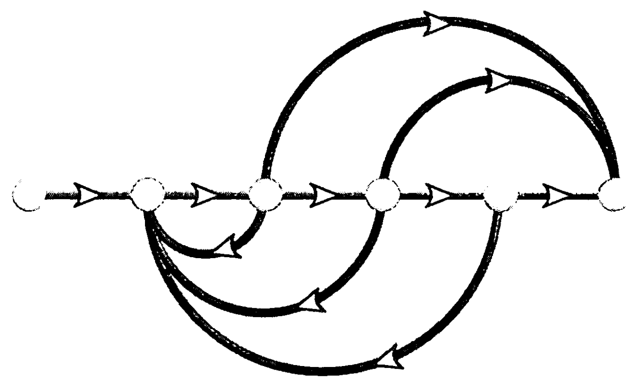
Design via Frequency Response 11



Chapter Learning Outcomes

After completing this chapter the student will be able to:

- Use frequency response techniques to adjust the gain to meet a transient response specification (Sections 11.1–11.2)
- Use frequency response techniques to design cascade compensators to improve the steady-state error (Section 11.3)
- Use frequency response techniques to design cascade compensators to improve the transient response (Section 11.4)
- Use frequency response techniques to design cascade compensators to improve both the steady-state error and the transient response (Section 11.5)

Case Study Learning Outcomes

You will be able to demonstrate your knowledge of the chapter objectives with case studies as follows:

- Given the antenna azimuth position control system shown on the front endpapers, you will be able to use frequency response techniques to design the gain to meet a transient response specification.
- Given the antenna azimuth position control system shown on the front endpapers, you will be able to use frequency response techniques to design a cascade compensator to meet both transient and steady-state error specifications.

11.1 Introduction

In Chapter 8, we designed the transient response of a control system by adjusting the gain along the root locus. The design process consisted of finding the transient response specification on the root locus, setting the gain accordingly, and settling for the resulting steady-state error. The disadvantage of design by gain adjustment is that only the transient response and steady-state error represented by points along the root locus are available.

In order to meet transient response specifications represented by points not on the root locus and, independently, steady-state error requirements, we designed cascade compensators in Chapter 9. In this chapter, we use Bode plots to parallel the root locus design process from Chapters 8 and 9.

Let us begin by drawing some general comparisons between root locus and frequency response design.

Stability and transient response design via gain adjustment. Frequency response design methods, unlike root locus methods, can be implemented conveniently without a computer or other tool except for testing the design. We can easily draw Bode plots using asymptotic approximations and read the gain from the plots. Root locus requires repeated trials to find the desired design point from which the gain can be obtained. For example, in designing gain to meet a percent overshoot requirement, root locus requires the search of a radial line for the point where the open-loop transfer function yields an angle of 180°. To evaluate the range of gain for stability, root locus requires a search of the $j\omega$-axis for 180°. Of course, if one uses a computer program, such as MATLAB, the computational disadvantage of root locus vanishes.

Transient response design via cascade compensation. Frequency response methods are not as intuitive as the root locus, and it is something of an art to design cascade compensation with the methods of this chapter. With root locus, we can identify a specific point as having a desired transient response characteristic. We can then design cascade compensation to operate at that point and meet the transient response specifications. In Chapter 10, we learned that phase margin is related to percent overshoot (Eq. (10.73)) and bandwidth is related to both damping ratio and settling time or peak time (Eqs. (10.55) and (10.56)). These equations are rather complicated. When we design cascade compensation using frequency response methods to improve the transient response, we strive to reshape the open-loop transfer function's frequency response to meet both the phase-margin requirement (percent overshoot) and the bandwidth requirement (settling or peak time). There is no easy way to relate all the requirements prior to the reshaping task. Thus, the reshaping of the open-loop transfer function's frequency response can lead to several trials until all transient response requirements are met.

Steady-state error design via cascade compensation. An advantage of using frequency design techniques is the ability to design derivative compensation, such as lead compensation, to speed up the system and at the same time build in a desired steady-state error requirement that can be met by the lead compensator alone. Recall that in using root locus there are an infinite number of possible solutions to the design of a lead compensator. One of the differences between these solutions is the steady-state error. We must make numerous tries to arrive at the solution that yields the required steady-state error performance. With frequency response techniques, we build the steady-state error requirement right into the design of the lead compensator.

You are encouraged to reflect on the advantages and disadvantages of root locus and frequency response techniques as you progress through this chapter. Let us take a closer look at frequency response design.

When designing via frequency response methods, we use the concepts of stability, transient response, and steady-state error that we learned in Chapter 10. First, the Nyquist criterion tells us how to determine if a system is stable. Typically, an open-loop stable system is stable in closed-loop if the open-loop magnitude frequency response has a gain of less than 0 dB at the frequency where the phase frequency response is 180°. Second, percent overshoot is reduced by increasing the phase margin, and the speed of the response is increased by increasing the bandwidth. Finally, steady-state error is improved by increasing the low-frequency magnitude responses, even if the high-frequency magnitude response is attenuated.

These, then, are the basic facts underlying our design for stability, transient response, and steady-state error using frequency response methods, where the Nyquist criterion and the Nyquist diagram compose the underlying theory behind the design process. Thus, even though we use the Bode plots for ease in obtaining the frequency response, the design process can be verified with the Nyquist diagram when questions arise about interpreting the Bode plots. In particular, when the structure of the system is changed with additional compensator poles and zeros, the Nyquist diagram can offer a valuable perspective.

