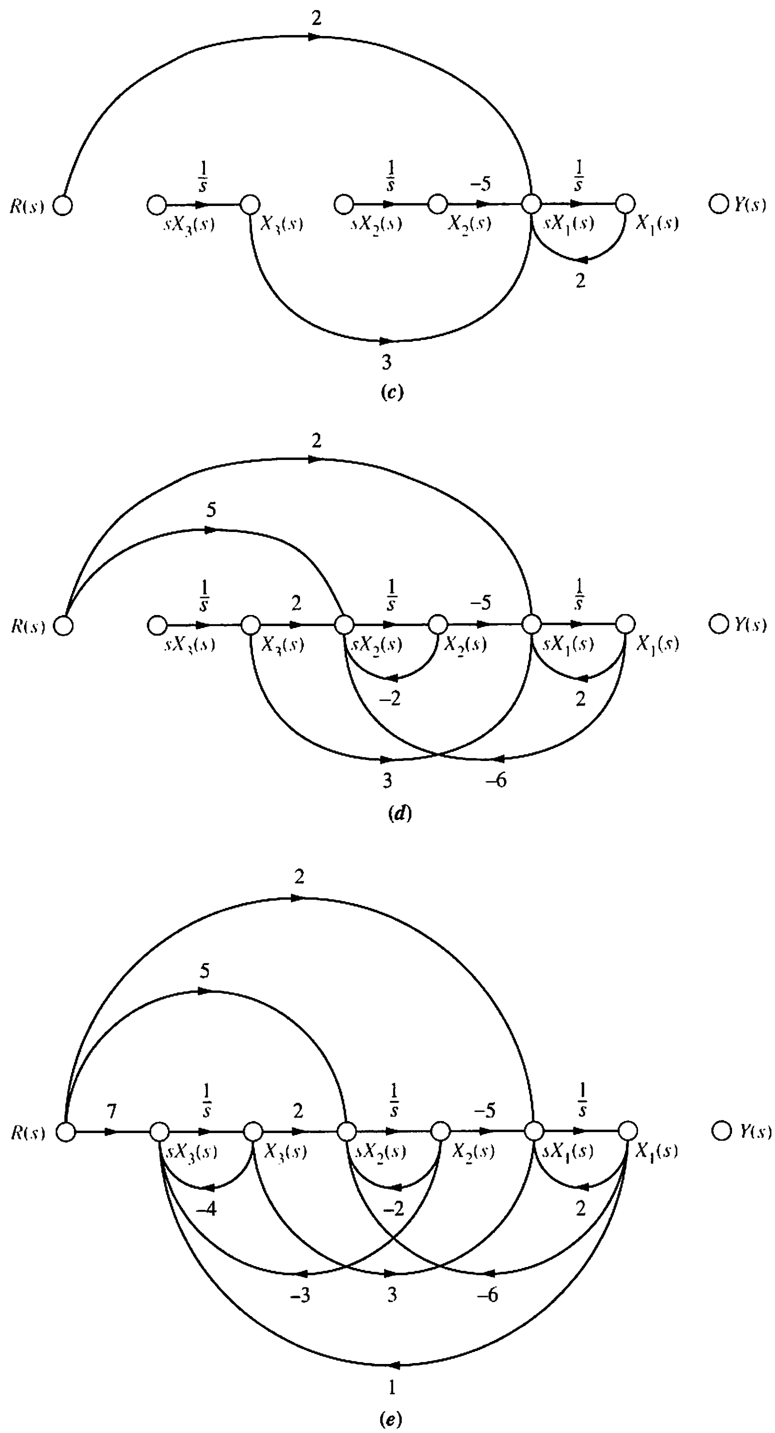
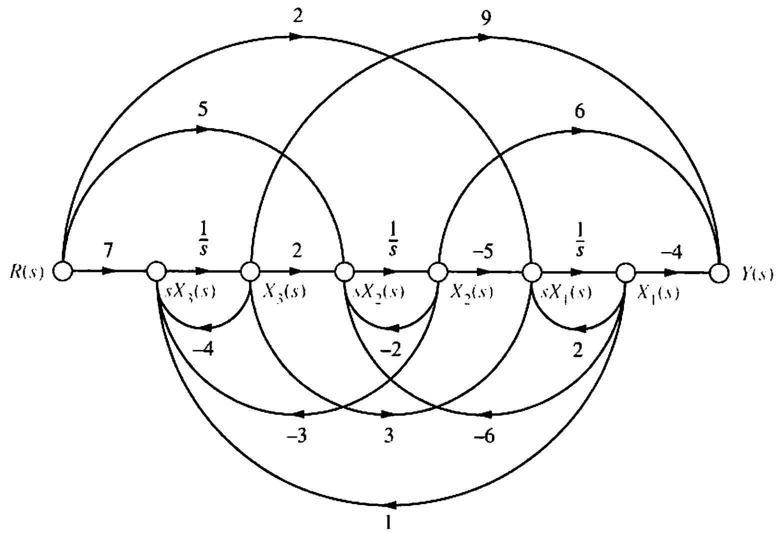


FIGURE 5.22 (*Continued*) **c.** form dx_1/dt; **d.** form dx_2/dt; **e.** form dx_3/dt; (*figure continues*)



(*f*)

FIGURE 5.22 (*Continued*) **f.** form output (*figure end*)

Skill-Assessment Exercise 5.5

PROBLEM: Draw a signal-flow graph for the following state and output equations:

$$\dot{\mathbf{x}} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -3 & 1 \\ -3 & -4 & -5 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

$$y = [0 \quad 1 \quad 0]\mathbf{x}$$

ANSWER: The complete solution is at www.wiley.com/college/nise.

In the next section, the signal-flow model will help us visualize the process of determining alternative representations in state space of the same system. We will see that even though a system can be the same with respect to its input and output terminals, the state-space representations can be many and varied.

5.7 Alternative Representations in State Space

State Space
SS

In Chapter 3, systems were represented in state space in phase-variable form. However, system modeling in state space can take on many representations other than the phase-variable form. Although each of these models yields the same output for a given input, an engineer may prefer a particular one for several reasons. For example, one set of state variables, with its unique representation, can model actual physical variables of a system, such as amplifier and filter outputs.

Another motive for choosing a particular set of state variables and state-space model is ease of solution. As we will see, a particular choice of state variables can decouple the system of simultaneous differential equations. Here each equation is written in terms of only one state variable, and the solution is effected by solving n first-order differential equations individually.

Ease of modeling is another reason for a particular choice of state variables. Certain choices may facilitate converting the subsystem to the state-variable representation by using recognizable features of the model. The engineer learns quickly how to write the state and output equations and draw the signal-flow graph, both by inspection. These converted subsystems generate the definition of the state variables.

We will now look at a few representative forms and show how to generate the state-space representation for each.

Cascade Form

We have seen that systems can be represented in state space with the state variables chosen to be the phase variables, that is, variables that are successive derivatives of each other. This is by no means the only choice. Returning to the system of Figure 3.10(a), the transfer function can be represented alternately as

$$\frac{C(s)}{R(s)} = \frac{24}{(s+2)(s+3)(s+4)} \tag{5.37}$$

Figure 5.23 shows a block diagram representation of this system formed by cascading each term of Eq. (5.37). The output of each first-order system block has been labeled as a state variable. These state variables are not the phase variables.

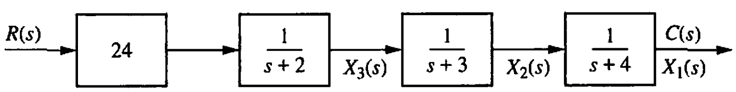


FIGURE 5.23 Representation of Figure 3.10 system as cascaded first-order systems

We now show how the signal-flow graph can be used to obtain a state-space representation of this system. In order to write the state equations with our new set of state variables, it is helpful to draw a signal-flow graph first, using Figure 5.23 as a guide. The signal flow for each first-order system of Figure 5.23 can be found by transforming each block into an equivalent differential equation. Each first-order block is of the form

$$\frac{C_i(s)}{R_i(s)} = \frac{1}{(s+a_i)} \tag{5.38}$$

Cross-multiplying, we get

$$(s+a_i)C_i(s) = R_i(s) \tag{5.39}$$

After taking the inverse Laplace transform, we have

$$\frac{dc_i(t)}{dt} + a_i c_i(t) = r_i(t) \tag{5.40}$$

Solving for $dc_i(t)/dt$ yields

$$\frac{dc_i(t)}{dt} = -a_i c_i(t) + r_i(t) \tag{5.41}$$

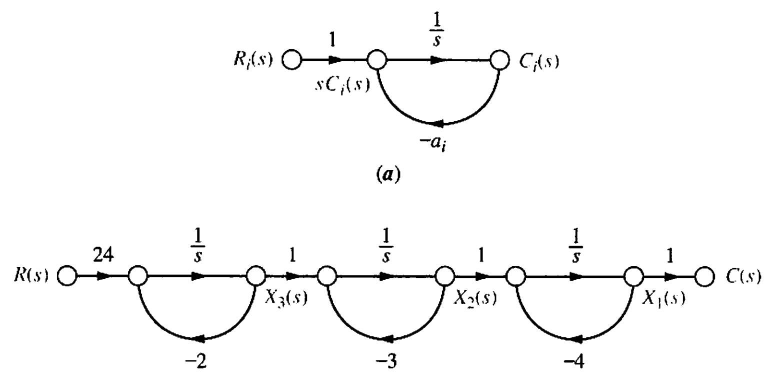


FIGURE 5.24 **a.** First-order subsystem; **b.** Signal-flow graph for Figure 5.23 system

Figure 5.24(*a*) shows the implementation of Eq. (5.41) as a signal-flow graph. Here again, a node was assumed for $c_i(t)$ at the output of an integrator, and its derivative was formed at the input.

Cascading the transfer functions shown in Figure 5.24(*a*), we arrive at the system representation shown in Figure 5.24(*b*).[2] Now write the state equations for the new representation of the system. Remember that the derivative of a state variable will be at the input to each integrator:

$$\dot{x}_1 = -4x_1 + x_2 \tag{5.42a}$$

$$\dot{x}_2 = -3x_2 + x_3 \tag{5.42b}$$

$$\dot{x}_3 = -2x_3 + 24r \tag{5.42c}$$

The output equation is written by inspection from Figure 5.24(*b*):

$$y = c(t) = x_1 \tag{5.43}$$

The state-space representation is completed by rewriting Eqs. (5.42) and (5.43) in vector-matrix form:

$$\dot{\mathbf{x}} = \begin{bmatrix} -4 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix} r \tag{5.44a}$$

$$y = [1 \quad 0 \quad 0]\mathbf{x} \tag{5.44b}$$

Comparing Eqs. (5.44) with Figure 5.24(*b*), you can form a vivid picture of the meaning of some of the components of the state equation. For the following discussion, please refer back to the general form of the state and output equations, Eqs. (3.18) and (3.19).

For example, the **B** matrix is the input matrix since it contains the terms that couple the input, $r(t)$, to the system. In particular, the constant 24 appears in both the signal-flow graph at the input, as shown in Figure 5.24(*b*), and the input matrix in Eqs. (5.44). The **C** matrix is the output matrix since it contains the constant that couples the state variable, x_1, to the output, $c(t)$. Finally, the **A** matrix is the system

[2] Note that node $X_3(s)$ and the following node cannot be merged, or else the input to the first integrator would be changed by the feedback from $X_2(s)$, and the signal $X_3(s)$ would be lost. A similar argument can be made for $X_2(s)$ and the following node.

matrix since it contains the terms relative to the internal system itself. In the form of Eqs. (5.44), the system matrix actually contains the system poles along the diagonal.

Compare Eqs. (5.44) to the phase-variable representation in Eqs. (3.59). In that representation, the coefficients of the system's characteristic polynomial appeared along the last row, whereas in our current representation, the roots of the characteristic equation, the system poles, appear along the diagonal.

Parallel Form

Another form that can be used to represent a system is the parallel form. This form leads to an **A** matrix that is purely diagonal, provided that no system pole is a repeated root of the characteristic equation.

Whereas the previous form was arrived at by cascading the individual first-order subsystems, the parallel form is derived from a partial-fraction expansion of the system transfer function. Performing a partial-fraction expansion on our example system, we get

$$\frac{C(s)}{R(s)} = \frac{24}{(s+2)(s+3)(s+4)} = \frac{12}{(s+2)} - \frac{24}{(s+3)} + \frac{12}{(s+4)} \quad (5.45)$$

Equation (5.45) represents the sum of the individual first-order subsystems. To arrive at a signal-flow graph, first solve for $C(s)$,

$$C(s) = R(s)\frac{12}{(s+2)} - R(s)\frac{24}{(s+3)} + R(s)\frac{12}{(s+4)} \quad (5.46)$$

and recognize that $C(s)$ is the sum of three terms. Each term is a first-order subsystem with $R(s)$ as the input. Formulating this idea as a signal-flow graph renders the representation shown in Figure 5.25.

Once again, we use the signal-flow graph as an aid to obtaining the state equations. By inspection the state variables are the outputs of each integrator, where the derivatives of the state variables exist at the integrator inputs. We write the state equations by summing the signals at the integrator inputs:

$$\begin{aligned} \dot{x}_1 &= -2x_1 \qquad\qquad\qquad + 12r \quad &(5.47a)\\ \dot{x}_2 &= \qquad -3x_2 \qquad - 24r \quad &(5.47b)\\ \dot{x}_3 &= \qquad\qquad -4x_3 + 12r \quad &(5.47c) \end{aligned}$$

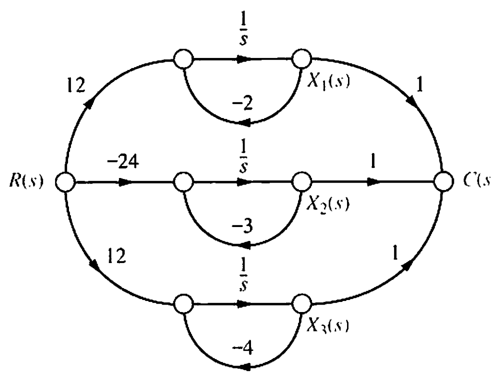


FIGURE 5.25 Signal-flow representation of Eq. (5.45)

The output equation is found by summing the signals that give $c(t)$:

$$y = c(t) = x_1 + x_2 + x_3 \quad (5.48)$$

In vector-matrix form, Eqs. (5.47) and (5.48) become

$$\dot{\mathbf{x}} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -4 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 12 \\ -24 \\ 12 \end{bmatrix} r \quad (5.49)$$

and

$$y = [1 \quad 1 \quad 1]\mathbf{x} \quad (5.50)$$

Thus, our third representation of the system of Figure 3.10(a) yields a diagonal system matrix. What is the advantage of this representation? Each equation is a first-order differential equation in only one variable. Thus, we would solve these equations independently. The equations are said to be *decoupled.*

MATLAB ML

`Students who are using MATLAB should now run ch5p3 in Appendix B. You will learn how to use MATLAB to convert a transfer function to state space in a specified form. The exercise solves the previous example by representing the transfer function in Eq. (5.45) by the state-space representation in parallel form of Eq. (5.49).`

If the denominator of the transfer function has repeated real roots, the parallel form can still be derived from a partial-fraction expansion. However, the system matrix will not be diagonal. For example, assume the system

$$\frac{C(s)}{R(s)} = \frac{(s+3)}{(s+1)^2(s+2)} \tag{5.51}$$

which can be expanded as partial fractions:

$$\frac{C(s)}{R(s)} = \frac{2}{(s+1)^2} - \frac{1}{(s+1)} + \frac{1}{(s+2)} \tag{5.52}$$

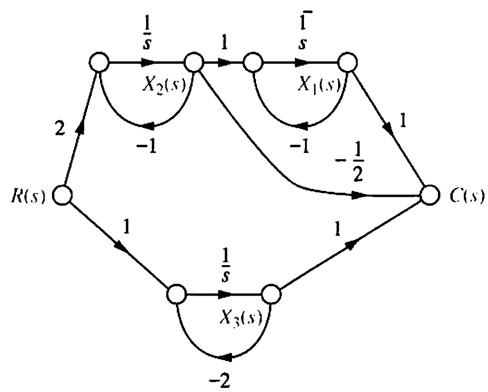


FIGURE 5.26 Signal-flow representation of Eq. (5.52)

Proceeding as before, the signal-flow graph for Eq. (5.52) is shown in Figure 5.26. The term $-1/(s+1)$ was formed by creating the signal flow from $X_2(s)$ to $C(s)$. Now the state and output equations can be written by inspection from Figure 5.26 as follows:

$$\dot{x}_1 = -x_1 + x_2 \tag{5.53a}$$

$$\dot{x}_2 = -x_2 + 2r \tag{5.53b}$$

$$\dot{x}_3 = -2x_3 + r \tag{5.53c}$$

$$y = c(t) = x_1 - \frac{1}{2}x_2 + x_3 \tag{5.53d}$$

or, in vector-matrix form,

$$\dot{\mathbf{x}} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} r \tag{5.54a}$$

$$y = \begin{bmatrix} 1 & -\frac{1}{2} & 1 \end{bmatrix}\mathbf{x} \tag{5.54b}$$

This system matrix, although not diagonal, has the system poles along the diagonal. Notice the 1 off the diagonal for the case of the repeated root. The form of the system matrix is known as the *Jordan canonical form.*

Controller Canonical Form

Another representation that uses phase variables is called the *controller canonical form*, so named for its use in the design of controllers, which is covered in Chapter 12. This form is obtained from the phase-variable form simply by ordering the phase variables in the reverse order. For example, consider the transfer function

$$G(s) = \frac{C(s)}{R(s)} = \frac{s^2 + 7s + 2}{s^3 + 9s^2 + 26s + 24} \tag{5.55}$$

The phase-variable form was derived in Example 3.5 as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -24 & -26 & -9\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}r \tag{5.56a}$$

$$y=\begin{bmatrix}2 & 7 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} \tag{5.56b}$$

where $y = c(t)$. Renumbering the phase variables in reverse order yields

$$\begin{bmatrix}\dot{x}_3\\ \dot{x}_2\\ \dot{x}_1\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -24 & -26 & -9\end{bmatrix}\begin{bmatrix}x_3\\ x_2\\ x_1\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}r \tag{5.57a}$$

$$y=\begin{bmatrix}2 & 7 & 1\end{bmatrix}\begin{bmatrix}x_3\\ x_2\\ x_1\end{bmatrix} \tag{5.57b}$$

Finally, rearranging Eqs. (5.57) in ascending numerical order yields the controller canonical form[3] as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}-9 & -26 & -24\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}1\\ 0\\ 0\end{bmatrix}r \tag{5.58a}$$

$$y=\begin{bmatrix}1 & 7 & 2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} \tag{5.58b}$$

TryIt 5.3

Use the following MATLAB and Control System Toolbox statements to convert the transfer function of Eq. (5.55) to the controller canonical state-space representation of Eqs. (5.58).

```
numg=[1 7 2];
deng=[1 9 26 24];
[Acc,Bcc,Ccc,Dcc]...
 =tf2ss(numg,deng)
```

Figure 5.27 shows the steps we have taken on a signal-flow graph. Notice that the controller canonical form is obtained simply by renumbering the phase variables in the opposite order. Equations (5.56) can be obtained from Figure 5.27(*a*), and Eqs. (5.58) from Figure 5.27(*b*).