The emphasis in this chapter is on the design of lag, lead, and lag-lead compensation. General design concepts are presented first, followed by step-by-step procedures. These procedures are only suggestions, and you are encouraged to develop other procedures to arrive at the same goals. Although the concepts in general apply to the design of PI, PD, and PID controllers, in the interest of brevity, detailed procedures and examples will not be presented. You are encouraged to extrapolate the concepts and designs covered and apply them to problems involving PI, PD, and PID compensation presented at the end of this chapter. Finally, the compensators developed in this chapter can be implemented with the realizations discussed in Section 9.6.

11.2 Transient Response via Gain Adjustment

Let us begin our discussion of design via frequency response methods by discussing the link between phase margin, transient response, and gain. In Section 10.10, the relationship between damping ratio (equivalently percent overshoot) and phase margin was derived for $G(s) = \omega_n^2/s(s + 2\zeta\omega_n)$. Thus, if we can vary the phase margin, we can vary the percent overshoot. Looking at Figure 11.1, we see that if we desire a phase margin, Φ_M, represented by CD, we would have to raise the magnitude curve by AB. Thus, a simple gain adjustment can be used to design phase margin and, hence, percent overshoot.

We now outline a procedure by which we can determine the gain to meet a percent overshoot requirement using the open-loop frequency response and assuming dominant second-order closed-loop poles.

Design Procedure

1. Draw the Bode magnitude and phase plots for a convenient value of gain.
2. Using Eqs. (4.39) and (10.73), determine the required phase margin from the percent overshoot.

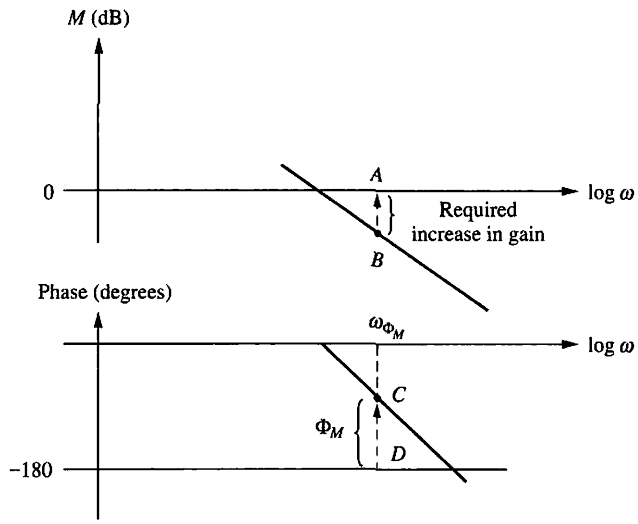


FIGURE 11.1 Bode plots showing gain adjustment for a desired phase margin

3. Find the frequency, ω_{Φ_M}, on the Bode phase diagram that yields the desired phase margin, *CD*, as shown on Figure 11.1.
4. Change the gain by an amount AB to force the magnitude curve to go through 0 dB at ω_{Φ_M}. The amount of gain adjustment is the additional gain needed to produce the required phase margin.

We now look at an example of designing the gain of a third-order system for percent overshoot.

Example 11.1

Transient Response Design via Gain Adjustment

Design D

PROBLEM: For the position control system shown in Figure 11.2, find the value of preamplifier gain, *K*, to yield a 9.5% overshoot in the transient response for a step input. Use only frequency response methods.

SOLUTION: We will now follow the previously described gain adjustment design procedure.

1. Choose $K = 3.6$ to start the magnitude plot at 0 dB at $\omega = 0.1$ in Figure 11.3.
2. Using Eq. (4.39), a 9.5% overshoot implies $\zeta = 0.6$ for the closed-loop dominant poles. Equation (10.73) yields a 59.2° phase margin for a damping ratio of 0.6.

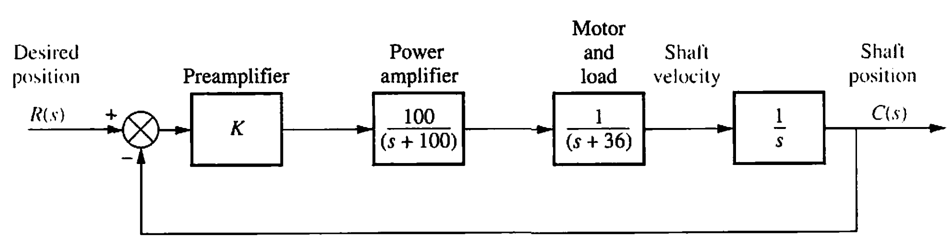


FIGURE 11.2 System for Example 11.1

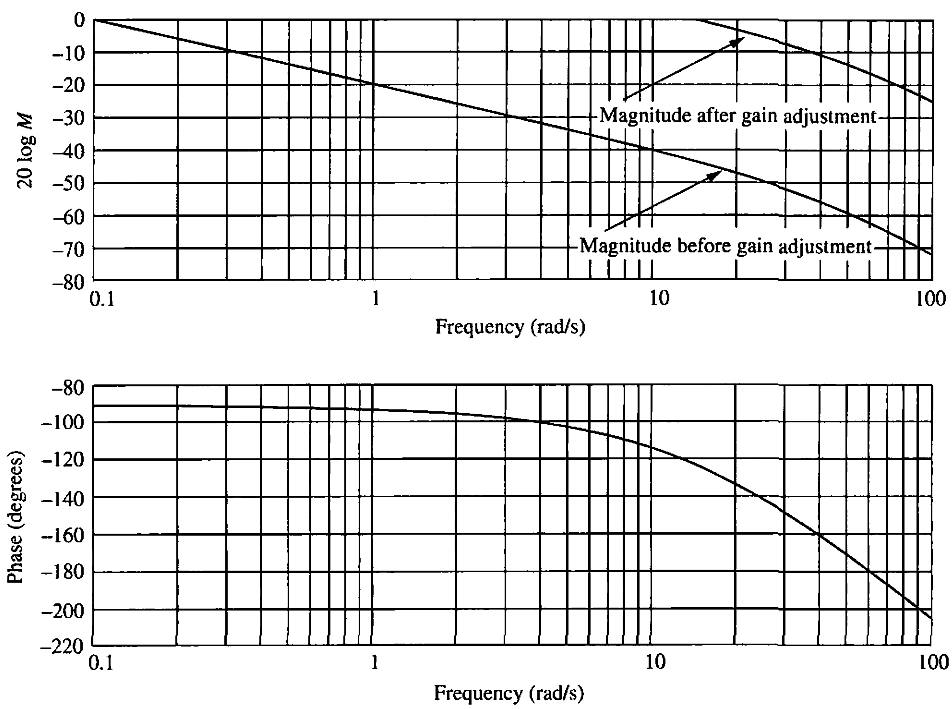


FIGURE 11.3 Bode magnitude and phase plots for Example 11.1

3. Locate on the phase plot the frequency that yields a 59.2° phase margin. This frequency is found where the phase angle is the difference between −180° and 59.2°, or −120.8°. The value of the phase-margin frequency is 14.8 rad/s.
4. At a frequency of 14.8 rad/s on the magnitude plot, the gain is found to be −44.2 dB. This magnitude has to be raised to 0 dB to yield the required phase margin. Since the log-magnitude plot was drawn for $K = 3.6$, a 44.2 dB increase, or $K = 3.6 \times 162.2 = 583.9$, would yield the required phase margin for 9.48% overshoot.

   The gain-adjusted open-loop transfer function is

$$G(s) = \frac{58{,}390}{s(s+36)(s+100)} \tag{11.1}$$

Table 11.1 summarizes a computer simulation of the gain-compensated system.

TABLE 11.1 Characteristic of gain-compensated system of Example 11.1

Parameter	Proposed specification	Actual value
K_v	–	16.22
Phase margin	59.2°	59.2°
Phase-margin frequency	–	14.8 rad/s
Percent overshoot	9.5	10
Peak time	–	0.18 second

MATLAB ML

Students who are using MATLAB should now run ch11p1 in Appendix B. You will learn how to use MATLAB to design a gain to meet a percent overshoot specification using Bode plots. This exercise solves Example 11.1 using MATLAB.

Skill-Assessment Exercise 11.1

WileyPLUS
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Control Solutions

PROBLEM: For a unity feedback system with a forward transfer function

$$G(s) = \frac{K}{s(s+50)(s+120)}$$

use frequency response techniques to find the value of gain, K, to yield a closed-loop step response with 20% overshoot.

ANSWER: $K = 194{,}200$

The complete solution is located at www.wiley.com/college/nise.

TryIt 11.1

Use MATLAB, the Control System Toolbox, and the following statements to solve Skill-Assessment Exercise 11.1.

```
pos=20
z=(-log(pos/100))/...
 (sqrt(pi^2+...
 log(pos/100)^2))
Pm=atan(2*z/...
 (sqrt(-2*z^2+...
 sqrt(1+4*z^4))))*...
 (180/pi)
G=zpk([],...
 [0.-50,-120],1)
sisotool
```

In the SISOTOOL Window:

1. Select **Import** . . . in the **File** menu.
2. Click on **G** in the **System Data Window** and click **Browse** . . .
3. In the **Model Import Window** select radio button Workspace and select **G** in **Available Models**. Click Import, then Close.
4. Click Ok in the **System Data Window.**
5. Right-click in the Bode graph area and be sure all selections under **Show** are checked.
6. Grab the stability margin point in the magnitude diagram and raise the magnitude curve until the phase curve shows the phase margin calculated by the program and shown in the MATLAB **Command Window** as Pm.
7. Right-click in the Bode plot area, select **Edit Compensator** . . . and read the gain under **Compensator** in the resulting window.

In this section, we paralleled our work in Chapter 8 with a discussion of transient response design through gain adjustment. In the next three sections, we parallel the root locus compensator design in Chapter 9 and discuss the design of lag, lead, and lag-lead compensation via Bode diagrams.

11.3 Lag Compensation

In Chapter 9, we used the root locus to design lag networks and PI controllers. Recall that these compensators permitted us to design for steady-state error without *appreciably affecting the transient response. In this section, we provide a parallel* development using the Bode diagrams.

Visualizing Lag Compensation

The function of the lag compensator as seen on Bode diagrams is to (1) improve the static error constant by increasing only the low-frequency gain without any resulting instability, and (2) increase the phase margin of the system to yield the desired transient response. These concepts are illustrated in Figure 11.4.

The uncompensated system is unstable since the gain at 180° is greater than 0 dB. The lag compensator, while not changing the low-frequency gain, does reduce