Notice that the phase-variable form and the controller canonical form contain the coefficients of the characteristic polynomial in the bottom row and in the top row,

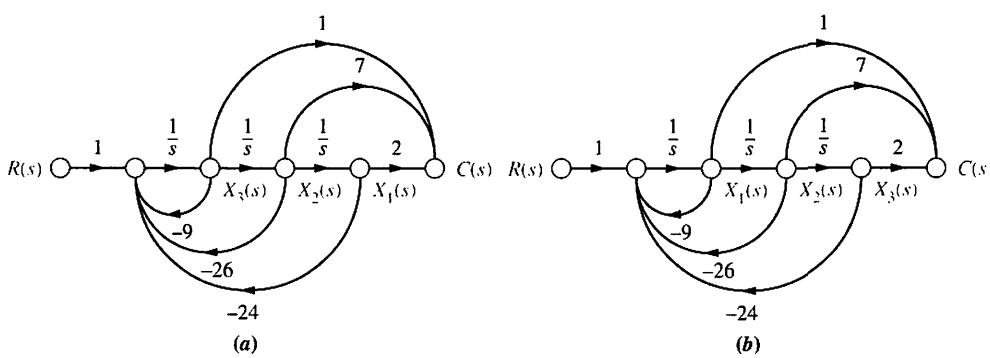


FIGURE 5.27 Signal-flow graphs for obtaining forms for $G(s) = C(s)/R(s) = (s^2 + 7s + 2)/(s^3 + 9s^2 + 26s + 24)$: **a.** phase-variable form; **b.** controller canonical form

[3] Students who are using MATLAB to convert from transfer functions to state space using the command **tf2ss** will notice that MATLAB reports the results in controller canonical form.

respectively. System matrices that contain the coefficients of the characteristic polynomial are called *companion matrices* to the characteristic polynomial. The phase-variable and controller canonical forms result in a lower and an upper companion system matrix, respectively. Companion matrices can also have the coefficients of the characteristic polynomial in the left or right column. In the next subsection, we discuss one of these representations.

Observer Canonical Form

The *observer canonical form*, so named for its use in the design of observers (covered in Chapter 12), is a representation that yields a left companion system matrix. As an example, the system modeled by Eq. (5.55) will be represented in this form. Begin by dividing all terms in the numerator and denominator by the highest power of s, s^3, and obtain

$$\frac{C(s)}{R(s)} = \frac{\frac{1}{s}+\frac{7}{s^2}+\frac{2}{s^3}}{1+\frac{9}{s}+\frac{26}{s^2}+\frac{24}{s^3}} \tag{5.59}$$

Cross-multiplying yields

$$\left[\frac{1}{s}+\frac{7}{s^2}+\frac{2}{s^3}\right]R(s) = \left[1+\frac{9}{s}+\frac{26}{s^2}+\frac{24}{s^3}\right]C(s) \tag{5.60}$$

Combining terms of like powers of integration gives

$$C(s) = \frac{1}{s}[R(s) - 9C(s)] + \frac{1}{s^2}[7R(s) - 26C(s)] + \frac{1}{s^3}[2R(s) - 24C(s)] \tag{5.61}$$

or

$$C(s) = \frac{1}{s}\left[[R(s) - 9C(s)] + \frac{1}{s}\left([7R(s) - 26C(s)] + \frac{1}{s}[2R(s) - 24C(s)]\right)\right] \tag{5.62}$$

Equation (5.61) or (5.62) can be used to draw the signal-flow graph. Start with three integrations, as shown in Figure 5.28(a).

Using Eq. (5.61), the first term tells us that output $C(s)$ is formed, in part, by integrating $[R(s) - 9C(s)]$. We thus form $[R(s) - 9C(s)]$ at the input to the integrator closest to the output, $C(s)$, as shown in Figure 5.28(b). The second term tells us that the

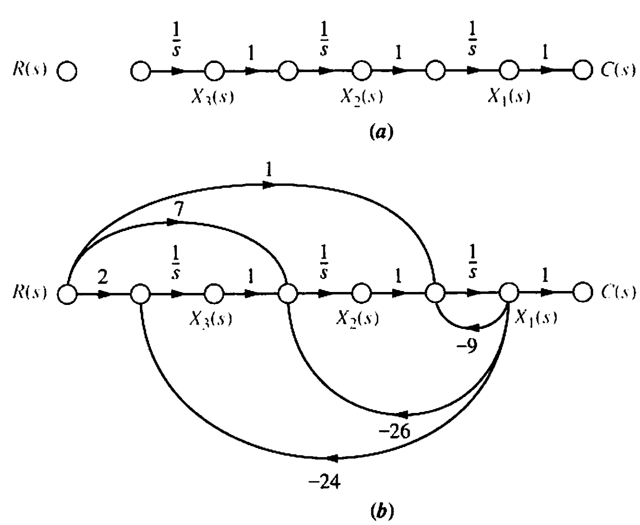


FIGURE 5.28 Signal-flow graph for observer canonical form variables: **a.** planning; **b.** implementation

term $[7R(s) - 26C(s)]$ must be integrated twice. Now form $[7R(s) - 26C(s)]$ at the input to the second integrator. Finally, the last term of Eq. (5.61) says $[2R(s) - 24C(s)]$ must be integrated three times. Form $[2R(s) - 24C(s)]$ at the input to the first integrator.

Identifying the state variables as the outputs of the integrators, we write the following state equations:

$$\dot{x}_1 = -9x_1 + x_2 + r \quad (5.63a)$$

$$\dot{x}_2 = -26x_1 + x_3 + 7r \quad (5.63b)$$

$$\dot{x}_3 = -24x_1 + 2r \quad (5.63c)$$

The output equation from Figure 5.28(b) is

$$y = c(t) = x_1 \quad (5.64)$$

In vector-matrix form, Eqs. (5.63) and (5.64) become

$$\dot{\mathbf{x}} = \begin{bmatrix} -9 & 1 & 0 \\ -26 & 0 & 1 \\ -24 & 0 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix} r \quad (5.65a)$$

$$y = [1 \quad 0 \quad 0]\mathbf{x} \quad (5.65b)$$

Notice that the form of Eqs. (5.65) is similar to the phase-variable form, except that the coefficients of the denominator of the transfer function are in the first column, and the coefficients of the numerator form the input matrix, **B**. Also notice that the observer canonical form has an **A** matrix that is the transpose of the controller canonical form, a **B** vector that is the transpose of the controller canonical form's **C** vector, and a **C** vector that is the transpose of the controller canonical form's **B** vector. We therefore say that these two forms are *duals*. Thus, if a system is described by **A, B,** and **C**, its dual is described by $\mathbf{A_D} = \mathbf{A}^T$, $\mathbf{B_D} = \mathbf{C}^T$, $\mathbf{C_D} = \mathbf{B}^T$. You can verify the significance of duality by comparing the signal-flow graphs of a system and its dual, Figures 5.27(b) and 5.28(b), respectively. The signal-flow graph of the dual can be obtained from that of the original by reversing all arrows, changing state variables to their derivatives and vice versa, and interchanging $C(s)$ and $R(s)$, thus reversing the roles of the input and the output.

We conclude this section with an example that demonstrates the application of the previously discussed forms to a feedback control system.

TryIt 5.4

Use the following MATLAB and Control System Toolbox statements to convert the transfer function of Eq. (5.55) to the observer canonical state-space representation of Eqs. (5.65).

```
numg=[1 7 2];
deng=[1 9 26 24];
[Acc,Bcc,Ccc,Dcc]...
 =tf2ss(numg,deng);
Aoc=transpose(Acc)
Boc=transpose(Ccc)
Coc=transpose(Bcc)
```

Example 5.8

State-Space Representation of Feedback Systems

PROBLEM: Represent the feedback control system shown in Figure 5.29 in state space. Model the forward transfer function in cascade form.

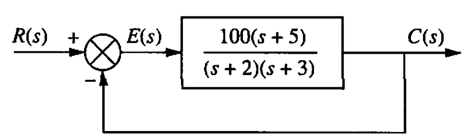


FIGURE 5.29 Feedback control system for Example 5.8

SOLUTION: First we model the forward transfer function in cascade form. The gain of 100, the pole at −2, and the pole at −3 are shown cascaded in Figure 5.30(a). The zero at −5 was obtained using the method for implementing zeros for a system represented in phase-variable form, as discussed in Section 3.5.

Next add the feedback and input paths, as shown in Figure 5.30(b). Now, by inspection, write the state equations:

$$\dot{x}_1 = -3x_1 + x_2 \quad (5.66a)$$

$$\dot{x}_2 = -2x_2 + 100(r - c) \quad (5.66b)$$

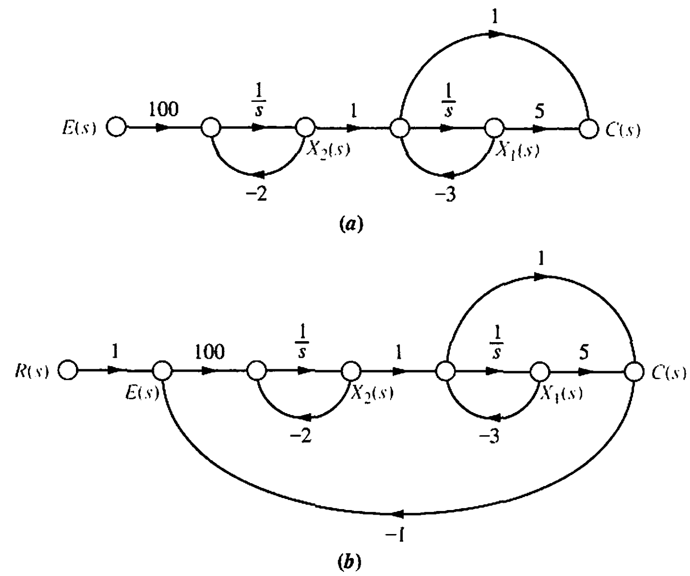


FIGURE 5.30 Creating a signal-flow graph for the Figure 5.29 system: **a.** forward transfer function; **b.** complete system

But, from Figure 5.30*(b)*,

$$c = 5x_1 + (x_2 - 3x_1) = 2x_1 + x_2 \tag{5.67}$$

Substituting Eq. (5.67) into (5.66b), we find the state equations for the system:

$$\dot{x}_1 = -3x_1 + x_2 \tag{5.68a}$$

$$\dot{x}_2 = -200x_1 - 102x_2 + 100r \tag{5.68b}$$

The output equation is the same as Eq. (5.67), or

$$y = c(t) = 2x_1 + x_2 \tag{5.69}$$

In vector-matrix form

$$\dot{\mathbf{x}} = \begin{bmatrix} -3 & 1 \\ -200 & -102 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 100 \end{bmatrix} r \tag{5.70a}$$

$$y = [2 \quad 1]\mathbf{x} \tag{5.70b}$$

Skill-Assessment Exercise 5.6

WileyPLUS
WPCS
Control Solutions

PROBLEM: Represent the feedback control system shown in Figure 5.29 in state space. Model the forward transfer function in controller canonical form.

ANSWER:

$$\dot{\mathbf{x}} = \begin{bmatrix} -105 & -506 \\ 1 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r$$

$$y = [100 \quad 500]\mathbf{x}$$

The complete solution is at www.wiley.com/college/nise.

In this section, we used transfer functions and signal-flow graphs to represent systems in parallel, cascade, controller canonical, and observer canonical forms, in addition to the phase-variable form. Using the transfer function $C(s)/R(s) = (s+3)/[(s+4)(s+6)]$ as an example, Figure 5.31 compares the aforementioned forms. Notice the duality of the controller and observer canonical forms, as demonstrated by their respective signal-flow graphs and state equations. In the next section, we will explore the possibility of transforming between representations without using transfer functions and signal-flow graphs.

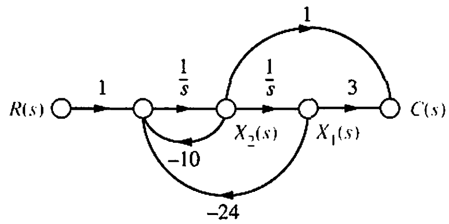
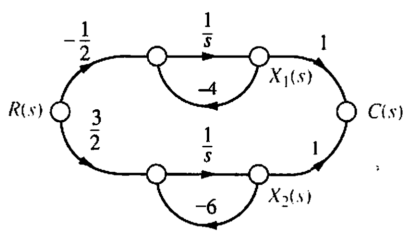
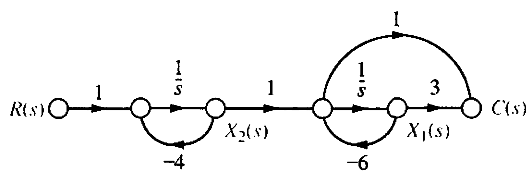
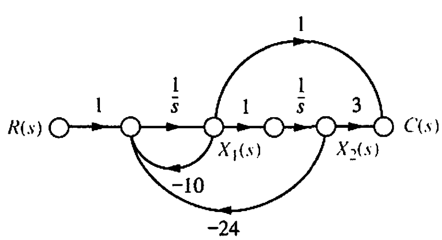
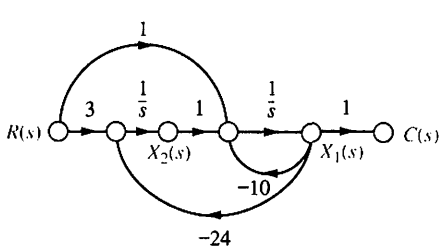
Form	Transfer function	Signal-flow diagram	State equations
Phase variable	$\frac{1}{(s^2+10s+24)} * (s+3)$		$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -24 & -10 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$ $y = [3 \;\; 1]\mathbf{x}$
Parallel	$\frac{-1/2}{(s+4)} + \frac{3/2}{s+6}$		$\dot{\mathbf{x}} = \begin{bmatrix} -4 & 0 \\ 0 & -6 \end{bmatrix}\mathbf{x} + \begin{bmatrix} -\frac{1}{2} \\ \frac{3}{2} \end{bmatrix} r$ $y = [1 \;\; 1]\mathbf{x}$
Cascade	$\frac{1}{(s+4)} * \frac{(s+3)}{(s+6)}$		$\dot{\mathbf{x}} = \begin{bmatrix} -6 & 1 \\ 0 & -4 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$ $y = [-3 \;\; 1]\mathbf{x}$
Controller canonical	$\frac{1}{(s^2+10s+24)} * (s+3)$		$\dot{\mathbf{x}} = \begin{bmatrix} -10 & -24 \\ 1 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r$ $y = [1 \;\; 3]\mathbf{x}$
Observer canonical	$\frac{\frac{1}{s} + \frac{3}{s^2}}{1 + \frac{10}{s} + \frac{24}{s^2}}$		$\dot{\mathbf{x}} = \begin{bmatrix} -10 & 1 \\ -24 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 3 \end{bmatrix} r$ $y = [1 \;\; 0]\mathbf{x}$

FIGURE 5.31 State-space forms for $C(s)/R(s) = (s+3)/[(s+4)(s+6)]$. Note: $y = c(t)$

5.8 Similarity Transformations

State Space
SS

In Section 5.7, we saw that systems can be represented with different state variables even though the transfer function relating the output to the input remains the same. The various forms of the state equations were found by manipulating the transfer function, drawing a signal-flow graph, and then writing the state equations from the signal-flow graph. These systems are called *similar systems*. Although their state-space representations are different, similar systems have the same transfer function and hence the same poles and eigenvalues.

We can make transformations between similar systems from one set of state equations to another without using the transfer function and signal-flow graphs. The results are presented in this section along with examples. Students who have not broached this subject in the past or who wish to refresh their memories are encouraged to study Appendix L at www.wiley.com/college/nise for the derivation. The result of the derivation states: A system represented in state space as

$$\boxed{\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}} \tag{5.71a}$$

$$\boxed{\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}} \tag{5.71b}$$

can be transformed to a similar system,

$$\boxed{\mathbf{z} = \mathbf{P}^{-1}\mathbf{A}\mathbf{P}\mathbf{z} + \mathbf{P}^{-1}\mathbf{B}\mathbf{u}} \tag{5.72a}$$

$$\boxed{\mathbf{y} = \mathbf{C}\mathbf{P}\mathbf{z} + \mathbf{D}\mathbf{u}} \tag{5.72b}$$

where, for 2-space,

$$\boxed{\mathbf{P} = [\mathbf{U}_{z_1}\mathbf{U}_{z_2}] = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}} \tag{5.72c}$$

$$\boxed{\mathbf{x} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \mathbf{P}\mathbf{z}} \tag{5.72d}$$

and

$$\boxed{\mathbf{z} = \mathbf{P}^{-1}\mathbf{x}} \tag{5.72e}$$

Thus, **P** is a transformation matrix whose columns are the coordinates of the basis vectors of the z_1z_2 space expressed as linear combinations of the x_1x_2 space. Let us look at an example.

Example 5.9

Similarity Transformations on State Equations

PROBLEM: Given the system represented in state space by Eqs. (5.73),

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -5 & -7 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}u \tag{5.73a}$$

$$y = [1 \quad 0 \quad 0]\mathbf{x} \tag{5.73b}$$

transform the system to a new set of state variables, **z**, where the new state variables are related to the original state variables, **x**, as follows:

$$z_1 = 2x_1 \tag{5.74a}$$

$$z_2 = 3x_1 + 2x_2 \tag{5.74b}$$

$$z_3 = x_1 + 4x_2 + 5x_3 \tag{5.74c}$$

SOLUTION: Expressing Eqs. (5.74) in vector-matrix form,

$$\mathbf{z} = \begin{bmatrix} 2 & 0 & 0 \\ 3 & 2 & 0 \\ 1 & 4 & 5 \end{bmatrix}\mathbf{x} = \mathbf{P}^{-1}\mathbf{x} \tag{5.75}$$

Using Eqs. (5.72) as a guide,

$$\begin{aligned} \mathbf{P}^{-1}\mathbf{AP} &= \begin{bmatrix} 2 & 0 & 0 \\ 3 & 2 & 0 \\ 1 & 4 & 5 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -5 & -7 \end{bmatrix}\begin{bmatrix} 0.5 & 0 & 0 \\ -0.75 & 0.5 & 0 \\ 0.5 & -0.4 & 0.2 \end{bmatrix} \\ &= \begin{bmatrix} -1.5 & 1 & 0 \\ -1.25 & 0.7 & 0.4 \\ -2.5 & 0.4 & -6.2 \end{bmatrix} \end{aligned} \tag{5.76}$$

$$\mathbf{P}^{-1}\mathbf{B} = \begin{bmatrix} 2 & 0 & 0 \\ 3 & 2 & 0 \\ 1 & 4 & 5 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} \tag{5.77}$$

$$\mathbf{CP} = [1 \quad 0 \quad 0]\begin{bmatrix} 0.5 & 0 & 0 \\ -0.75 & 0.5 & 0 \\ 0.5 & -0.4 & 0.2 \end{bmatrix} = [0.5 \quad 0 \quad 0] \tag{5.78}$$

Therefore, the transformed system is

$$\dot{\mathbf{z}} = \begin{bmatrix} -1.5 & 1 & 0 \\ -1.25 & 0.7 & 0.4 \\ -2.55 & 0.4 & -6.2 \end{bmatrix}\mathbf{z} + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}u \tag{5.79a}$$

$$y = [0.5 \quad 0 \quad 0]\mathbf{z} \tag{5.79b}$$

MATLAB ML

`Students who are using MATLAB should now run ch5p4 in Appendix B. You will learn how to perform similarity transformations. This exercise uses MATLAB to do Example 5.9.`

Thus far we have talked about transforming systems between basis vectors in a different state space. One major advantage of finding these similar systems is apparent in the transformation to a system that has a diagonal matrix.