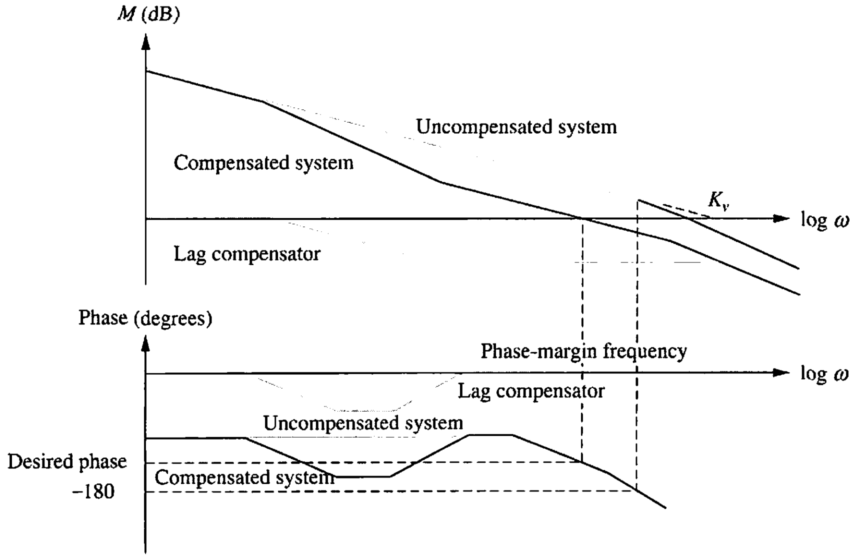


FIGURE 11.4 Visualizing lag compensation

the high-frequency gain.[1] Thus, the low-frequency gain of the system can be made high to yield a large K_v without creating instability. This stabilizing effect of the lag network comes about because the gain at 180° of phase is reduced below 0 dB. Through judicious design, the magnitude curve can be reshaped, as shown in Figure 11.4, to go through 0 dB at the desired phase margin. Thus, both K_v and the desired transient response can be obtained. We now enumerate a design procedure.

Design Procedure

1. Set the gain, K, to the value that satisfies the steady-state error specification and plot the Bode magnitude and phase diagrams for this value of gain.
2. Find the frequency where the phase margin is 5° to 12° greater than the phase margin that yields the desired transient response (*Ogata, 1990*). This step compensates for the fact that the phase of the lag compensator may still contribute anywhere from −5° to − 12° of phase at the phase-margin frequency.
3. Select a lag compensator whose magnitude response yields a composite Bode magnitude diagram that goes through 0 dB at the frequency found in Step 2 as follows: Draw the compensator's high-frequency asymptote to yield 0 dB for the compensated system at the frequency found in Step 2. Thus, if the gain at the frequency found in Step 2 is 20 log K_{PM}, then the compensator's high-frequency asymptote will be set at $-20 \log K_{PM}$; select the upper break frequency to be 1 decade below the frequency found in Step 2;[2] select the low-frequency asymptote to be at 0 dB; connect the compensator's high- and low-frequency asymptotes with a −20 dB/decade line to locate the lower break frequency.
4. Reset the system gain, K, to compensate for any attenuation in the lag network in order to keep the static error constant the same as that found in Step 1.

[1] The name *lag compensator* comes from the fact that the typical phase angle response for the compensator, as shown in Figure 11.4, is always negative, or *lagging* in phase angle.

[2] This value of break frequency ensures that there will be only −5° to − 12° phase contribution from the compensator at the frequency found in Step 2.

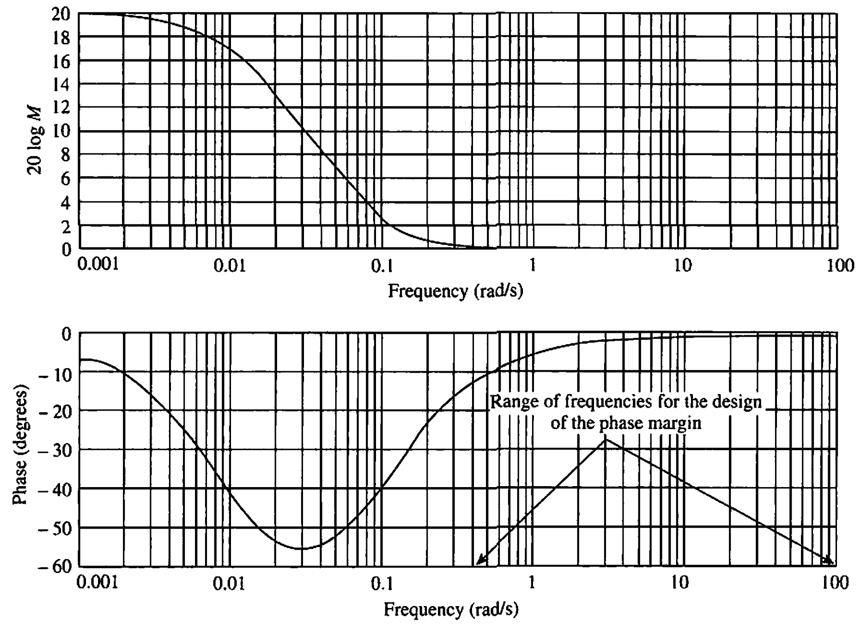


FIGURE 11.5 Frequency response plots of a lag compensator, $G_c(s) = (s + 0.1)/(s + 0.01)$

From these steps, you see that we are relying upon the initial gain setting to meet the steady-state requirements and then relying upon the lag compensator's −20 dB/decade slope to meet the transient response requirement by setting the 0 dB crossing of the magnitude plot.

The transfer function of the lag compensator is

$$G_c(s) = \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} \tag{11.2}$$

where $\alpha > 1$.

Figure 11.5 shows the frequency response curves for the lag compensator. The range of high frequencies shown in the phase plot is where we will design our phase margin. This region is after the second break frequency of the lag compensator, where we can rely on the attenuation characteristics of the lag network to reduce the total open-loop gain to unity at the phase-margin frequency. Further, in this region the phase response of the compensator will have minimal effect on our design of the phase margin. Since there is still some effect, approximately 5° to 12°, we will add this amount to our phase margin to compensate for the phase response of the lag compensator (see Step 2).

Example 11.2

Lag Compensation Design

Design
D

PROBLEM: Given the system of Figure 11.2, use Bode diagrams to design a lag compensator to yield a tenfold improvement in steady-state error over the gain-compensated system while keeping the percent overshoot at 9.5%.

SOLUTION: We will follow the previously described lag compensation design procedure.

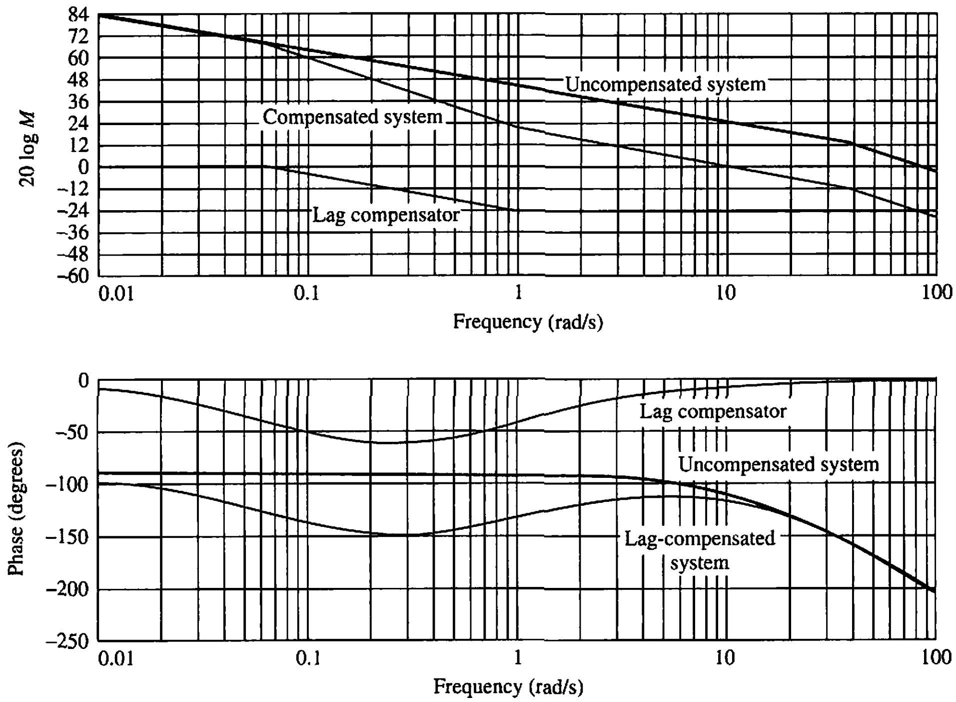


FIGURE 11.6 Bode plots for Example 11.2.

1. From Example 11.1 a gain, K, of 583.9 yields a 9.5% overshoot. Thus, for this system, $K_v = 16.22$. For a tenfold improvement in steady-state error, K_v must increase by a factor of 10, or $K_v = 162.2$. Therefore, the value of K in Figure 11.2 equals 5839, and the open-loop transfer function is

$$G(s) = \frac{583{,}900}{s(s+36)(s+100)} \tag{11.3}$$

The Bode plots for $K = 5839$ are shown in Figure 11.6.

2. The phase margin required for a 9.5% overshoot ($\zeta = 0.6$) is found from Eq. (10.73) to be 59.2°. We increase this value of phase margin by 10° to 69.2° in order to compensate for the phase angle contribution of the lag compensator. Now find the frequency where the phase margin is 69.2°. This frequency occurs at a phase angle of $-180° + 69.2° = -110.8°$ and is 9.8 rad/s. At this frequency, the magnitude plot must go through 0 dB. The magnitude at 9.8 rad/s is now +24 dB (exact, that is, nonasymptotic). Thus, the lag compensator must provide −24 dB attenuation at 9.8 rad/s.

3.&4. We now design the compensator. First draw the high-frequency asymptote at −24 dB. Arbitrarily select the higher break frequency to be about one decade below the phase-margin frequency, or 0.98 rad/s. Starting at the intersection of this frequency with the lag compensator's high-frequency asymptote, draw a −20 dB/decade line until 0 dB is reached. The compensator must have a dc gain of unity to retain the value of K_v that we have already designed by setting $K = 5839$. The lower break frequency is found to be 0.062 rad/s. Hence, the lag compensator's transfer function is

$$G_c(s) = \frac{0.063(s+0.98)}{(s+0.062)} \tag{11.4}$$

where the gain of the compensator is 0.063 to yield a dc gain of unity.

The compensated system's forward transfer function is thus

$$G(s)G_c(s) = \frac{36{,}786(s+0.98)}{s(s+36)(s+100)(s+0.062)} \tag{11.5}$$

The characteristics of the compensated system, found from a simulation and exact frequency response plots, are summarized in Table 11.2.

TABLE 11.2 Characteristics of the lag-compensated system of Example 11.2

Parameter	Proposed specification	Actual value
K_v	162.2	161.5
Phase margin	59.2°	62°
Phase-margin frequency	—	11 rad/s
Percent overshoot	9.5	10
Peak time	—	0.25 second

MATLAB

ML

Students who are using MATLAB should now run ch11p2 in Appendix B. You will learn how to use MATLAB to design a lag compensator. You will enter the value of gain to meet the steady-state error requirement as well as the desired percent overshoot. MATLAB then designs a lag compensator using Bode plots, evaluates K_v, and generates a closed-loop step response. This exercise solves Example 11.2 using MATLAB.

Skill-Assessment Exercise 11.2

PROBLEM: Design a lag compensator for the system in Skill-Assessment Exercise 11.1 that will improve the steady-state error tenfold, while still operating with 20% overshoot.

ANSWER:

$$G_{lag}(s) = \frac{0.0691(s+2.04)}{(s+0.141)}; \quad G(s) = \frac{1{,}942{,}000}{s(s+50)(s+120)}$$

The complete solution is at www.wiley.com/college/nise.