Diagonalizing a System Matrix

In Section 5.7, we saw that the parallel form of a signal-flow graph can yield a diagonal system matrix. A diagonal system matrix has the advantage that each state equation is a function of only one state variable. Hence, each differential equation can be solved independently of the other equations. We say that the equations are *decoupled.*

Rather than using partial fraction expansion and signal-flow graphs, we can decouple a system using matrix transformations. If we find the correct matrix, **P**, the transformed system matrix, $\mathbf{P}^{-1}\mathbf{AP}$, will be a diagonal matrix. Thus, we are looking for a transformation to another state space that yields a diagonal matrix in that space. This new state space also has basis vectors that lie along its state variables. We give a special name to any vectors that are collinear with the basis vectors of the new system that yields a diagonal system matrix: they are called *eigenvectors*. Thus, the coordinates of the eigenvectors form the columns of the transformation matrix, **P**, as we demonstrate in Eq. L.7 in Appendix L at www.wiley.com/college/nise.

First, let us formally define eigenvectors from another perspective and then show that they have the property just described. Then we will define eigenvalues. Finally, we will show how to diagonalize a matrix.

Definitions

Eigenvector. The eigenvectors of the matrix **A** are all vectors, $\mathbf{x_i} \neq \mathbf{0}$, which under the transformation **A** become multiples of themselves; that is,

$$\boxed{\mathbf{Ax_i} = \lambda_i \mathbf{x_i}} \tag{5.80}$$

where λ_i's are constants.

Figure 5.32 shows this definition of eigenvectors. If **Ax** is not collinear with **x** after the transformation, as in Figure 5.32(*a*), **x** is not an eigenvector. If **Ax** is collinear with **x** after the transformation, as in Figure 5.32(*b*), **x** is an eigenvector.

Eigenvalue. The eigenvalues of the matrix **A** are the values of λ_i that satisfy Eq. (5.80) for $\mathbf{x_i} \neq \mathbf{0}$.

To find the eigenvectors, we rearrange Eq. (5.80). Eigenvectors, $\mathbf{x_i}$, satisfy

$$\mathbf{0} = (\lambda_i \mathbf{I} - \mathbf{A})\mathbf{x_i} \tag{5.81}$$

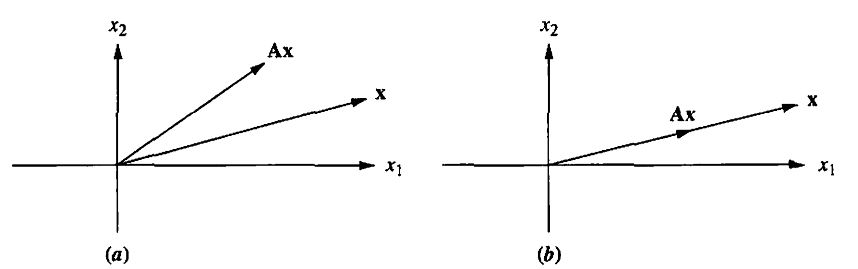


FIGURE 5.32 To be an eigenvector, the transformation **Ax** must be collinear with **x**; thus, in (*a*), **x** is not an eigenvector; in (*b*), it is

Solving for $\mathbf{x_i}$ by premultiplying both sides by $(\lambda_i\mathbf{I} - \mathbf{A})^{-1}$ yields

$$\mathbf{x_i} = (\lambda_i\mathbf{I} - \mathbf{A})^{-1}\mathbf{0} = \frac{\operatorname{adj}(\lambda_i\mathbf{I} - \mathbf{A})}{\det(\lambda_i\mathbf{I} - \mathbf{A})}\mathbf{0} \tag{5.82}$$

Since $\mathbf{x_i} \neq \mathbf{0}$, a nonzero solution exists if

$$\boxed{\det(\lambda_i\mathbf{I} - \mathbf{A}) = \mathbf{0}} \tag{5.83}$$

from which λ_i, the eigenvalues, can be found.

We are now ready to show how to find the eigenvectors, $\mathbf{x_i}$. First we find the eigenvalues, λ_i, using $\det(\lambda_i\mathbf{I} - \mathbf{A}) = \mathbf{0}$, and then we use Eq. (5.80) to find the eigenvectors.

Example 5.10

Finding Eigenvectors

PROBLEM: Find the eigenvectors of the matrix

$$\mathbf{A} = \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix} \tag{5.84}$$

SOLUTION: The eigenvectors, $\mathbf{x_i}$, satisfy Eq. (5.81). First, use $\det(\lambda_i\mathbf{I} - \mathbf{A}) = 0$ to find the eigenvalues, λ_i, for Eq. (5.81):

$$\begin{aligned} \det(\lambda\mathbf{I} - \mathbf{A}) &= \left| \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix} \right| \\ &= \begin{vmatrix} \lambda + 3 & -1 \\ -1 & \lambda + 3 \end{vmatrix} \\ &= \lambda^2 + 6\lambda + 8 \end{aligned} \tag{5.85}$$

from which the eigenvalues are $\lambda = -2$, and -4.

Using Eq. (5.80) successively with each eigenvalue, we have

$$\mathbf{Ax_i} = \lambda\mathbf{x_i}$$

$$\begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -2\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{5.86}$$

or

$$-3x_1 + x_2 = -2x_1 \tag{5.87a}$$
$$x_1 - 3x_2 = -2x_2 \tag{5.87b}$$

from which $x_1 = x_2$. Thus,

$$\mathbf{x} = \begin{bmatrix} c \\ c \end{bmatrix} \tag{5.88}$$

Using the other eigenvalue, -4, we have

$$\mathbf{x} = \begin{bmatrix} c \\ -c \end{bmatrix} \tag{5.89}$$

Using Eqs. (5.88) and (5.89), one choice of eigenvectors is

$$\mathbf{x_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ and } \mathbf{x_2} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \tag{5.90}$$

We now show that if the eigenvectors of the matrix **A** are chosen as the basis vectors of a transformation, **P**, the resulting system matrix will be diagonal. Let the transformation matrix **P** consist of the eigenvectors of **A**, $\mathbf{x_i}$.

$$\mathbf{P} = [\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \ldots, \mathbf{x_n}] \tag{5.91}$$

Since $\mathbf{x_i}$ are eigenvectors, $\mathbf{Ax_i} = \lambda_i \mathbf{x_i}$, which can be written equivalently as a set of equations expressed by

$$\mathbf{AP} = \mathbf{PD} \tag{5.92}$$

where **D** is a diagonal matrix consisting of λ_i's, the eigenvalues, along the diagonal, and **P** is as defined in Eq. (5.91). Solving Eq. (5.92) for **D** by premultiplying by $\mathbf{P}^{-1}$, we get

$$\mathbf{D} = \mathbf{P}^{-1}\mathbf{AP} \tag{5.93}$$

which is the system matrix of Eq. (5.72).

In summary, under the transformation **P**, consisting of the eigenvectors of the system matrix, the transformed system is diagonal, with the eigenvalues of the system along the diagonal. The transformed system is identical to that obtained using partial-fraction expansion of the transfer function with distinct real roots.

In Example 5.10, we found eigenvectors of a second-order system. Let us continue with this problem and diagonalize the system matrix.

Example 5.11

Diagonalizing a System in State Space

PROBLEM: Given the system of Eqs. (5.94), find the diagonal system that is similar.

$$\dot{\mathbf{x}} = \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}u \tag{5.94a}$$

$$y = [2 \quad 3]\mathbf{x} \tag{5.94b}$$

SOLUTION: First find the eigenvalues and the eigenvectors. This step was performed in Example 5.10. Next form the transformation matrix **P**, whose columns consist of the eigenvectors.

$$\mathbf{P} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \tag{5.95}$$

Finally, form the similar system's system matrix, input matrix, and output matrix, respectively.

$$\mathbf{P}^{-1}\mathbf{AP} = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{bmatrix}\begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -4 \end{bmatrix} \tag{5.96a}$$

$$\mathbf{P}^{-1}\mathbf{B} = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3/2 \\ -1/2 \end{bmatrix} \tag{5.96b}$$

$$\mathbf{CP} = [2 \quad 3]\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = [5 \quad -1] \tag{5.96c}$$

Substituting Eqs. (5.96) into Eqs. (5.72), we get

$$\dot{\mathbf{z}} = \begin{bmatrix} -2 & 0 \\ 0 & -4 \end{bmatrix}\mathbf{z} + \begin{bmatrix} 3/2 \\ -1/2 \end{bmatrix}u \quad (5.97a)$$

$$y = \begin{bmatrix} 5 & -1 \end{bmatrix}\mathbf{z} \quad (5.97b)$$

Notice that the system matrix is diagonal, with the eigenvalues along the diagonal.

MATLAB ML

Students who are using MATLAB should now run ch5p5 in Appendix B. This problem, which uses MATLAB to diagonalize a system, is similar (but not identical) to Example 5.11.

Skill-Assessment Exercise 5.7

WileyPLUS WPCS Control Solutions

PROBLEM: For the system represented in state space as follows:

$$\dot{\mathbf{x}} = \begin{bmatrix} 1 & 3 \\ -4 & -6 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 3 \end{bmatrix}u$$

$$y = \begin{bmatrix} 1 & 4 \end{bmatrix}\mathbf{x}$$

convert the system to one where the new state vector, $\mathbf{z}$, is

$$\mathbf{z} = \begin{bmatrix} 3 & -2 \\ 1 & -4 \end{bmatrix}\mathbf{x}$$

ANSWER:

$$\dot{\mathbf{z}} = \begin{bmatrix} 6.5 & -8.5 \\ 9.5 & -11.5 \end{bmatrix}\mathbf{z} + \begin{bmatrix} -3 \\ -11 \end{bmatrix}u$$

$$y = \begin{bmatrix} 0.8 & -1.4 \end{bmatrix}\mathbf{z}$$

The complete solution is at www.wiley.com/college/nise.

Skill-Assessment Exercise 5.8

PROBLEM: For the original system of Skill-Assessment Exercise 5.7, find the diagonal system that is similar.

ANSWER:

$$\dot{\mathbf{z}} = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix}\mathbf{z} + \begin{bmatrix} 18.39 \\ 20 \end{bmatrix}u$$

$$y = \begin{bmatrix} -2.121 & 2.6 \end{bmatrix}\mathbf{z}$$

The complete solution is at www.wiley.com/college/nise.

TryIt 5.5

Use the following MATLAB and Control System Toolbox statements to do Skill-Assessment Exercise 5.8.

```
A=[1 3;-4 -6];
B=[1;3];
C=[1 4];
D=0;S=ss(A,B,C,D);
Sd=canon(S,'modal')
```

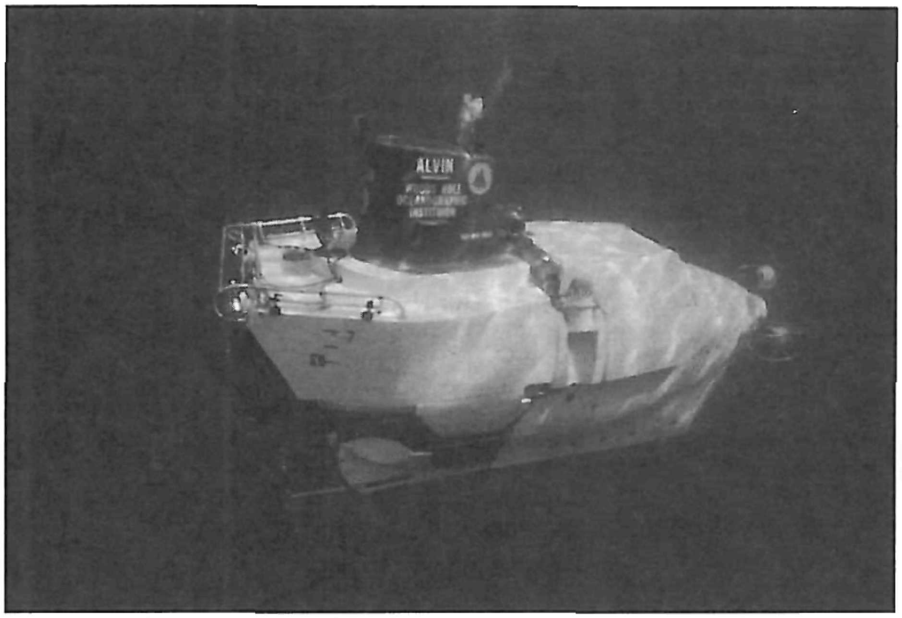


FIGURE 5.33 *Alvin*, a manned submersible, explored the wreckage of the *Titanic* with a tethered robot, *Jason Junior*.

In this section, we learned how to move between different state-space representations of the same system via matrix transformations rather than transfer function manipulation and signal-flow graphs. These different representations are called *similar*. The characteristics of similar systems are that the transfer functions relating the output to the input are the same, as are the eigenvalues and poles. A particularly useful transformation was converting a system with distinct, real eigenvalues to a diagonal system matrix.

We now summarize the concepts of block diagram and signal-flow representations of systems, first through case study problems and then in a written summary. Our case studies include the antenna azimuth position control system and the Unmanned Free-Swimming Submersible vehicle (UFSS). Block diagram reduction is important for the analysis and design of these systems as well as the control systems on board *Alvin* (Figure 5.33), used to explore the wreckage of the *Titanic* 13,000 feet under the Atlantic in July 1986 (*Ballard, 1987*).

Case Studies

Antenna Control: Designing a Closed-Loop Response

Design
D

This chapter has shown that physical subsystems can be modeled mathematically with transfer functions and then interconnected to form a feedback system. The interconnected mathematical models can be reduced to a single transfer function representing the system from input to output. This transfer function, the closed-loop transfer function, is then used to determine the system response.

The following case study shows how to reduce the subsystems of the antenna azimuth position control system to a single, closed-loop transfer function in order to analyze and design the transient response characteristics.

PROBLEM: Given the antenna azimuth position control system shown on the front endpapers, Configuration 1, do the following:

a. Find the closed-loop transfer function using block diagram reduction.

b. Represent each subsystem with a signal-flow graph and find the state-space representation of the closed-loop system from the signal-flow graph.

State Space
SS

c. Use the signal-flow graph found in **b** along with Mason's rule to find the closed-loop transfer function.

d. Replace the power amplifier with a transfer function of unity and evaluate the closed-loop peak time, percent overshoot, and settling time for $K = 1000$.

e. For the system of **d**, derive the expression for the closed-loop step response of the system.

f. For the simplified model of **d**, find the value of K that yields a 10% overshoot.

SOLUTION: Each subsystem's transfer function was evaluated in the case study in Chapter 2. We first assemble them into the closed-loop, feedback control system block diagram shown in Figure 5.34(a).

a. The steps taken to reduce the block diagram to a single, closed-loop transfer function relating the output angular displacement to the input angular displacement are shown in Figure 5.34(a–d). In Figure 5.34(b), the input potentiometer was pushed to the right past the summing junction, creating a unity feedback

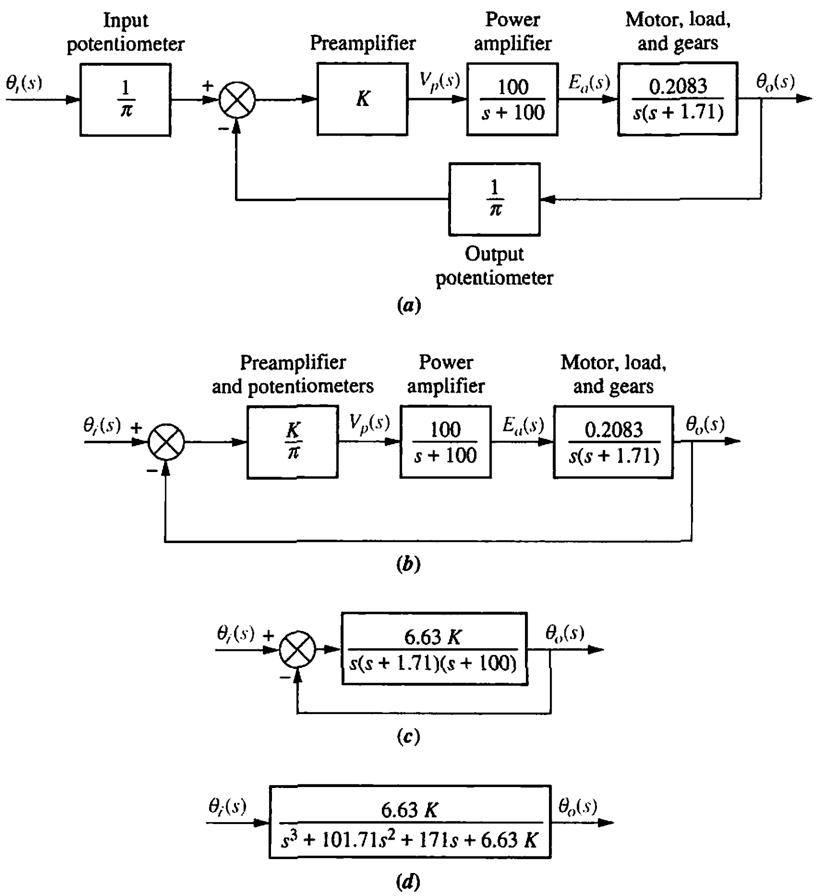


FIGURE 5.34 Block diagram reduction for the antenna azimuth position control system: **a.** original; **b.** pushing input potentiometer to the right past the summing junction; **c.** showing equivalent forward transfer function; **d.** final closed-loop transfer function

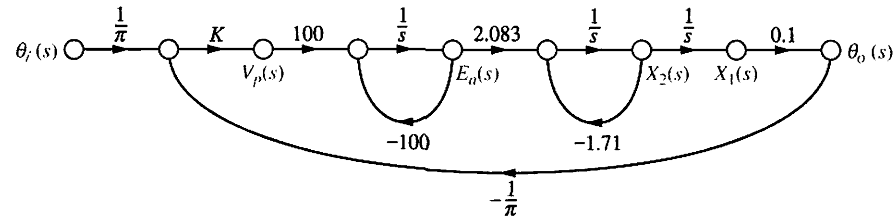


FIGURE 5.35 Signal-flow graph for the antenna azimuth position control system

system. In Figure 5.34(*c*), all the blocks of the forward transfer function are multiplied together, forming the equivalent forward transfer function. Finally, the feedback formula is applied, yielding the closed-loop transfer function in Figure 5.34(*d*).

State Space
SS

b. In order to obtain the signal-flow graph of each subsystem, we use the state equations derived in the case study of Chapter 3. The signal-flow graph for the power amplifier is drawn from the state equations of Eqs. (3.87) and (3.88), and the signal-flow graph of the motor and load is drawn from the state equation of Eq. (3.98). Other subsystems are pure gains. The signal-flow graph for Figure 5.34(*a*) is shown in Figure 5.35 and consists of the interconnected subsystems.

The state equations are written from Figure 5.35. First define the state variables as the outputs of the integrators. Hence, the state vector is

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ e_a \end{bmatrix} \tag{5.98}$$

Using Figure 5.35, we write the state equations by inspection:

$$\dot{x}_1 = +x_2 \tag{5.99a}$$

$$\dot{x}_2 = -1.71x_2 + 2.083e_a \tag{5.99b}$$

$$\dot{e}_a = -3.18Kx_1 - 100e_a + 31.8K\theta_i \tag{5.99c}$$

along with the output equation,

$$y = \theta_o = 0.1x_1 \tag{5.100}$$

where $1/\pi = 0.318$.