TryIt 11.2

Use MATLAB, the Control System Toolbox, and the following statements to solve Skill-Assessment Exercise 11.2.

```
pos=20
Ts=0.2
z=(-log(pos/100))/(sqrt(pi^2+log(pos/100)^2))
Pm=atan(2*z/(sqrt(-2*z^2+sqrt(1+4*z^4))))*(180/pi)
Wbw=(4/(Ts*z))*sqrt((1-2*z^2)+sqrt(4*z^4-4*z^2+2))
K=1942000
G=zpk([],[0,-50,-120],K)
sisotool(G,1)
```

(TryIt continues)

(*TryIt Continued*)
When the **SISO Design for SISO Design Task Window** appears:

1. Right-click on the Bode plot area and select **Grid.**
2. Note the phase margin shown in the MATLAB **Command Window.**
3. Using the Bode phase plot, estimate the frequency at which the phase margin from Step 2 occurs.
4. On the **SISO Design for SISO Design Task Window toolbar**, click on the red zero.
5. Place the zero of the compensator by clicking on the gain plot at a frequency that is 1/10 that found in Step 3.
6. On the **SISO Design for SISO Design Task Window toolbar**, click on the red pole.
7. Place the pole of the compensator by clicking on the gain plot to the left of the compensator zero.
8. Grab the pole with the mouse and move it until the phase plot shows a P.M. equal to that found in Step 2.
9. Right-click in the Bode plot area and select **Edit Compensator...**
10. Read the lag compensator in the **Control and Estimation Tools Manager Window.**

In this section, we showed how to design a lag compensator to improve the steady-state error while keeping the transient response relatively unaffected. We next discuss how to improve the transient response using frequency response methods.

11.4 Lead Compensation

For second-order systems, we derived the relationship between phase margin and percent overshoot as well as the relationship between closed-loop bandwidth and other time-domain specifications, such as settling time, peak time, and rise time. When we designed the lag network to improve the steady-state error, we wanted a minimal effect on the phase diagram in order to yield an imperceptible change in the transient response. However, in designing lead compensators via Bode plots, we want to change the phase diagram, increasing the phase margin to reduce the percent overshoot, and increasing the gain crossover to realize a faster transient response.

Visualizing Lead Compensation

The lead compensator increases the bandwidth by increasing the gain crossover frequency. At the same time, the phase diagram is raised at higher frequencies. The result is a larger phase margin and a higher phase-margin frequency. In the time domain, lower percent overshoots (larger phase margins) with smaller peak times (higher phase-margin frequencies) are the results. The concepts are shown in Figure 11.7.

The uncompensated system has a small phase margin (B) and a low phase-margin frequency (A). Using a phase lead compensator, the phase angle plot (compensated system) is raised for higher frequencies.[3] At the same time, the gain crossover frequency in the magnitude plot is increased from A rad/s to C rad/s. These effects yield a larger phase margin (D), a higher phase-margin frequency (C), and a larger bandwidth.

One advantage of the frequency response technique over the root locus is that we can implement a steady-state error requirement and then design a transient response. This specification of transient response with the constraint of a steady-state error is easier to implement with the frequency response technique than with the root locus. Notice that the initial slope, which determines the steady-state error, is not affected by the design for the transient response.

[3] The name *lead compensator* comes from the fact that the typical phase angle response shown in Figure 11.7 is always positive, or *leading* in phase angle.

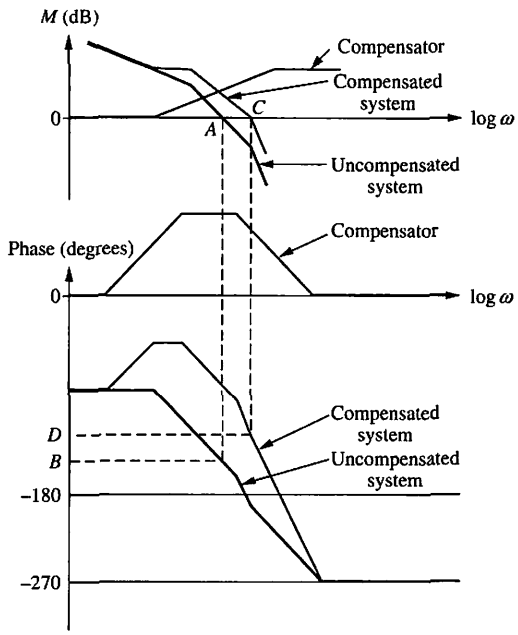


FIGURE 11.7 Visualizing lead compensation

Lead Compensator Frequency Response

Let us first look at the frequency response characteristics of a lead network and derive some valuable relationships that will help us in the design process. Figure 11.8 shows plots of the lead network

$$\boxed{G_c(s) = \frac{1}{\beta}\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}} \tag{11.6}$$

for various values of β, where $\beta < 1$. Notice that the peaks of the phase curve vary in maximum angle and in the frequency at which the maximum occurs. The dc gain of the compensator is set to unity with the coefficient $1/\beta$, in order not to change the dc gain designed for the static error constant when the compensator is inserted into the system.

In order to design a lead compensator and change both the phase margin and phase-margin frequency, it is helpful to have an analytical expression for the maximum value of phase and the frequency at which the maximum value of phase occurs, as shown in Figure 11.8.