In vector-matrix form,

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -1.71 & 2.083 \\ -3.18K & 0 & -100 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 31.8K \end{bmatrix}\theta_i \tag{5.101a}$$

$$y = [0.1 \quad 0 \quad 0]\mathbf{x} \tag{5.101b}$$

c. We now apply Mason's rule to Figure 5.35 to derive the closed-loop transfer function of the antenna azimuth position control system. First find the forward-path gains. From Figure 5.35 there is only one forward-path gain:

$$T_1 = \left(\frac{1}{\pi}\right)(K)(100)\left(\frac{1}{s}\right)(2.083)\left(\frac{1}{s}\right)\left(\frac{1}{s}\right)(0.1) = \frac{6.63K}{s^3} \tag{5.102}$$

Next identify the closed-loop gains. There are three: the power amplifier loop, $G_{L1}(s)$, with e_a at the output; the motor loop, $G_{L2}(s)$, with x_2 at the output; and the entire system loop, $G_{L3}(s)$, with θ_0 at the output.

$$G_{L1}(s) = \frac{-100}{s} \tag{5.103a}$$

$$G_{L2}(s) = \frac{-1.71}{s} \tag{5.103b}$$

$$G_{L3}(s) = (K)(100)\left(\frac{1}{s}\right)(2.083)\left(\frac{1}{s}\right)\left(\frac{1}{s}\right)(0.1)\left(\frac{-1}{\pi}\right) = \frac{-6.63K}{s^3} \tag{5.103c}$$

Only $G_{L1}(s)$ and $G_{L2}(s)$ are nontouching loops. Thus, the nontouching-loop gain is

$$G_{L1}(s)G_{L2}(s) = \frac{171}{s^2} \tag{5.104}$$

Forming Δ and Δ_k in Eq. (5.28), we have

$$\begin{aligned}\Delta &= 1 - [G_{L1}(s) + G_{L2}(s) + G_{L3}(s)] + [G_{L1}(s)G_{L2}(s)] \\ &= 1 + \frac{100}{s} + \frac{1.71}{s} + \frac{6.63K}{s^3} + \frac{171}{s^2}\end{aligned} \tag{5.105}$$

and

$$\Delta_1 = 1 \tag{5.106}$$

Substituting Eqs. (5.102), (5.105), and (5.106) into Eq. (5.28), we obtain the closed-loop transfer function as

$$T(s) = \frac{C(s)}{R(s)} = \frac{T_1\Delta_1}{\Delta} = \frac{6.63K}{s^3 + 101.71s^2 + 171s + 6.63K} \tag{5.107}$$

d. Replacing the power amplifier with unity gain and letting the preamplifier gain, K, in Figure 5.34(b) equal 1,000 yield a forward transfer function, $G(s)$, of

$$G(s) = \frac{66.3}{s(s+1.71)} \tag{5.108}$$

Using the feedback formula to evaluate the closed-loop transfer function, we obtain

$$T(s) = \frac{66.3}{s^2 + 1.71s + 66.3} \tag{5.109}$$

From the denominator, $\omega_n = 8.14$, $\zeta = 0.105$. Using Eqs. (4.34), (4.38), and (4.42), the peak time = 0.388 second, the percent overshoot = 71.77%, and the settling time = 4.68 seconds.

e. The Laplace transform of the step response is found by first multiplying' Eq. (5.109) by $1/s$, a unit-step input, and expanding into partial fractions:

$$\begin{aligned}C(s) &= \frac{66.3}{s(s^2 + 1.71s + 66.3)} = \frac{1}{s} - \frac{s + 1.71}{s^2 + 1.71s + 66.3} \\ &= \frac{1}{s} - \frac{(s + 0.855) + 0.106(8.097)}{(s + 0.855)^2 + (8.097)^2}\end{aligned} \tag{5.110}$$

Taking the inverse Laplace transform, we find

$$c(t) = 1 - e^{-0.855r}(\cos 8.097t + 0.106 \sin 8.097t) \tag{5.111}$$

f. For the simplified model we have

$$G(s) = \frac{0.0663K}{s(s+1.71)} \tag{5.112}$$

from which the closed-loop transfer function is calculated to be

$$T(s) = \frac{0.0663K}{s^2 + 1.71s + 0.0663K} \tag{5.113}$$

From Eq. (4.39) a 10% overshoot yields $\zeta = 0.591$. Using the denominator of Eq. (5.113), $\omega_n = \sqrt{0.0663K}$ and $2\zeta\omega_n = 1.71$. Thus,

$$\zeta = \frac{1.71}{2\sqrt{0.0663K}} = 0.591 \tag{5.114}$$

from which $K = 31.6$.

CHALLENGE: You are now given a problem to test your knowledge of this chapter's objectives: Referring to the antenna azimuth position control system shown on the front endpapers, Configuration 2, do the following:

State Space
SS

a. Find the closed-loop transfer function using block diagram reduction.

b. Represent each subsystem with a signal-flow graph and find the state-space representation of the closed-loop system from the signal-flow graph.

c. Use the signal-flow graph found in (b) along with Mason's rule to find the closed-loop transfer function.

d. Replace the power amplifier with a transfer function of unity and evaluate the closed-loop percent overshoot, settling time, and peak time for $K = 5$.

e. For the system used for (d), derive the expression for the closed-loop step response.

f. For the simplified model in (d), find the value of preamplifier gain, $\boldsymbol{K}$, to yield 15% overshoot.

UFSS Vehicle: Pitch-Angle Control Representation

State Space
SS

We return to the Unmanned Free-Swimming Submersible (UFSS) vehicle introduced in the case studies in Chapter 4 (*Johnson, 1980*). We will represent in state space the pitch-angle control system that is used for depth control.

PROBLEM: Consider the block diagram of the pitch control loop of the UFSS vehicle shown on the back endpapers. The pitch angle, θ, is controlled by a commanded pitch angle, θ_e, which along with pitch-angle and pitch-rate feedback determines the elevator deflection, δ_e, which acts through the vehicle dynamics to determine the pitch angle. Let $K_1 = K_2 = 1$ and do the following:

a. Draw the signal-flow graph for each subsystem, making sure that pitch angle, pitch rate, and elevator deflection are represented as state variables. Then interconnect the subsystems.

b. Use the signal-flow graph obtained in **a** to represent the pitch control loop in state space.

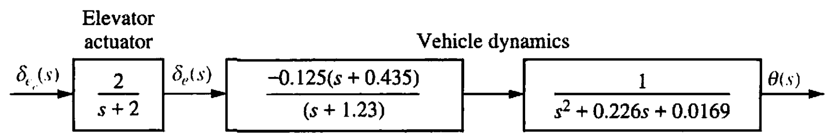


FIGURE 5.36 Block diagram of the UFSS vehicle's elevator and vehicle dynamics, from which a signal-flow graph can be drawn

SOLUTION:

a. The vehicle dynamics are split into two transfer functions, from which the signal-flow graph is drawn. Figure 5.36 shows the division along with the elevator actuator. Each block is drawn in phase-variable form to meet the requirement that particular system variables be state variables. This result is shown in Figure 5.37(*a*). The feedback paths are then added to complete the signal-flow graph, which is shown in Figure 5.37(*b*).

b. By inspection, the derivatives of state variables x_1 through x_4 are written as

$$\dot{x}_1 = x_2 \tag{5.115a}$$

$$\dot{x}_2 = -0.0169x_1 - 0.226x_2 + 0.435x_3 - 1.23x_3 - 0.125x_4 \tag{5.115b}$$

$$\dot{x}_3 = -1.23x_3 - 0.125x_4 \tag{5.115c}$$

$$\dot{x}_4 = 2x_1 + 2x_2 - 2x_4 - 2\theta c \tag{5.115d}$$

Finally, the output $y = x_1$.

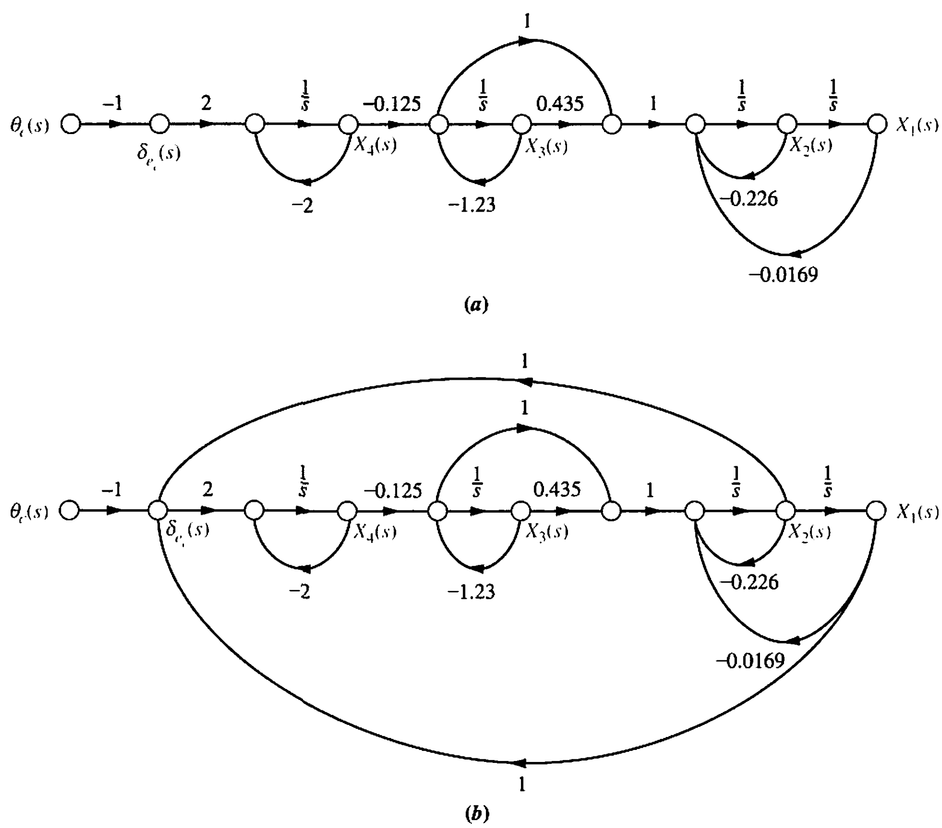


FIGURE 5.37 Signal-flow graph representation of the UFSS vehicle's pitch control system: **a.** without position and rate feedback; **b.** with position and rate feedback. (Note: Explicitly required variables are $x_1 = \theta, x_2 = d\theta/dt$, and $x_4 = \delta_e$.)

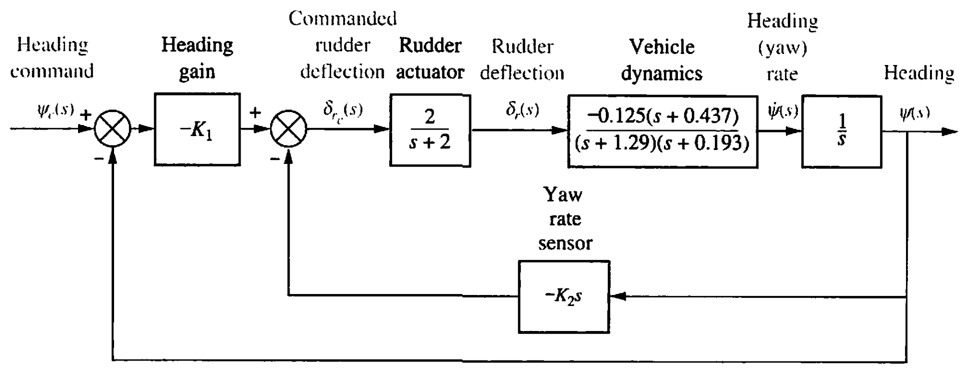


FIGURE 5.38 Block diagram of the heading control system for the UFSS vehicle

In vector-matrix form the state and output equations are

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -0.0169 & -0.226 & -0.795 & -0.125 \\ 0 & 0 & -1.23 & -0.125 \\ 2 & 2 & 0 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -2 \end{bmatrix} \theta_c \tag{5.116a}$$

$$y = [1 \quad 0 \quad 0 \quad 0]\mathbf{x} \tag{5.116b}$$

CHALLENGE: We now give you a problem to test your knowledge of this chapter's objectives. The UFSS vehicle steers via the heading control system shown in Figure 5.38 and repeated on the back endpapers. A heading command is the input. The input and feedback from the submersible's heading and yaw rate are used to generate a rudder command that steers the submersible (*Johnson, 1980*). Let $K_1 = K_2 = 1$ and do the following:

a. Draw the signal-flow graph for each subsystem, making sure that heading angle, yaw rate, and rudder deflection are represented as state variables. Then interconnect the subsystems.

b. Use the signal-flow graph obtained in **a** to represent the heading control loop in state space.

MATLAB ML

c. `Use MATLAB to represent the closed-loop UFSS heading control system in state space in controller canonical form.`

Summary

One objective of this chapter has been for you to learn how to represent multiple subsystems via block diagrams or signal-flow graphs. Another objective has been to be able to reduce either the block diagram representation or the signal-flow graph representation to a single transfer function.

We saw that the block diagram of a linear, time-invariant system consisted of four elements: *signals*, *systems*, *summing junctions*, and *pickoff points*. These elements were assembled into three basic forms: *cascade*, *parallel*, and *feedback*. Some basic operations were then derived: moving systems across summing junctions and across pickoff points.

Once we recognized the basic forms and operations, we could reduce a complicated block diagram to a single transfer function relating input to output. Then we applied the methods of Chapter 4 for analyzing and designing a second-order system for transient behavior. We saw that adjusting the gain of a feedback control system gave us partial control of the transient response.

The signal-flow representation of linear, time-invariant systems consists of two elements: nodes, which represent signals, and lines with arrows, which represent subsystems. Summing junctions and pickoff points are implicit in signal-flow graphs. These graphs are helpful in visualizing the meaning of the state variables. Also, they can be drawn first as an aid to obtaining the state equations for a system.

Mason's rule was used to derive the system's transfer function from the signal-flow graph. This formula replaced block diagram reduction techniques. Mason's rule seems complicated, but its use is simplified if there are no nontouching loops. In many of these cases, the transfer function can be written by inspection, with less labor than in the block diagram reduction technique.

Finally, we saw that systems in state space can be represented using different sets of variables. In the last three chapters, we have covered *phase-variable*, *cascade*, *parallel*, *controller canonical*, and *observer canonical* forms. A particular representation may be chosen because one set of state variables has a different physical meaning than another set, or because of the ease with which particular state equations can be solved.

In the next chapter, we discuss system stability. Without stability we cannot begin to design a system for the desired transient response. We will find out how to tell whether a system is stable and what effect parameter values have on a system's stability.

Review Questions

1. Name the four components of a block diagram for a linear, time-invariant system.
2. Name three basic forms for interconnecting subsystems.
3. For each of the forms in Question 2, state (respectively) how the equivalent transfer function is found.
4. Besides knowing the basic forms as discussed in Questions 2 and 3, what other equivalents must you know in order to perform block diagram reduction?
5. For a simple, second-order feedback control system of the type shown in Figure 5.14, describe the effect that variations of forward-path gain, K, have on the transient response.
6. For a simple, second-order feedback control system of the type shown in Figure 5.14, describe the changes in damping ratio as the gain, K, is increased over the underdamped region.
7. Name the two components of a signal-flow graph.
8. How are summing junctions shown on a signal-flow graph?
9. If a forward path touched all closed loops, what would be the value of Δ_k?
10. Name five representations of systems in state space.

State Space
SS

State Space SS 11. Which two forms of the state-space representation are found using the same method?

State Space SS 12. Which form of the state-space representation leads to a diagonal matrix?

State Space SS 13. When the system matrix is diagonal, what quantities lie along the diagonal?

State Space SS 14. What terms lie along the diagonal for a system represented in Jordan canonical form?

State Space SS 15. What is the advantage of having a system represented in a form that has a diagonal system matrix?

State Space SS 16. Give two reasons for wanting to represent a system by alternative forms.

State Space SS 17. For what kind of system would you use the observer canonical form?

State Space SS 18. Describe state-vector transformations from the perspective of different bases.

State Space SS 19. What is the definition of an eigenvector?

State Space SS 20. Based upon your definition of an eigenvector, what is an eigenvalue?

State Space SS 21. What is the significance of using eigenvectors as basis vectors for a system transformation?

Problems

1. Reduce the block diagram shown in Figure P5.1 to a single transfer function, $T(s) = C(s)/R(s)$ Use the following methods:
   a. Block diagram reduction [Section: 5.2]
   b. MATLAB (MATLAB ML)

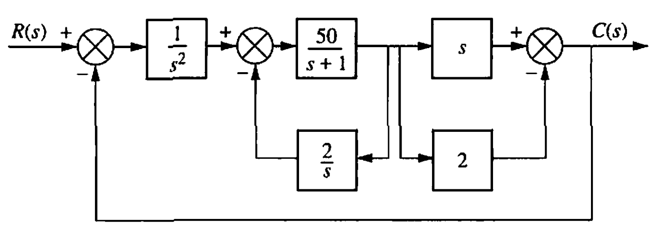


FIGURE P5.1

2. Find the closed-loop transfer function, $T(s) = C(s)/R(s)$ for the system shown in Figure P5.2, using block diagram reduction. [Section: 5.2]

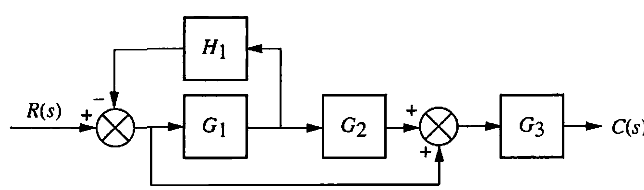


FIGURE P5.2

3. Find the equivalent transfer function, $T(s) = C(s)/R(s)$, for the system shown in Figure P5.3. [Section: 5.2] (WileyPLUS WPCS Control Solutions)

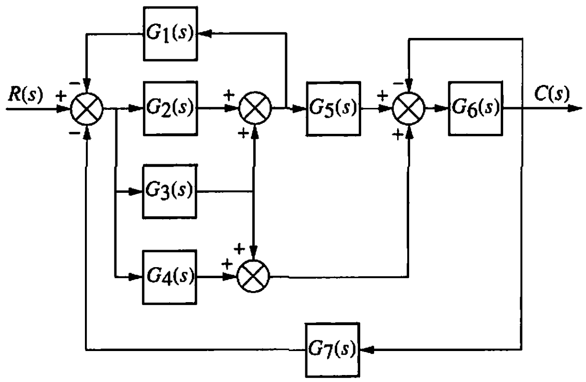


FIGURE P5.3

4. Reduce the system shown in Figure P5.4 to a single transfer function, $T(s) = C(s)/R(s)$. [Section: 5.2]

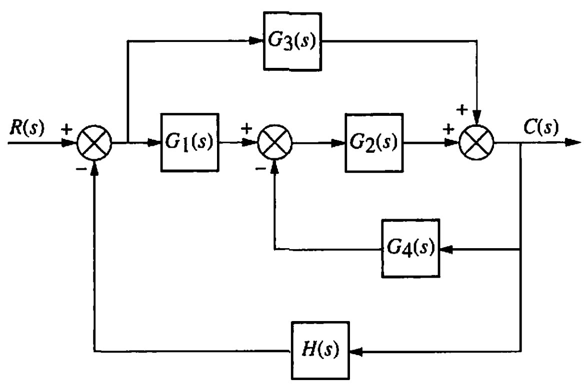


FIGURE P5.4

5. Find the transfer function, $T(s) = C(s)/R(s)$, for the system shown in Figure P5.5. Use the following methods:
   a. Block diagram reduction [Section: 5.2]
   b. MATLAB. Use the following transfer functions: MATLAB ML
   $G_1(s) = 1/(s+7)$, $G_2(s) = 1/(s^2+2s+3)$,
   $G_3(s) = 1/(s+4)$, $G_4(s) = 1/s$,
   $G_5(s) = 5/(s+7)$, $G_6(s) = 1/(s^2+5s+10)$,
   $G_7(s) = 3/(s+2)$, $G_8(s) = 1/(s+6)$.
   Hint: Use the **append** and **connect** commands in MATLAB's Control System Toolbox.

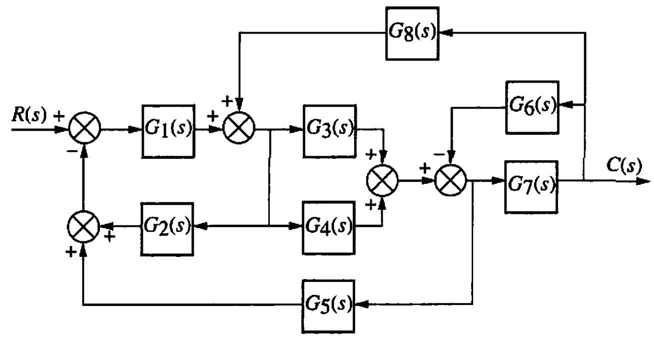


FIGURE P5.5

6. Reduce the block diagram shown in Figure P5.6 to a single block, $T(s) = C(s)/R(s)$. [Section: 5.2]

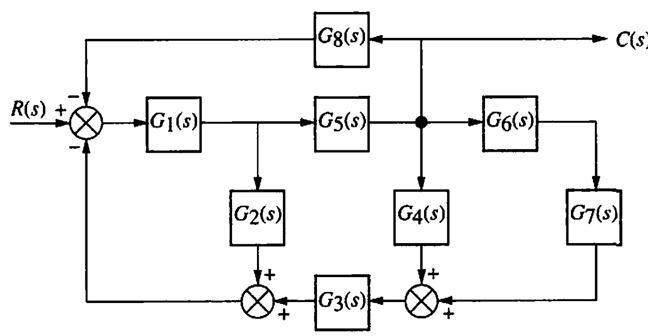


FIGURE P5.6

7. Find the unity feedback system that is equivalent to the system shown in Figure P5.7. [Section: 5.2].

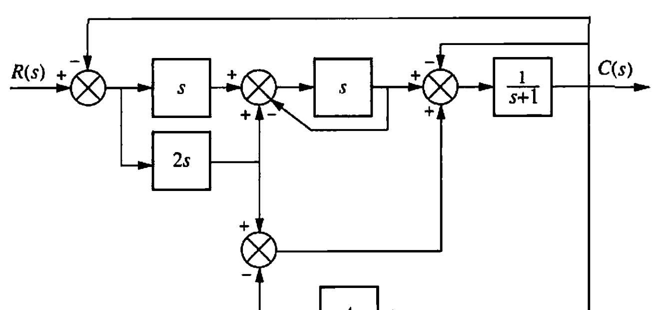


FIGURE P5.7

8. Given the block diagram of a system shown in Figure P5.8, find the transfer function $G(s) = \theta_{22}(s)/\theta_{11}(s)$. [Section: 5.2]

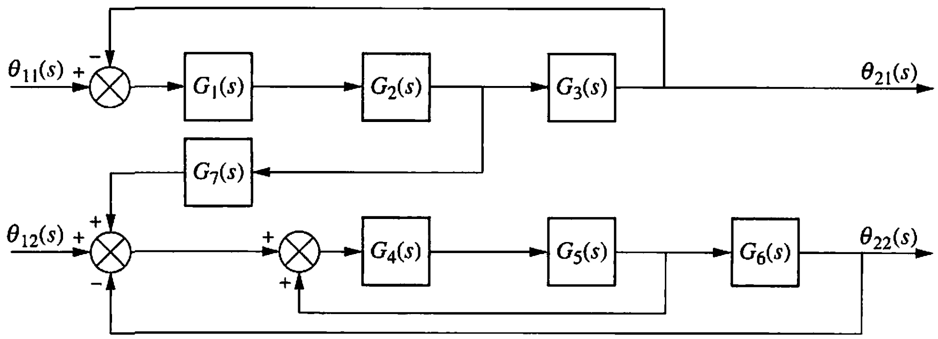


FIGURE P5.8

9. Reduce the block diagram shown in Figure P5.9 to a single transfer function, $T(s) = C(s)/R(s)$. [Section: 5.2]

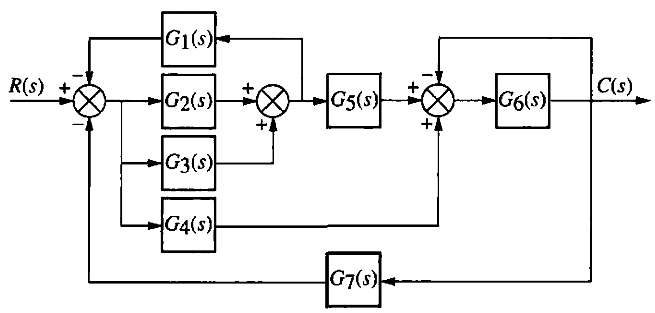


FIGURE P5.9

10. Reduce the block diagram shown in Figure P5.10 to a single block representing the transfer function, $T(s) = C(s)/R(s)$. [Section: 5.2]

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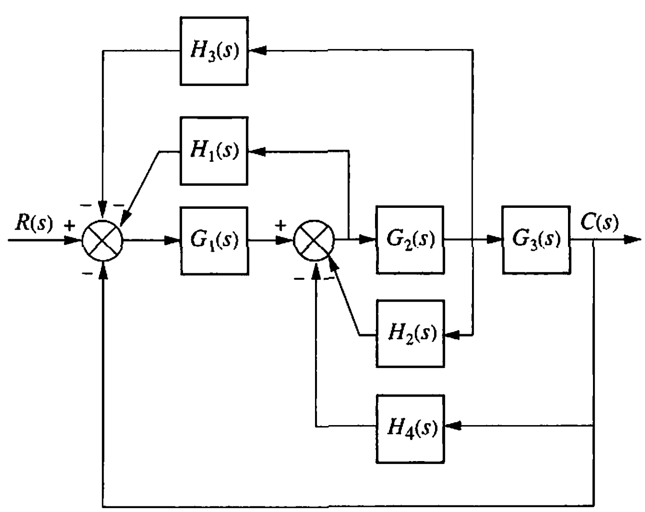

FIGURE P5.10

11. For the system shown in Figure P5.11, find the percent overshoot, settling time, and peak time for a step input if the system's response is underdamped. (Is it? Why?) [Section: 5.3]

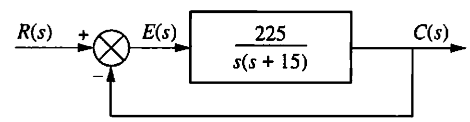

FIGURE P5.11

12. For the system shown in Figure P5.12, find the output, $c(t)$, if the input, $r(t)$, is a unit step. [Section: 5.3]

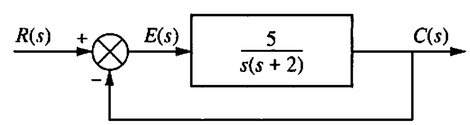

FIGURE P5.12

13. For the system shown in Figure P5.13, find the poles of the closed-loop transfer function, $T(s) = C(s)/R(s)$. [Section: 5.3]

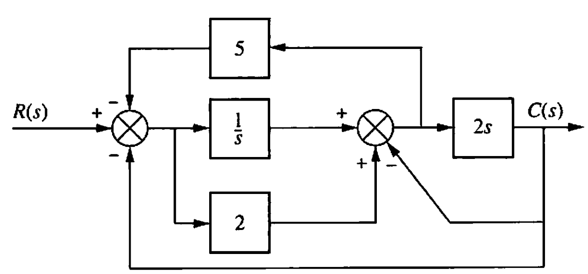

FIGURE P5.13

14. For the system of Figure P5.14, find the value of K that yields 10% overshoot for a step input. [Section: 5.3]

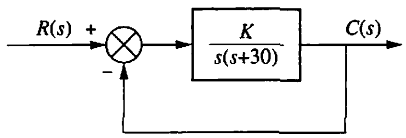

FIGURE P5.14

15. For the system shown in Figure P5.15, find K and α to yield a settling time of 0.15 second and a 30% overshoot. [Section: 5.3]

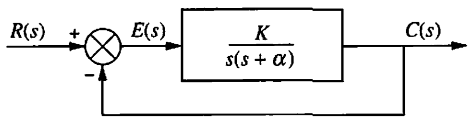

FIGURE P5.15

16. For the system of Figure P5.16, find the values of K_1 and K_2 to yield a peak time of 1.5 second and a settling time of 3.2 seconds for the closed-loop system's step response. [Section: 5.3]

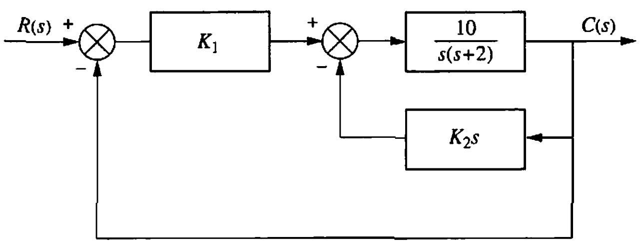

FIGURE P5.16

17. Find the following for the system shown in Figure P5.17: [Section: 5.3]

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a. The equivalent single block that represents the transfer function, $T(s) = C(s)/R(s)$.
b. The damping ratio, natural frequency, percent overshoot, settling time, peak time, rise time, and damped frequency of oscillation.

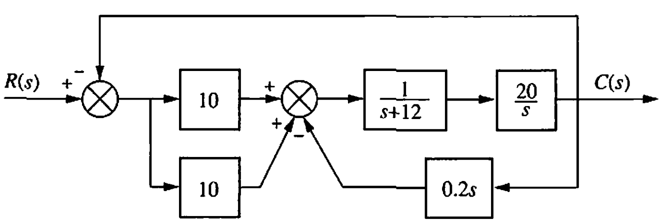

FIGURE P5.17

18. For the system shown in Figure P5.18, find ζ, ω_n, percent overshoot, peak time, rise time, and settling time. [Section: 5.3]

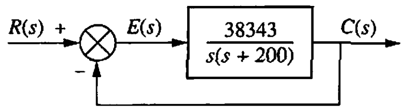

FIGURE P5.18

19. A motor and generator are set up to drive a load as shown in Figure P5.19. If the generator output voltage is $e_g(t) = K_f i_f(t)$, where $i_f(t)$ is the generator's field current, find the transfer function $G(s) = \theta_o(s)/E_i(s)$. For the generator, $K_f = 2\,\Omega$. For the motor, $K_t = 1$ N-m/A, and $K_b = 1$ V-s/rad.

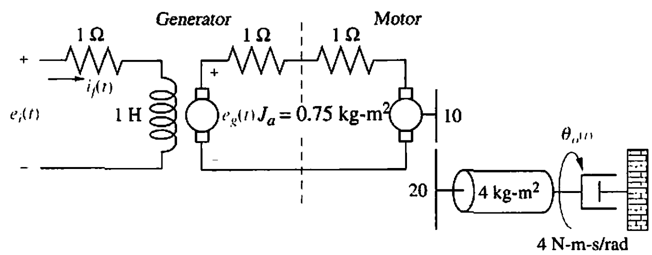

FIGURE P5.19

20. Find $G(s) = E_0(s)/T(s)$ for the system shown in Figure P5.20.

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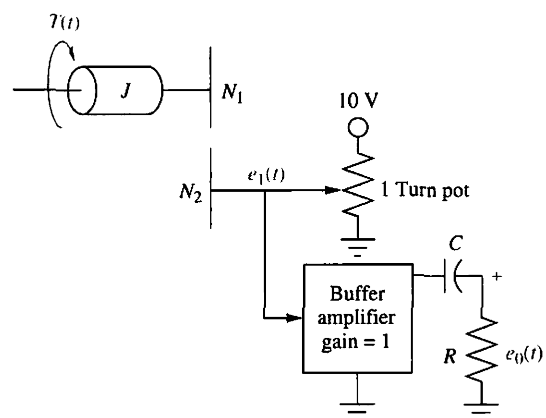

FIGURE P5.20

21. Find the transfer function $G(s) = E_o(s)/T(s)$ for the system shown in Figure P5.21.

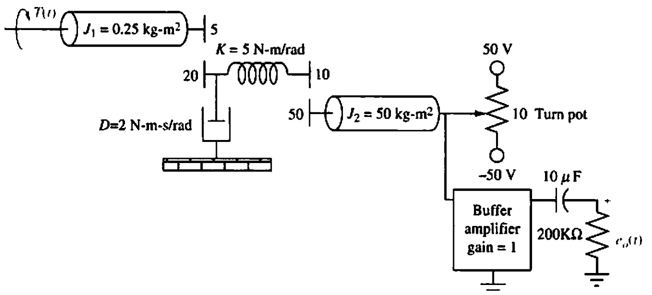

FIGURE P5.21

22. Label signals and draw a signal-flow graph for each of the block diagrams shown in the following problems: [Section: 5.4]
   a. Problem 1
   b. Problem 3
   c. Problem 5

23. Draw a signal-flow graph for each of the following state equations: [Section: 5.6] State Space SS

   a. $\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -4 & -6 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$

   $y = [1 \quad 1 \quad 0]\mathbf{x}$

   b. $\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -3 & 1 \\ -3 & -4 & -5 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} r$

   $y = [1 \quad 2 \quad 0]\mathbf{x}$

   c. $\dot{\mathbf{x}} = \begin{bmatrix} 7 & 1 & 0 \\ -3 & 2 & -1 \\ -1 & 0 & 2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} r$

   $y = [1 \quad 3 \quad 2]\mathbf{x}$

24. Given the system below, draw a signal-flow graph and represent the system in state space in the following forms: [Section: 5.7] State Space SS WileyPLUS WPCS Control Solutions
   a. Phase-variable form
   b. Cascade form

$$G(s) = \frac{10}{(s+7)(s+8)(s+9)}$$

25. Repeat Problem 24 for State Space SS

$$G(s) = \frac{20}{s(s-2)(s+5)(s+8)}$$

[Section: 5.7]

26. Using Mason's rule, find the transfer function, $T(s) = C(s)/R(s)$, for the system represented in Figure P5.22. [Section: 5.5]

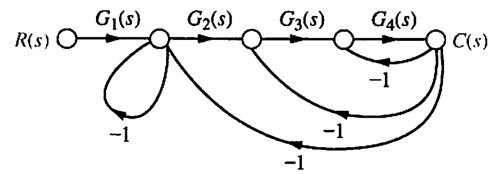

FIGURE P5.22

27. Using Mason's rule, find the transfer function, $T(s) = C(s)/R(s)$, for the system represented by Figure P5.23. [Section: 5.5]

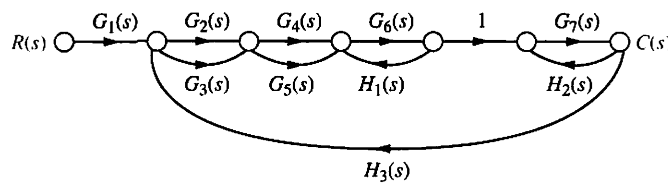

FIGURE P5.23

28. Use Mason's rule to find the transfer function of Figure 5.13 in the text. [Section: 5.5]

29. Use block diagram reduction to find the transfer function of Figure 5.21 in the text, and compare your answer with that obtained by Mason's rule. [Section: 5.5]

30. Represent the following systems in state space in Jordan canonical form. Draw the signal-flow graphs. [Section: 5.7] State Space SS

   a. $G(s) = \dfrac{(s+1)(s+2)}{(s+3)^2(s+4)}$

   b. $G(s) = \dfrac{(s+2)}{(s+5)^2(s+7)^2}$

   c. $G(s) = \dfrac{(s+3)}{(s+2)^2(s+4)(s+5)}$

31. Represent the systems below in state space in phase-variable form. Draw the signal-flow graphs. [Section: 5.7] State Space SS WileyPLUS WPCS Control Solutions

   a. $G(s) = \dfrac{s+3}{s^2+2s+7}$

b. $G(s) = \dfrac{s^2 + 2s + 6}{s^3 + 5s^2 + 2s + 1}$

c. $G(s) = \dfrac{s^3 + 2s^2 + 7s + 1}{s^4 + 3s^3 + 5s^2 + 6s + 4}$

32. Repeat Problem 31 and represent each system in controller canonical and observer canonical forms. [Section: 5.7] State Space SS

33. Represent the feedback control systems shown in Figure P5.24 in state space. When possible, represent the open-loop transfer functions separately in cascade and complete the feedback loop with the signal path from output to input. Draw your signal-flow graph to be in one-to-one correspondence to the block diagrams (as close as possible). [Section: 5.7] State Space SS

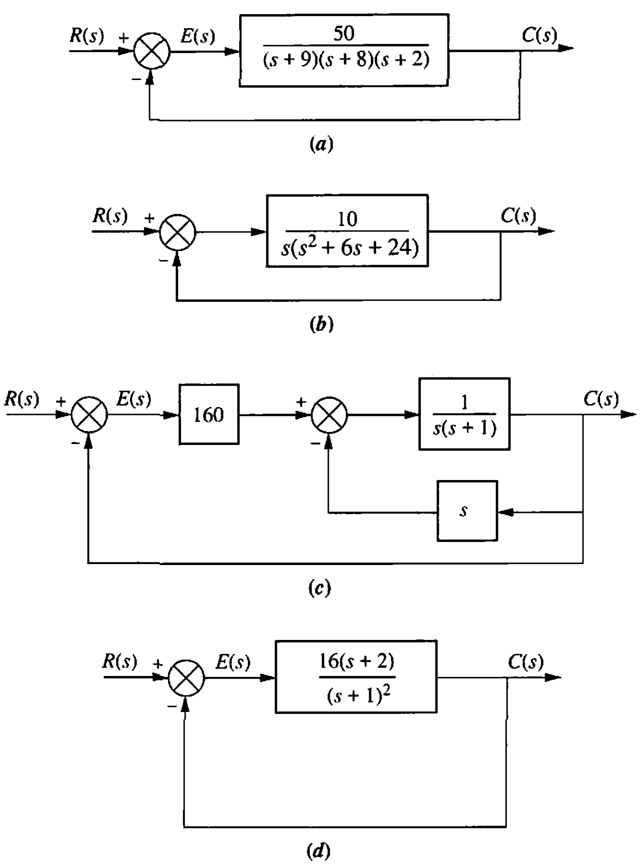

FIGURE P5.24

34. You are given the system shown in Figure P5.25. [Section: 5.7] State Space SS

a. Represent the system in state space in phase-variable form.

b. Represent the system in state space in any other form besides phase-variable.