From Eq. (11.6) the phase angle of the lead compensator, ϕ_c, is

$$\phi_c = \tan^{-1}\omega T - \tan^{-1}\omega\beta T \tag{11.7}$$

Differentiating with respect to ω, we obtain

$$\frac{d\phi_c}{d\omega} = \frac{T}{1+(\omega T)^2} - \frac{\beta T}{1+(\omega\beta T)^2} \tag{11.8}$$

Setting Eq. (11.8) equal to zero, we find that the frequency, ω_{max}, at which the maximum phase angle, ϕ_{max}, occurs is

$$\boxed{\omega_{max} = \frac{1}{T\sqrt{\beta}}} \tag{11.9}$$

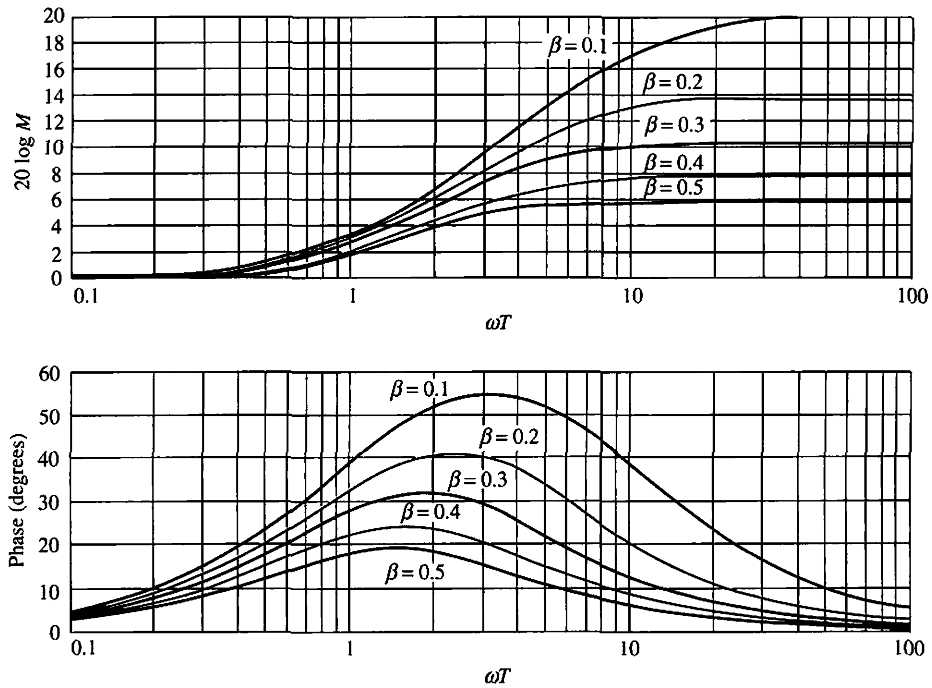


FIGURE 11.8 Frequency response of a lead compensator, $G_c(s) = [1/\beta][(s + 1/T)/(s + 1/\beta T)]$

Substituting Eq. (11.9) into Eq. (11.6) with $s = j\omega_{\max}$,

$$G_c(j\omega_{\max}) = \frac{1}{\beta}\frac{j\omega_{\max} + \dfrac{1}{T}}{j\omega_{\max} + \dfrac{1}{\beta T}} = \frac{j\dfrac{1}{\sqrt{\beta}} + 1}{j\sqrt{\beta} + 1} \tag{11.10}$$

Making use of $\tan(\phi_1 - \phi_2) = (\tan\phi_1 - \tan\phi_2)/(1 + \tan\phi_1 \tan\phi_2)$, the maximum phase shift of the compensator, $\phi_{\max}$, is

$$\boxed{\phi_{\max} = \tan^{-1}\frac{1-\beta}{2\sqrt{\beta}} = \sin^{-1}\frac{1-\beta}{1+\beta}} \tag{11.11}$$

and the compensator's magnitude at $\omega_{\max}$ is

$$\boxed{|G_c(j\omega_{\max})| = \frac{1}{\sqrt{\beta}}} \tag{11.12}$$

We are now ready to enumerate a design procedure.

Design Procedure

1. Find the closed-loop bandwidth required to meet the settling time, peak time, or rise time requirement (see Eqs. (10.54) through (10.56)).
2. Since the lead compensator has negligible effect at low frequencies, set the gain, K, of the uncompensated system to the value that satisfies the steady-state error requirement.

3. Plot the Bode magnitude and phase diagrams for this value of gain and determine the uncompensated system's phase margin.
4. Find the phase margin to meet the damping ratio or percent overshoot requirement. Then evaluate the additional phase contribution required from the compensator.[4]
5. Determine the value of β (see Eqs. (11.6) and (11.11)) from the lead compensator's required phase contribution.
6. Determine the compensator's magnitude at the peak of the phase curve (Eq. (11.12)).
7. Determine the new phase-margin frequency by finding where the uncompensated system's magnitude curve is the negative of the lead compensator's magnitude at the peak of the compensator's phase curve.
8. Design the lead compensator's break frequencies, using Eqs. (11.6) and (11.9) to find T and the break frequencies.
9. Reset the system gain to compensate for the lead compensator's gain.
10. Check the bandwidth to be sure the speed requirement in Step 1 has been met.
11. Simulate to be sure all requirements are met.
12. Redesign if necessary to meet requirements.

From these steps, we see that we are increasing both the amount of phase margin (improving percent overshoot) and the gain crossover frequency (increasing the speed). Now that we have enumerated a procedure with which we can design a lead compensator to improve the transient response, let us demonstrate.

Example 11.3

Design D

Lead Compensation Design

PROBLEM: Given the system of Figure 11.2, design a lead compensator to yield a 20% overshoot and $K_v = 40$, with a peak time of 0.1 second.

SOLUTION: The uncompensated system is $G(s) = 100K/[s(s+36)(s+100)]$. We will follow the outlined procedure.