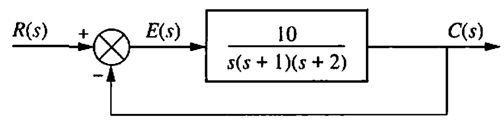

FIGURE P5.25

35. Repeat Problem 34 for the system shown in Figure P5.26. [Section: 5.7]

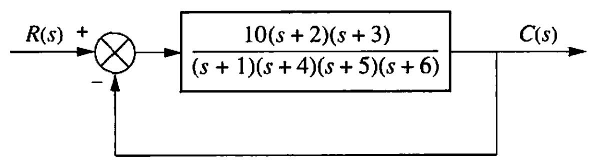

FIGURE P5.26

36. `Use MATLAB to solve Problem 35.` MATLAB ML

37. Represent the system shown in Figure P5.27 in state space where $x_1(t)$, $x_3(t)$, and $x_4(t)$, as shown, are among the state variables, $c(t)$ is the output, and $x_2(t)$ is internal to $X_1(s)/X_3(s)$. [Section: 5.7] State Space SS

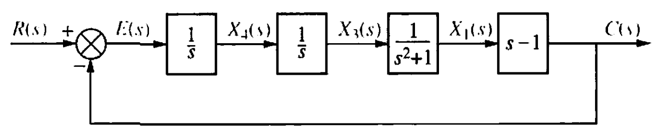

FIGURE P5.27

38. Consider the rotational mechanical system shown in Figure P5.28. State Space SS

a. Represent the system as a signal-flow graph.

b. Represent the system in state space if the output is $\theta_2(t)$.

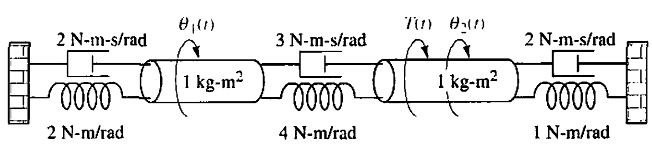

FIGURE P5.28

39. `Given a unity feedback system with the forward-path transfer function` MATLAB ML State Space SS

$$G(s) = \frac{7}{s(s+9)(s+12)}$$

use MATLAB to represent the closed loop system in state space in

a. phase-variable form;

b. parallel form.

40. Consider the cascaded subsystems shown in Figure P5.29. If $G_1(s)$ is represented in state space as

State Space SS | WileyPLUS WPCS Control Solutions

$$\dot{\mathbf{x}}_1 = \mathbf{A}_1\mathbf{x}_1 + \mathbf{B}_1 r$$
$$y_1 = \mathbf{C}_1\mathbf{x}_1$$

and $G_2(s)$ is represented in state space as

$$\dot{\mathbf{x}}_2 = \mathbf{A}_2\mathbf{x}_2 + \mathbf{B}_2 y_1$$
$$y_2 = \mathbf{C}_2\mathbf{x}_2$$

show that the entire system can be represented in state space as

$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \cdots \\ \dot{\mathbf{x}}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 & \vdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{B}_2\mathbf{C}_1 & \vdots & \mathbf{A}_2 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \cdots \\ \mathbf{x}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{B}_1 \\ \cdots \\ \mathbf{0} \end{bmatrix} r$$

$$y_2 = \begin{bmatrix} \mathbf{0} & \vdots & \mathbf{C}_2 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \cdots \\ \mathbf{x}_2 \end{bmatrix}$$

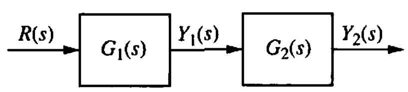

FIGURE P5.29

41. Consider the parallel subsystems shown in Figure P5.30. If $G_1(s)$ is represented in state space as

State Space SS

$$\dot{\mathbf{x}}_1 = \mathbf{A}_1\mathbf{x}_1 + \mathbf{B}_1 r$$
$$y_1 = \mathbf{C}_1\mathbf{x}_1$$

and $G_2(s)$ is represented in state space as

$$\dot{\mathbf{x}}_2 = \mathbf{A}_2\mathbf{x}_2 + \mathbf{B}_2 r$$
$$y_2 = \mathbf{C}_2\mathbf{x}_2$$

show that the entire system can be represented in state space as

$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \cdots \\ \dot{\mathbf{x}}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 & \vdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \vdots & \mathbf{A}_2 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \cdots \\ \mathbf{x}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{B}_1 \\ \cdots \\ \mathbf{B}_2 \end{bmatrix} r$$

$$y = \begin{bmatrix} \mathbf{C}_1 & \vdots & \mathbf{C}_2 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \cdots \\ \mathbf{x}_2 \end{bmatrix}$$

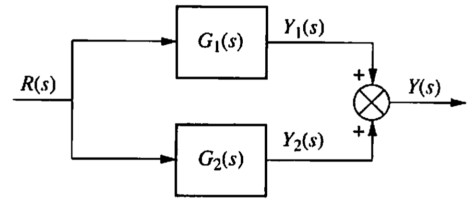

FIGURE P5.30

42. Consider the subsystems shown in Figure P5.31 and connected to form a feedback system. If $G(s)$ is represented in state space as

State Space SS

$$\dot{\mathbf{x}}_1 = \mathbf{A}_1\mathbf{x}_1 + \mathbf{B}_1 e$$
$$y = \mathbf{C}_1\mathbf{x}_1$$

and $H_2(s)$ is represented in state space as

$$\dot{\mathbf{x}}_2 = \mathbf{A}_2\mathbf{x}_2 + \mathbf{B}_2 y$$
$$p = \mathbf{C}_2\mathbf{x}_2$$

show that the closed-loop system can be represented in state space as

$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \cdots \\ \dot{\mathbf{x}}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 & \vdots & -\mathbf{B}_1\mathbf{C}_2 \\ \cdots & \cdots & \cdots \\ \mathbf{B}_2\mathbf{C}_1 & \vdots & \mathbf{A}_2 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \cdots \\ \mathbf{x}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{B}_1 \\ \cdots \\ \mathbf{0} \end{bmatrix} r$$

$$y = \begin{bmatrix} \mathbf{C}_1 & \vdots & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \cdots \\ \mathbf{x}_2 \end{bmatrix}$$

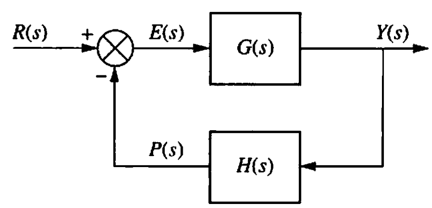

FIGURE P5.31

43. Given the system represented in state space as follows: [Section: 5.8]

State Space SS

$$\dot{\mathbf{x}} = \begin{bmatrix} -1 & -7 & 6 \\ -8 & 4 & 8 \\ 4 & 7 & -8 \end{bmatrix}\mathbf{x} + \begin{bmatrix} -5 \\ -7 \\ 5 \end{bmatrix} r$$
$$y = \begin{bmatrix} -9 & -9 & -8 \end{bmatrix}\mathbf{x}$$

convert the system to one where the new state vector, $\mathbf{z}$, is

$$\mathbf{z} = \begin{bmatrix} -4 & 9 & -3 \\ 0 & -4 & 7 \\ -1 & -4 & -9 \end{bmatrix}\mathbf{x}$$

44. Repeat Problem 43 for the following system: [Section: 5.8]

State Space SS

$$\dot{\mathbf{x}} = \begin{bmatrix} -5 & 1 & 1 \\ 9 & -9 & -9 \\ -9 & -1 & 8 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 9 \\ -4 \\ 0 \end{bmatrix} r$$

$$y = \begin{bmatrix} -2 & -4 & 1 \end{bmatrix}\mathbf{x}$$

and the following state-vector transformation:

$$\mathbf{z} = \begin{bmatrix} 5 & -4 & 9 \\ 6 & -7 & 6 \\ 6 & -5 & -3 \end{bmatrix}\mathbf{x}$$

45. Diagonalize the following system: [Section: 5.8]

State Space SS

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$$\dot{\mathbf{x}} = \begin{bmatrix} -5 & -5 & 4 \\ 2 & 0 & -2 \\ 0 & -2 & -1 \end{bmatrix}\mathbf{x} + \begin{bmatrix} -1 \\ 2 \\ -2 \end{bmatrix} r$$

$$y = \begin{bmatrix} -1 & 1 & 2 \end{bmatrix}\mathbf{x}$$

46. Repeat Problem 45 for the following system: [Section: 5.8]

State Space SS

$$\dot{\mathbf{x}} = \begin{bmatrix} -10 & -3 & 7 \\ 18.25 & 6.25 & -11.75 \\ -7.25 & -2.25 & 5.75 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1 & -2 & 4 \end{bmatrix}\mathbf{x}$$

47. `Diagonalize the system in Problem 46 using MATLAB.`

MATLAB ML

48. During ascent the space shuttle is steered by commands generated by the computer's guidance calculations. These commands are in the form of vehicle attitude, attitude rates, and attitude accelerations obtained through measurements made by the vehicle's inertial measuring unit, rate gyro assembly, and accelerometer assembly, respectively. The ascent digital autopilot uses the errors between the actual and commanded attitude, rates, and accelerations to gimbal the space shuttle main engines (called thrust vectoring) and the solid rocket boosters to effect the desired vehicle attitude. The space shuttle's attitude control system employs the same method in the pitch, roll, and yaw control systems. A simplified model of the pitch control system is shown in Figure P5.32.[4]

a. Find the closed-loop transfer function relating actual pitch to commanded pitch. Assume all other inputs are zero.

b. Find the closed-loop transfer function relating actual pitch rate to commanded pitch rate. Assume all other inputs are zero.

c. Find the closed-loop transfer function relating actual pitch acceleration to commanded pitch acceleration. Assume all other inputs are zero.

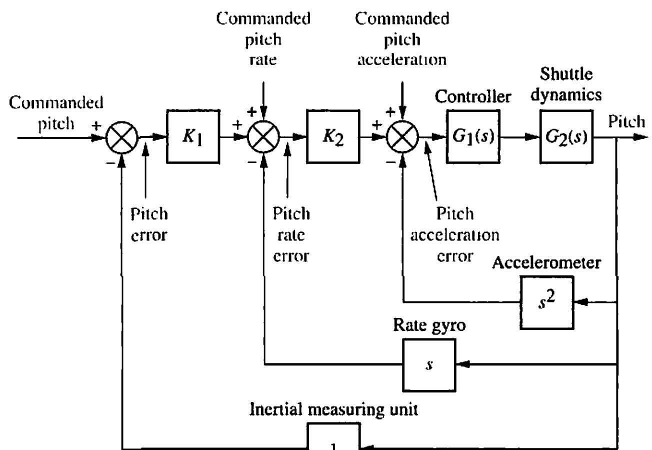

FIGURE P5.32 Space shuttle pitch control system (simplified)

49. An AM radio modulator generates the product of a carrier waveform and a message waveform, as shown in Figure P5.33 (*Kurland, 1971*). Represent the system in state space if the carrier is a sinusoid of frequency $\omega = a$, and the message is a sinusoid of frequency $\omega = b$. Note that this system is nonlinear because of the multiplier.

State Space SS

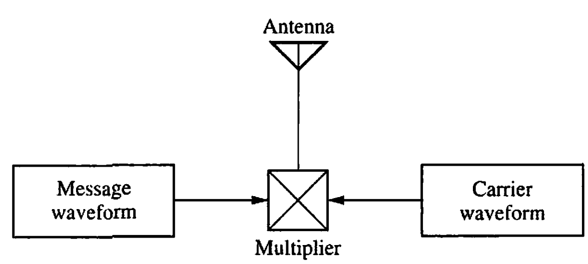

FIGURE P5.33 AM modulator

50. A model for human eye movement consists of the closed-loop system shown in Figure P5.34, where an

[4] Source of background information for this problem: Rockwell International.

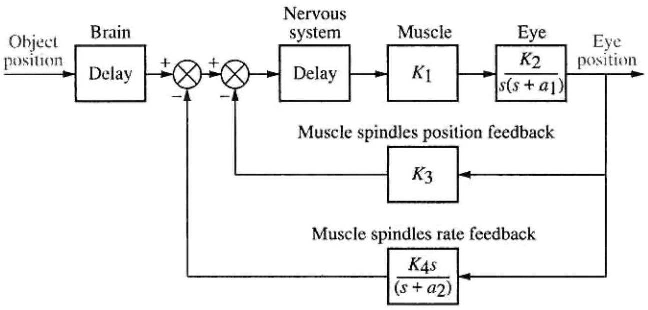

FIGURE P5.34 Feedback control system representing human eye movement

object's position is the input and the eye position is the output. The brain sends signals to the muscles that move the eye. These signals consist of the difference between the object's position and the position and rate information from the eye sent by the muscle spindles. The eye motion is modeled as an inertia and viscous damping and assumes no elasticity (spring) (*Milhorn, 1966*). Assuming that the delays in the brain and nervous system are negligible, find the closed-loop transfer function for the eye position control.

51. A HelpMate transport robot, shown in Figure P5.35(*a*), is used to deliver goods in a hospital setting. The robot can deliver food, drugs, laboratory materials, and patients' records (*Evans, 1992*). Given the simplified block diagram of the robot's bearing angle control system, as shown in Figure P5.35(*b*), do the following:

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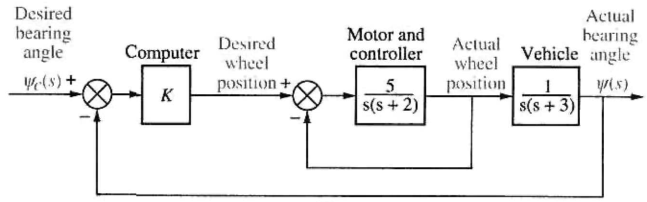
(*a*)



(*b*)

FIGURE P5.35 **a.** HelpMate robot used for in-hospital deliveries; **b.** simplified block diagram for bearing angle control

a. Find the closed-loop transfer function.

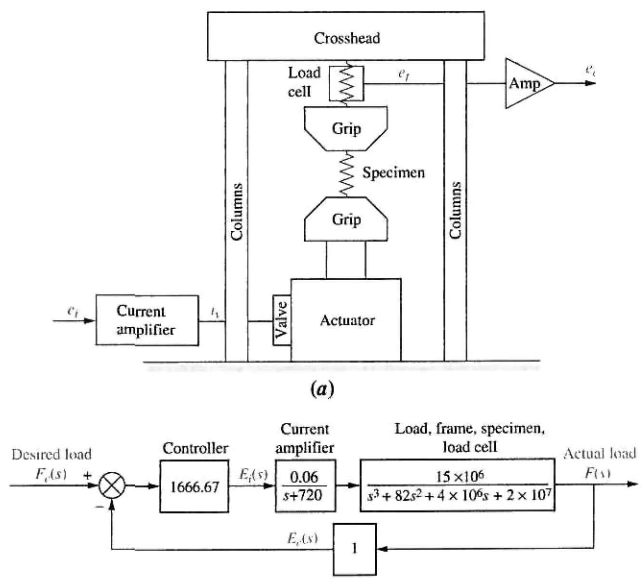
b. Represent the system in state space, where the input is the desired bearing angle, the output is the actual bearing angle, and the actual wheel position and actual bearing angle are among the state variables.

State Space **SS**

c. `Simulate the closed-loop system using MATLAB. Obtain the unit step response for different values of K that yield responses from overdamped to underdamped to unstable.`

MATLAB **ML**

52. Automatically controlled load testers can be used to test product reliability under real-life conditions. The tester consists of a load frame and specimen as shown in Figure P5.36(*a*). The desired load is



(*b*)

FIGURE P5.36 **a.** Load tester, (© 1992 IEEE) **b.** approximate block diagram input via a voltage, $e_i(t)$, to a current amplifier. The output load is measured via a voltage, $e_i(t)$, from a load cell measuring the load on the specimen. Figure P5.36(b) shows an approximate model of a load testing system without compensation (*Bailey, 1992*).

a. Model the system in state space. (State Space SS)

b. Simulate the step response using MATLAB. Is the response predominantly first or second order? Describe the characteristics of the response that need correction. (MATLAB ML)

53. Consider the F4-E aircraft of Problem 22, Chapter 3. If the open-loop transfer function relating normal acceleration, $A_n(s)$, to the input deflection command, $\delta_c(s)$, is approximated as (State Space SS)

$$\frac{A_n(s)}{\delta_c(s)} = \frac{-272(s^2 + 1.9s + 84)}{(s + 14)(s - 1.8)(s + 4.9)}$$

(*Cavallo, 1992*), find the state-space representation in

a. Phase-variable form

b. Controller canonical form

c. Observer canonical form

d. Cascade form

e. Parallel form

54. Find the closed-loop transfer function of the Unmanned Free-Swimming Submersible vehicle's pitch control system shown on the back endpapers (*Johnson, 1980*).

55. Repeat Problem 54 using MATLAB. (MATLAB ML)

56. Use Simulink to plot the effects of nonlinearities upon the closed-loop step response of the antenna azimuth position control system shown on the front endpapers, Configuration 1. In particular, consider individually each of the following nonlinearities: saturation (±5 volts), backlash (dead-band width 0.15), deadzone (−2 to +2), as well as the linear response. Assume the preamplifier gain is 100 and the step input is 2 radians. (Simulink SL)

57. Problem 12 in Chapter 1 describes a high-speed proportional solenoid valve. A subsystem of the valve is the solenoid coil shown in Figure P5.37. Current through the coil, L, generates a magnetic field that produces a force to operate the valve. Figure P5.37can be represented as a block diagram (*Vaughan, 1996*)

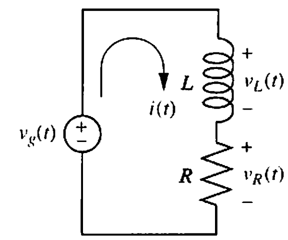


FIGURE P5.37 Solenoid coil circuit

a. Derive a block diagram of a feedback system that represents the coil circuit, where the applied voltage, $v_g(t)$, is the input, the coil voltage, $v_L(t)$, is the error voltage, and the current, $i(t)$, is the output.

b. For the block diagram found in Part **a**, find the Laplace transform of the output current, $I(s)$.

c. Solve the circuit of Figure P5.37 for $I(s)$, and compare to your result in Part **b**.

58. Ktesibios' water clock (see Section 1.2) is probably the first man-made system in which feedback was used in a deliberate manner. Its operations are shown in Figure P5.38(a). The clock indicates time progressively on scale D as water falls from orifice A toward vessel B. Clock accuracy depends mainly on water height h_f in the water reservoir G, which must be maintained at a constant level h_r by means of the conical float F that moves up or down to control the water inflow. Figure P5.38(b) shows a block diagram describing the system (*Lepschy, 1992*).

Let $q_i(t)$ and $q_o(t)$ represent the input and output water flow, respectively, and h_m the height of water in vessel B. Use Mason's rule to find the following transfer functions, assuming α and β are constants:

a. $\dfrac{H_m(s)}{H_r(s)}$

b. $\dfrac{H_f(s)}{H_r(s)}$

c. $\dfrac{Q_i(s)}{H_r(s)}$

d. $\dfrac{Q_o(s)}{H_r(s)}$

e. Using the above transfer functions, show that if $h_r(t) =$ constant, then $q_o(t) =$ constant and $h_m(t)$ increases at a constant speed.

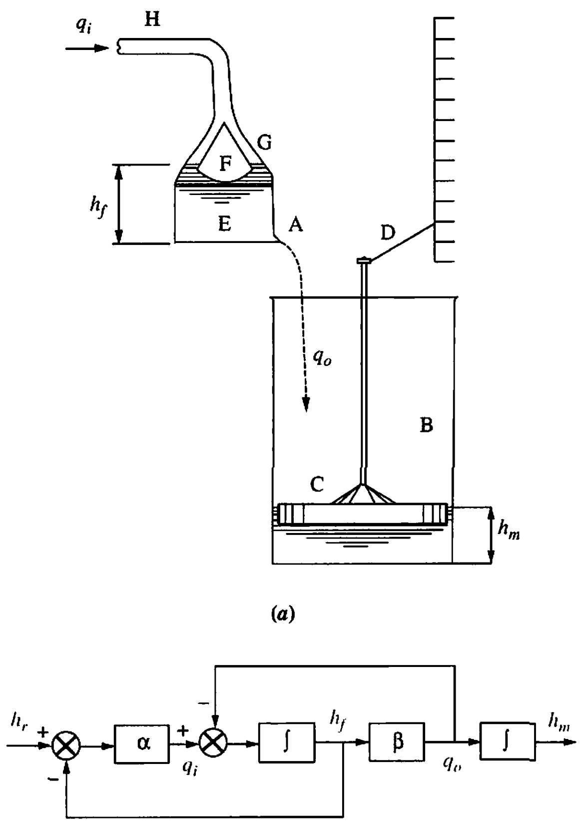


FIGURE P5.38 **a.** Ktesibios' water clock; **b.** water clock block diagram (© 1992 IEEE)

59. Some robotic applications can benefit from actuators in which load position as well as exerted force are controlled. Figure P5.39 shows the block diagram of such an actuator, where u_1 and u_2 are voltage inputs to two coils, each of which controls a pneumatic piston, and y represents the load displacement.

The system's output is u, the differential pressure acting on the load. The system also has a disturbance input f_{ext}, which represents external forces that are not system generated, but are acting on the load. A is a constant (*Ben-Dov, 1995*). Use any method to obtain:

a. An expression for the system's output in terms of the inputs u_1 and u_2 (Assume $f_{ext} = 0$.)

b. An expression for the effect of f_{ext} on the output u (Assume u_1 and $u_2 = 0$.)

c. What condition on the inputs u_1 and u_2 will result in $u = 0$?

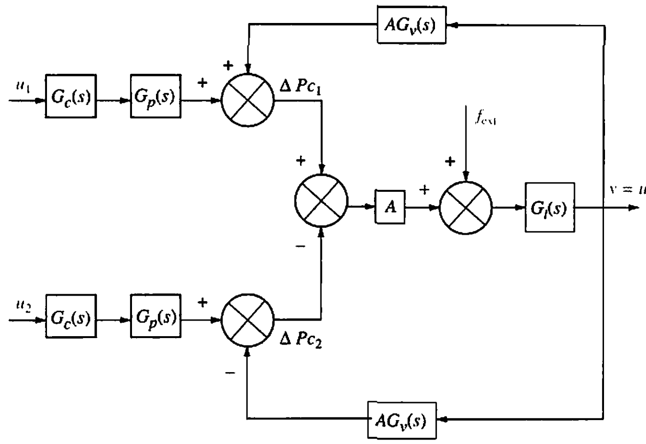


FIGURE P5.39 Actuator block diagram (© 1995 IEEE)

60. Figure P5.40 shows a noninverting operational amplifier.

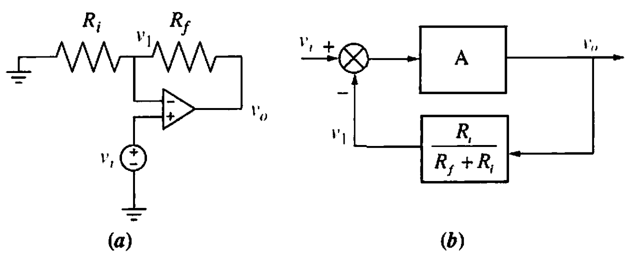


FIGURE P5.40 **a.** Noninverting amplifier; **b.** block diagram

Assuming the operational amplifier is ideal,

a. Verify that the system can be described by the following two equations:

$$v_o = A(v_i - v_o)$$

$$v_1 = \frac{R_i}{R_i + R_f} v_o$$

b. Check that these equations can be described by the block diagram of Figure P5.40(*b*)

c. Use Mason's rule to obtain the closed-loop system transfer function $\frac{V_o(s)}{V_i(s)}$

d. Show that when $A \to \infty$, $\frac{V_o(s)}{V_i(s)} = 1 + \frac{R_f}{R_i}$

61. Figure P5.41 shows the diagram of an inverting operational amplifier.

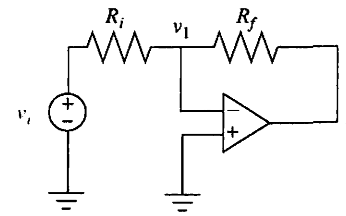


FIGURE P5.41 Inverting operational amplifier

a. Assuming an ideal operational amplifier, use a similar procedure to the one outlined in Problem 60 to find the system equations.

b. Draw a corresponding block diagram and obtain the transfer function $\frac{V_o(s)}{V_i(s)}$.

c. Show that when $A \to \infty$, $\frac{V_o(s)}{V_i(s)} = -\frac{R_f}{R_i}$.

62. Figure P5.42(a) shows an n-channel enhancement-mode MOSFET source follower circuit. Figure P5.42(b) shows its small-signal equivalent (where $R_i = R_1 \| R_2$) (*Neamen, 2001*).

a. Verify that the equations governing this circuit are

$$\frac{v_{in}}{v_i} = \frac{R_i}{R_i + R_s}; \quad v_{gs} = v_{in} - v_o; \quad v_o = g_m(R_s \| r_o)v_{gs}$$

b. Draw a block diagram showing the relations between the equations.

c. Use the block diagram in Part **b** to find $\frac{V_o(s)}{V_i(s)}$.

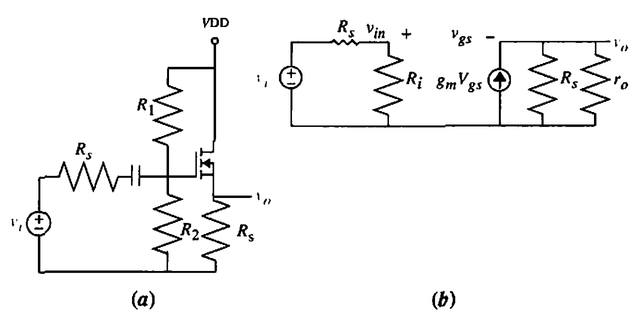


FIGURE P5.42 a. An n-channel enhancement-mode MOSFET source follower circuit; b. small-signal equivalent

63. A car active suspension system adds an active hydraulic actuator in parallel with the passive damper and spring to create a dynamic impedance that responds to road variations. The block diagram of Figure P5.43 depicts such an actuator with closed-loop control.

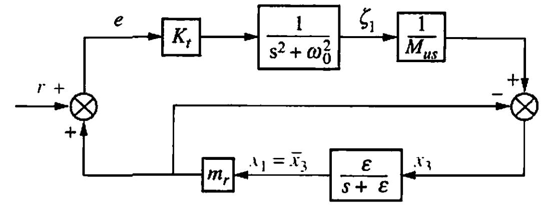


FIGURE P5.43 (© 1997 IEEE)

In the figure, K_t is the spring constant of the tire, M_{US} is the wheel mass, r is the road disturbance, x_1 is the vertical car displacement, x_3 is the wheel vertical displacement, $\omega_0^2 = \frac{K_t}{M_{US}}$ is the natural frequency of the unsprung system and ε is a filtering parameter to be judiciously chosen (*Lin, 1997*). Find the two transfer functions of interest:

a. $\frac{X_3(s)}{R(s)}$

b. $\frac{X_1(s)}{R(s)}$

64. The basic unit of skeletal and cardiac muscle cells is a *sarcomere*, which is what gives such cells a striated (parallel line) appearance. For example, one bicep cell has about 10^5 sarcomeres. In turn, sarcomeres are composed of protein complexes. Feedback mechanisms play an important role in sarcomeres and thus muscle contraction. Namely, Fenn's law says that the energy liberated during muscle contraction depends on the initial conditions and the load encountered. The following linearized model describing sarcomere contraction has been developed for cardiac muscle:

$$\begin{bmatrix} \dot{A} \\ \dot{T} \\ \dot{U} \\ \dot{SL} \end{bmatrix} = \begin{bmatrix} -100.2 & -20.7 & -30.7 & 200.3 \\ 40 & -20.22 & 49.95 & 526.1 \\ 0 & 10.22 & -59.95 & -526.1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} A \\ T \\ U \\ SL \end{bmatrix} + \begin{bmatrix} 208 \\ -208 \\ -108.8 \\ -1 \end{bmatrix} u(t)$$

$$y = [0 \quad 1570 \quad 1570 \quad 59400] \begin{bmatrix} A \\ T \\ U \\ SL \end{bmatrix} - 6240u(t)$$

where

A = density of regulatory units with bound calcium and adjacent weak cross bridges (μM)
T = density of regulatory units with bound calcium and adjacent strong cross bridges (M)
U = density of regulatory units without bound calcium and adjacent strong cross bridges (M)
SL = sarcomere length (m)

The system's input is $u(t)$ = the shortening muscle velocity in meters/second and the output is $y_{(t)}$ = muscle force output in Newtons (*Yaniv, 2006*).

Do the following:

a. `Use MATLAB to obtain the transfer function` $\frac{Y(s)}{U(s)}$. MATLAB ML

b. `Use MATLAB to obtain a partial-fraction expansion for` $\frac{Y(s)}{U(s)}$. MATLAB ML

c. Draw a signal-flow diagram of the system in parallel form. State Space SS

d. Use the diagram of Part **c** to express the system in state-variable form with decoupled equations. State Space SS

65. An electric ventricular assist device (EVAD) has been designed to help patients with diminished but still functional heart pumping action to work in parallel with the natural heart. The device consists of a brushless dc electric motor that actuates on a pusher plate. The plate movements help the ejection of blood in systole and sac filling in diastole. System dynamics during systolic mode have been found to be:

$$\begin{bmatrix} \dot{x} \\ \dot{v} \\ \dot{P}_{ao} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -68.3 & -7.2 \\ 0 & 3.2 & -0.7 \end{bmatrix} \begin{bmatrix} x \\ v \\ P_{ao} \end{bmatrix} + \begin{bmatrix} 0 \\ 425.4 \\ 0 \end{bmatrix} e_m$$

The state variables in this model are x, the pusher plate position, v, the pusher plate velocity, and P_{ao}, the aortic blood pressure. The input to the system is e_m, the motor voltage (*Tasch, 1990*).

a. `Use MATLAB to find a similarity transformation to diagonalize the system.` MATLAB ML

b. `Use MATLAB and the obtained similarity transformation of Part a to obtain a diagonalized expression for the system.` MATLAB ML

66. In an experiment to measure and identify postural arm reflexes, subjects hold with their hands a linear hydraulic manipulator. A load cell is attached to the actuator handle to measure resulting forces. At the application of a force, subjects try to maintain a fixed posture. Figure P5.44 shows a block diagram for the combined arm-environment system.

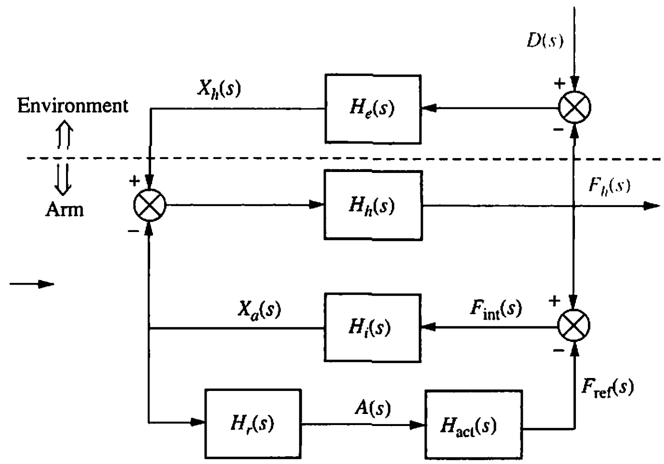


FIGURE P5.44

In the diagram, $H_r(s)$ represents the reflexive length and velocity feedback dynamics; $H_{act}(s)$ the activation dynamics, $H_i(s)$ the intrinsic act dynamics; $H_h(s)$ the hand dynamics; $H_e(s)$ the environmental dynamics; $X_a(s)$ the position of the arm; $X_h(s)$ the measured position of the hand; $F_h(s)$ the measured interaction force applied by the hand; $F_{int}(s)$ the intrinsic force; $F_{ref}(s)$ the reflexive force; $A(s)$ the reflexive activation; and $D(s)$ the external force perturbation (*de Vlugt, 2002*).

a. Obtain a signal-flow diagram from the block diagram.

b. Find $\frac{F_h(s)}{D(s)}$.

67. `Use LabVIEW's Control Design and Simulation Module to obtain the controller and the observer canonical forms for:` State Space SS LabVIEW LV

$$G(s) = \frac{s^2 + 7s + 2}{s^3 + 9s^2 + 26s + 24}$$

68. A virtual reality simulator with haptic (sense of touch) feedback was developed to simulate the control of a submarine driven through a joystick input. Operator haptic feedback is provided through joystick position constraints and simulator movement (*Karkoub, 2010*). Figure P5.45 shows the block diagram of the haptic feedback system in which the input u_h is the force exerted by the muscle of the human arm; and the outputs are y_s, the position of the simulator, and y_j, the position of the joystick.

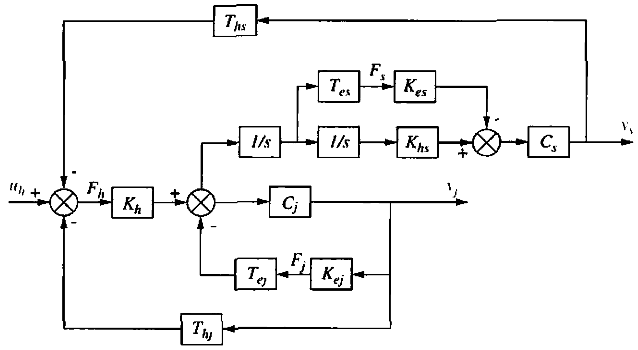


FIGURE P5.45

a. Find the transfer function $\frac{Y_s(s)}{U_h(s)}$.

b. Find the transfer function $\frac{Y_j(s)}{U_h(s)}$.

69. Some medical procedures require the insertion of a needle under a patient's skin using CT scan monitoring guidance for precision. CT scans emit radiation, posing some cumulative risks for medical personnel. To avoid this problem, a remote control robot has been developed (*Piccin, 2009*). The robot controls the needle in position and angle in the constraint space of a CT scan machine and also provides the physician with force feedback commensurate with the insertion opposition encountered by the type of tissue in which the needle is inserted. The robot has other features that give the operator the similar sensations and maneuverability as if the needle was inserted directly. Figure P5.46 shows the block diagram of the force insertion mechanism, where F_h is the input force and X_h is the output displacement. Summing junction inputs are positive unless indicated with a negative sign. By way of explanation, Z = impedance; G = transfer function; C_i = communication channel transfer functions; F = force; X = position. Subscripts h and m refer to the master manipulator. Subscripts s and e refer to the slave manipulator.

a. Assuming $Z_h = 0$, $C_1 = C_s$, $C_2 = 1 + C_6$ and $C_4 = -C_m$ use Mason's Rule to show that the transfer function from the operators force input F_h to needle displacement X_h is given by

$$Y(s) = \frac{X_h(s)}{F_h(s)} = \frac{Z_m^{-1}C_2(1 + G_sC_s)}{1 + G_sC_s + Z_m^{-1}(c_m + C_2Z_eG_sC_s)}$$

b. Now with $Z_h \neq 0$ show that $\frac{X_h(s)}{F_h(s)} = \frac{Y(s)}{1 + Y(s)Z_h}$

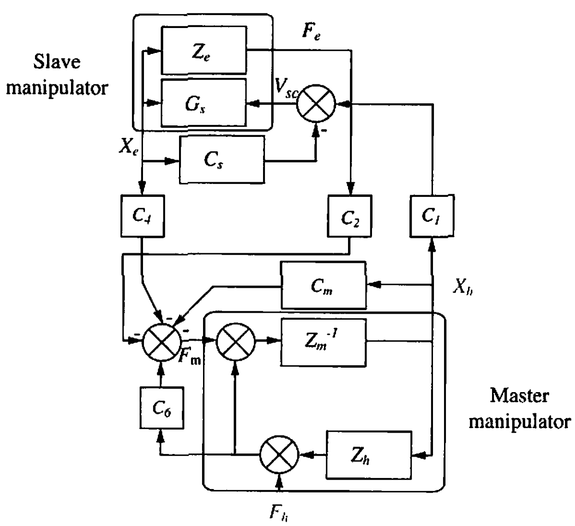


FIGURE P5.46

70. A hybrid solar cell and diesel power distribution system has been proposed and tested (*Lee, 2007*). The system has been shown to have a very good uninterruptible power supply as well as line voltage regulation capabilities. Figure P5.47 shows a signal-flow diagram of the system. The output, V_{Load}, is the voltage across the load. The two inputs are I_{Cf}, the reference current, and I_{Dist}, the disturbance representing current changes in the supply.

a. Refer to Figure P5.47 and find the transfer function $\frac{V_{Load}(s)}{I_{Cf}(s)}$.

b. Find the transfer function $\frac{V_{Load}(s)}{I_{Dist}(s)}$.

71. Continuous casting in steel production is essentially a solidification process by which molten steel is

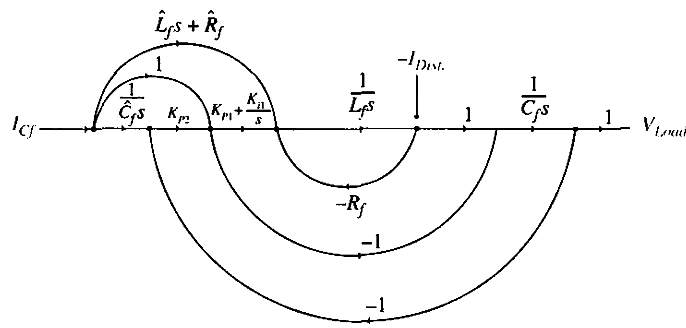


FIGURE P5.47

solidified into a steel slab after passing through a mold, as shown in Figure P5.48(a). Final product dimensions depend mainly on the casting speed V_p (in m/min), and on the stopper position X(in %) that controls the flow of molten material into the mold (*Kong, 1993*). A simplified model of a casting system is shown in Figure P5.48(b) (*Kong, 1993*) and (*Graebe, 1995*). In the model, H_m = mold level (in mm); H_t = assumed constant height of molten steel in the tundish; D_z = mold thickness = depth of nozzle immerged into molten steel; and W_t = weight of molten steel in the tundish.

For a specific setting let $A_m = 0.5$ and

$$G_x(s) = \frac{0.63}{s + 0.926}$$

Also assume that the valve positioning loop may be modeled by the following second-order transfer function:

$$G_V(s) = \frac{X(s)}{Y_C(s)} = \frac{100}{s^2 + 10s + 100}$$

and the controller is modeled by the following transfer function:

$$G_C(s) = \frac{1.6(s^2 + 1.25s + 0.25)}{s}$$

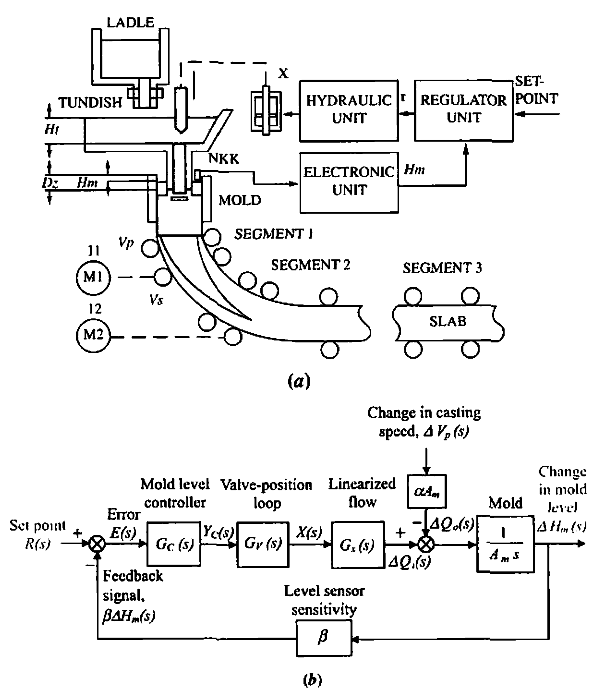


FIGURE P5.48 Steel mold process: **a.** process (© 1993 IEEE); **b.** block diagram

The sensitivity of the mold level sensor is $\beta = 0.5$ and the initial values of the system variables at $t = 0^-$ are: $R(0^-) = 0$; $Y_C(0^-) = X(0^-) = 41.2$; $\Delta H_m(0^-) = 0$; $H_m(0^-) = -75$; $\Delta V_p(0^-) = 0$; and $V_p(0^-) = 0$. Do the following:

a. Assuming $v_p(t)$ is constant $[\Delta v_p = 0]$, find the closed-loop transfer function $T(s) = \Delta H_m(s)/R(s)$.

b. For $r(t) = 5\,u(t)$, $v_p(t) = 0.97\,u(t)$, and $H_m(0^-) = -75$ mm, use Simulink to simulate the system. Record the time and mold level (in array format) by connecting them to **Workspace** sinks, each of which should carry the respective variable name. After the simulation ends, utilize MATLAB plot commands to obtain and edit the graph of $h_m(t)$ from $t = 0$ to 80 seconds.

Simulink **SL**

72. A simplified second-order transfer function model for bicycle dynamics is given by

State Space **SS**

$$\frac{\varphi(s)}{\delta(s)} = \frac{aV}{bh}\frac{\left(s + \frac{V}{a}\right)}{\left(s^2 - \frac{g}{h}\right)}$$

The input is $\delta(s)$, the steering angle, and the output is $\varphi(s)$, the tilt angle (between the floor and the bicycle longitudinal plane). In the model parameter a is the horizontal distance from the center of the back wheel to the bicycle center of mass; b is the horizontal distance between the centers of both wheels; h is the vertical distance from the center of mass to the floor; V is the rear wheel velocity (assumed constant); and g is the gravity constant. It is also assumed that the rider remains at a fixed position with respect to the bicycle so that the steer axis is vertical and that all angle deviations are small (*Åstrom, 2005*).

a. Obtain a state-space representation for the bicycle model in phase-variable form.

b. Find system eigenvalues and eigenvectors.

c. Find an appropriate similarity transformation matrix to diagonalize the system and obtain the state-space system's diagonal representation.

73. It is shown in Figure 5.6(c) that when negative feedback is used, the overall transfer function for the system of Figure 5.6(b) is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

Develop the block diagram of an alternative feedback system that will result in the same closed-loop transfer function, $C(s)/R(s)$, with $G(s)$ unchanged and unmoved. In addition, your new block diagram must have unity gain in the feedback path. You can add input transducers and/or controllers in the main forward path as required.

DESIGN PROBLEMS

74. The motor and load shown in Figure P5.49(a) are used as part of the unity feedback system shown in Figure P5.49(b). Find the value of the coefficient of viscous damping, D_L, that must be used in order to yield a closed-loop transient response having a 20% overshoot.

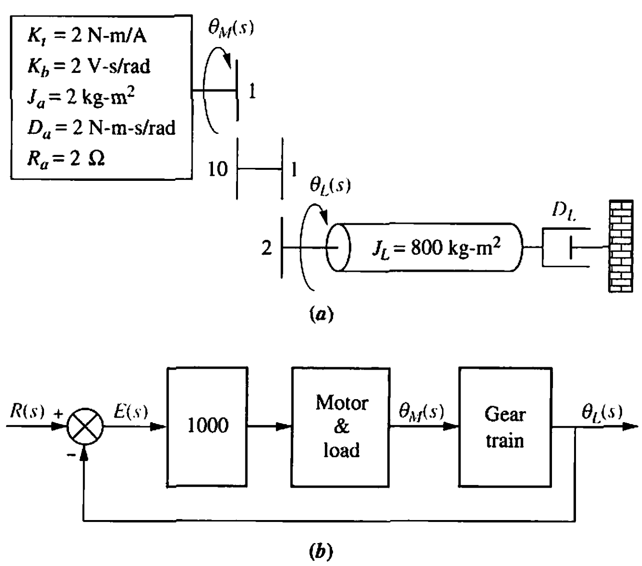


FIGURE P5.49 Position control: **a.** motor and load; **b.** block diagram

75. Assume that the motor whose transfer function is shown in Figure P5.50(a) is used as the forward path of a closed-loop, unity feedback system.
   a. Calculate the percent overshoot and settling time that could be expected.
   b. You want to improve the response found in Part **a**. Since the motor and the motor constants cannot be changed, an amplifier and a tachometer (voltage generator) are inserted into the loop, as shown in Figure P5.50. Find the values of K_1 and K_2 to yield a 16% overshoot and a settling time of 0.2 second.

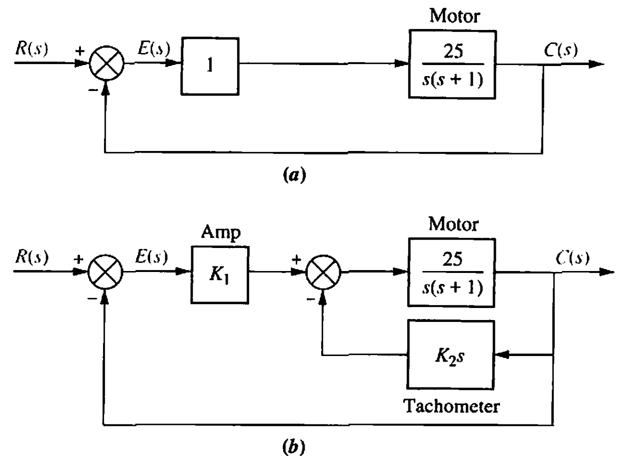


FIGURE P5.50 **a.** Position control; **b.** position control with tachometer

76. The system shown in Figure P5.51 will have its transient response altered by adding a tachometer. Design K and K_2 in the system to yield a damping ratio of 0.69. The natural frequency of the system before the addition of the tachometer is 10 rad/s.

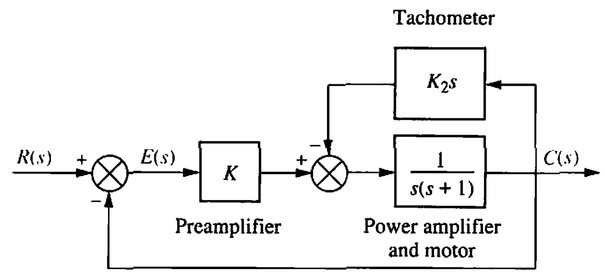


FIGURE P5.51 Position control

77. The mechanical system shown in Figure P5.52(a) is used as part of the unity feedback system shown in Figure P5.52(b). Find the values of M and D to yield 20% overshoot and 2 seconds settling time.

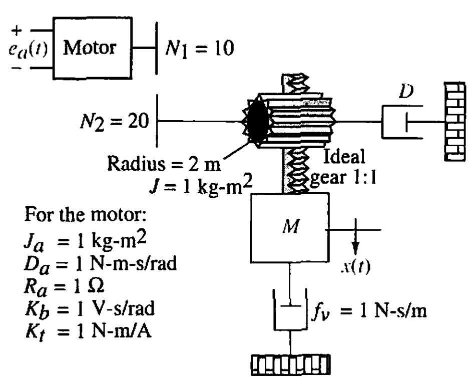
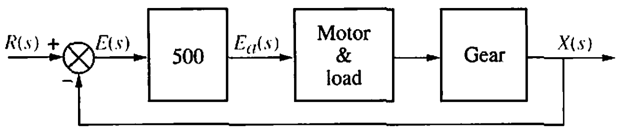


FIGURE P5.52 **a.** Motor and load; (*figure continues*)



(b)

FIGURE P5.52 (*Continued*) **b.** motor and load in feedback system

78. Assume ideal operational amplifiers in the circuit of Figure P5.53.
    a. Show that the leftmost operational amplifier works as a subtracting amplifier. Namely, $v_1 = v_o - v_{in}$.
    b. Draw a block diagram of the system, with the subtracting amplifier represented with a summing junction, and the circuit of the rightmost operational amplifier with a transfer function in the forward path. Keep R as a variable.
    c. Obtain the system's closed-loop transfer function.
    d. For a unit step input, obtain the value of R that will result in a settling time $T_s = 1$ msec.
    e. Using the value of R calculated in Part **d**, make a sketch of the resulting unit step response.

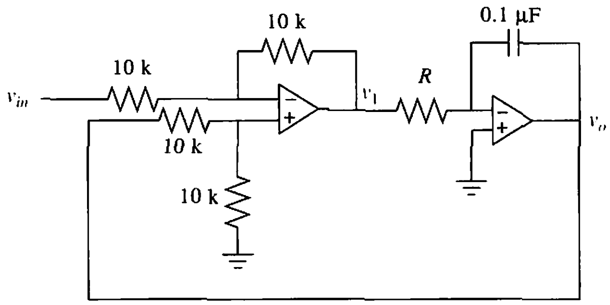


FIGURE P5.53

PROGRESSIVE ANALYSIS AND DESIGN PROBLEMS

79. **High-speed rail pantograph.** Problem 21 in Chapter 1 discusses the active control of a pantograph mechanism for high-speed rail systems. In this problem you found a functional block diagram relating the output force (*actual*) to the input force (*desired* output). In Problem 67, Chapter 2, you found the transfer function for the pantograph dynamics, that is, the transfer function relating the displacement of the spring that models the head to the applied force, or $G(s) = (Y_h(s) - Y_{cat}(s))/F_{up}(s)$ (*O'Connor, 1997*). We now create a pantograph active-control loop by adding the following components and following your functional block diagram found in Problem 21, Chapter 1: input transducer ($G_i(s) = 1/100$), controller ($G_c(s) = K$), actuator ($G_a(s) = 1/1000$), pantograph spring ($K_s = 82.3 \times 10^3$N/m), and sensor ($H_o(s) = 1/100$).
    a. Using the functional block diagram from your solution of Problem 21 in Chapter 1, and the pantograph dynamics, $G(s)$, found in Problem 67, Chapter 2, assemble a block diagram of the active pantograph control system.
    b. Find the closed-loop transfer function for the block diagram found in Part **a** if $K = 1000$.
    c. Represent the pantograph dynamics in phase-variable form and find a state-space representation for the closed-loop system if $K = 1000$). State Space SS

80. **Control of HIV/AIDS.** Given the HIV system of Problem 82 in Chapter 4 and repeated here for convenience (*Craig, 2004*): State Space SS

$$\begin{bmatrix} \dot{T} \\ \dot{T}^* \\ \dot{v} \end{bmatrix} = \begin{bmatrix} -0.04167 & 0 & -0.0058 \\ 0.0217 & -0.24 & 0.0058 \\ 0 & 100 & -2.4 \end{bmatrix} \begin{bmatrix} T \\ T^* \\ v \end{bmatrix} + \begin{bmatrix} 5.2 \\ -5.2 \\ 0 \end{bmatrix} u_1$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} T \\ T^* \\ v \end{bmatrix}$$

Express the system in the following forms:

a. Phase-variable form
b. Controller canonical form
c. Observer canonical form

Finally,

d. Use MATLAB to obtain the system's diagonalized representation. MATLAB ML

81. **Hybrid vehicle.** Figure P5.54 shows the block diagram of a possible cascade control scheme for an HEV driven by a dc motor (*Preitl, 2007*).

    Let the speed controller $G_{SC}(s) = 100 + \frac{40}{s}$, the torque controller and power amp $K_A G_{TC}(s) = 10 + \frac{6}{s}$, the current sensor sensitivity $K_{CS} = 0.5$, the speed sensor sensitivity $K_{SS} = 0.0433$. Also following the development in previous chapters $\frac{1}{R_a} = 1; \eta_{tot} K_t = 1.8; k_b = 2; D = k_f = 0.1; \frac{1}{J_{tot}} = \frac{1}{7.226}; \frac{r}{i_{tot}} = 0.0615$; and $\rho C_w A v_0 \frac{r}{i_{tot}} = 0.6154$.
    a. Substitute these values in the block diagram, and find the transfer function, $T(s) = V(s)/R_v(s)$, using block-diagram reduction rules. [Hint: Start

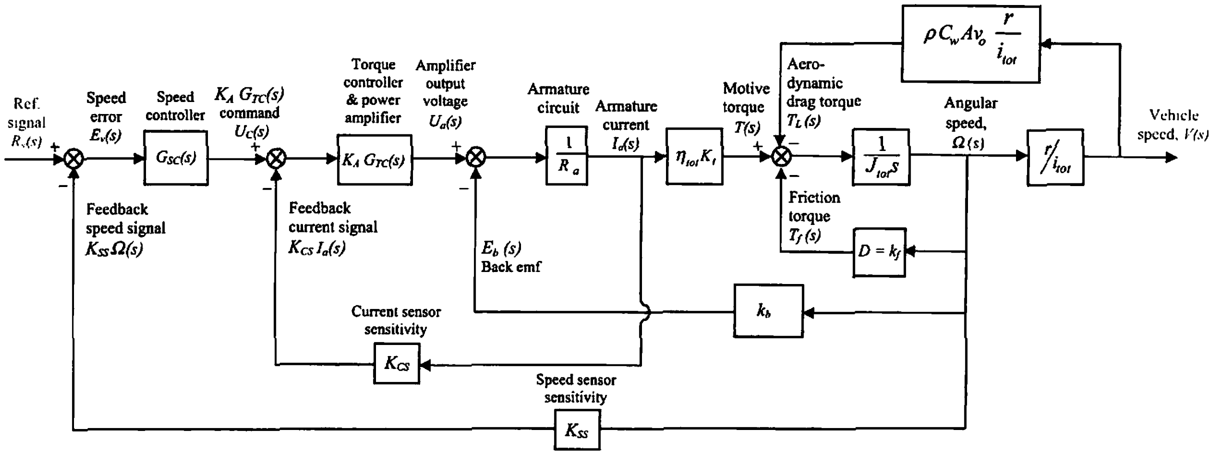


FIGURE P5.54

by moving the last $\frac{r}{i_{tot}}$ block to the right past the pickoff point.]

b. Develop a Simulink model for the original system in Figure P5.54. Set the reference signal input, $r_v(t) = 4\,u(t)$, as a step input with a zero initial value, a step time = 0 seconds, and a final value of 4 volts. Use X-Y graphs to display (over the period from 0 to 8 seconds) the response of the following variables to the step input: (1) change in car speed (m/s), (2) car acceleration (m/s^2), and (3) motor armature current (A).

Simulink SL MATLAB ML

To record the time and the above three variables (in array format), connect them to four **Workspace** sinks, each of which carry the respective variable name. After the simulation ends, utilize MATLAB plot commands to obtain and edit the three graphs of interest.

Cyber Exploration Laboratory

Experiment 5.1

Objectives To verify the equivalency of the basic forms, including cascade, parallel, and feedback forms. To verify the equivalency of the basic moves, including moving blocks past summing junctions, and moving blocks past pickoff points.

Minimum Required Software Packages MATLAB, Simulink, and the Control System Toolbox

Prelab

1. Find the equivalent transfer function of three cascaded blocks, $G_1(s) = \frac{1}{s+1}$, $G_2(s) = \frac{1}{s+4}$, and $G_3(s) = \frac{s+3}{s+5}$.
2. Find the equivalent transfer function of three parallel blocks, $G_1(s) = \frac{1}{s+4}$, $G_2(s) = \frac{1}{s+4}$, and $G_3(s) = \frac{s+3}{s+5}$.
3. Find the equivalent transfer function of the negative feedback system of Figure P5.55 if $G(s) = \frac{s+1}{s(s+2)}$, and $H(s) = \frac{s+3}{s+4}$.

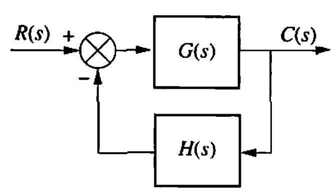


FIGURE P5.55

4. For the system of Prelab 3, push $H(s)$ to the left past the summing junction and draw the equivalent system.
5. For the system of Prelab 3, push $H(s)$ to the right past the pickoff point and draw the equivalent system.

Lab

1. Using Simulink, set up the cascade system of Prelab 1 and the equivalent single block. Make separate plots of the step response of the cascaded system and its equivalent single block. Record the values of settling time and rise time for each step response.
2. Using Simulink, set up the parallel system of Prelab 2 and the equivalent single block. Make separate plots of the step response of the parallel system and its equivalent single block. Record the values of settling time and rise time for each step response.
3. Using Simulink, set up the negative feedback system of Prelab 3 and the equivalent single block. Make separate plots of the step response of the negative feedback system and its equivalent single block. Record the values of settling time and rise time for each step response.
4. Using Simulink, set up the negative feedback systems of Prelabs 3, 4, and 5. Make separate plots of the step response of each of the systems. Record the values of settling time and rise time for each step response.

Postlab

1. Using your lab data, verify the equivalent transfer function of blocks in cascade.
2. Using your lab data, verify the equivalent transfer function of blocks in parallel.
3. Using your lab data, verify the equivalent transfer function of negative feedback systems.
4. Using your lab data, verify the moving of blocks past summing junctions and pickoff points.
5. Discuss your results. Were the equivalencies verified?

Experiment 5.2

Objective To use the various functions within LabVIEW's Control Design and Simulation Module to implement block diagram reduction.

Minimum Required Software Package LabVIEW with the Control Design Simulation Module

Prelab Given the block diagram from Example 5.2, replace $G_1, G_2, G_3, H_1, H_2, H_3$ with the following transfer functions and obtain an equivalent transfer function.

$$G_1 = \frac{1}{s+10}; G_2 = \frac{1}{s+1}; G_3 = \frac{s+1}{s^2+4s+4}; H_1 = \frac{s+1}{s+2}; H_2 = 2; H_3 = 1$$

Lab Use LabVIEW to implement the block diagram from Example 5.2 using the transfer functions given in the Prelab.

Postlab Verify your calculations from the Prelab with that of the equivalent transfer function obtained with LabVIEW.

Experiment 5.3

Objective To use the various functions within LabVIEW's Control Design and Simulation Module and the Mathematics/Polynomial palette to implement Mason's rule for block diagram reduction.

Minimum Required Software Package LabVIEW with Control Design and Simulation Module, Math Script RT Module, and the Mathematics/Polynomial palette.

Prelab Given the block diagram created in the Prelab of Cyber Exploration Laboratory 5.2, use Mason's rule to obtain an equivalent transfer function.

Lab Use LabVIEW's Control Design and Simulation Module as well as the Mathematics/Polynomial functions to implement block diagram reduction using Mason's rule.

Postlab Verify your calculations from the Prelab with that of the equivalent transfer function obtained with LabVIEW.

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