1. We first look at the closed-loop bandwidth needed to meet the speed requirement imposed by $T_p = 0.1$ second. From Eq. (10.56), with $T_p = 0.1$ second and $\zeta = 0.456$ (i.e., 20% overshoot), a closed-loop bandwidth of 46.6 rad/s is required.
2. In order to meet the specification of $K_v = 40$, K must be set at 1440, yielding $G(s) = 144{,}000/[s(s+36)(s+100)]$.
3. The uncompensated system's frequency response plots for $K = 1440$ are shown in Figure 11.9.
4. A 20% overshoot implies a phase margin of 48.1°. The uncompensated system with $K = 1440$ has a phase margin of 34° at a phase-margin frequency

[4] We know that the phase-margin frequency will be increased after the insertion of the compensator. At this new phase-margin frequency, the system's phase will be smaller than originally estimated, as seen by comparing points B and D in Figure 11.7. Hence, an additional phase should be added to that provided by the lead compensator to correct for the phase reduction caused by the original system.

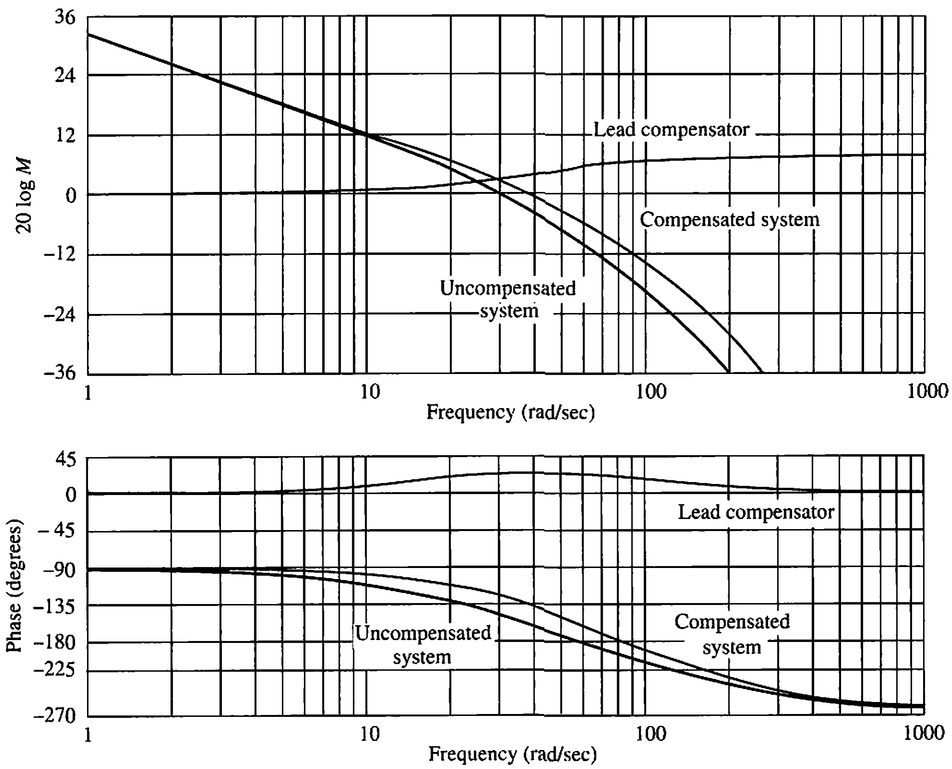


FIGURE 11.9 Bode plots for lead compensation in Example 11.3

of 29.6. To increase the phase margin, we insert a lead network that adds enough phase to yield a 48.1° phase margin. Since we know that the lead network will also increase the phase-margin frequency, we add a correction factor to compensate for the lower uncompensated system's phase angle at this higher phase-margin frequency. Since we do not know the higher phase-margin frequency, we assume a correction factor of 10°. Thus, the total phase contribution required from the compensator is $48.1° - 34° + 10° = 24.1°$. In summary, our compensated system should have a phase margin of 48.1° with a bandwidth of 46.6 rad/s. If the system's characteristics are not acceptable after the design, then a redesign with a different correction factor may be necessary.

5. Using Eq. (11.11), $\beta = 0.42$ for $\phi_{\max} = 24.1°$.
6. From Eq. (11.12), the lead compensator's magnitude is 3.76 dB at $\omega_{\max}$.
7. If we select $\omega_{\max}$ to be the new phase-margin frequency, the uncompensated system's magnitude at this frequency must be −3.76 dB to yield a 0 dB crossover at $\omega_{\max}$ for the compensated system. The uncompensated system passes through −3.76 dB at $\omega_{max} = 39$ rad/s. This frequency is thus the new phase-margin frequency.
8. We now find the lead compensator's break frequencies. From Eq. (11.9), $1/T = 25.3$ and $1/\beta T = 60.2$.
9. Hence, the compensator is given by

$$G_c(s) = \frac{1}{\beta}\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}} = 2.38\frac{s+25.3}{s+60.2} \tag{11.13}$$

where 2.38 is the gain required to keep the dc gain of the compensator at unity so that $K_v = 40$ after the compensator is inserted.

The final, compensated open-loop transfer function is then

$$G_c(s)G(s) = \frac{342{,}600(s+25.3)}{s(s+36)(s+100)(s+60.2)} \tag{11.14}$$

10. From Figure 11.9, the lead-compensated open-loop magnitude response is -7 dB at approximately 68.8 rad/s. Thus, we estimate the closed-loop bandwidth to be 68.8 rad/s. Since this bandwidth exceeds the requirement of 46.6 rad/s, we assume the peak time specification is met. This conclusion about the peak time is based upon a second-order and asymptotic approximation that will be checked via simulation.
11. Figure 11.9 summarizes the design and shows the effect of the compensation. Final results, obtained from a simulation and the actual (nonasymptotic) frequency response, are shown in Table 11.3. Notice the increase in phase margin, phase-margin frequency, and closed-loop bandwidth after the lead compensator was added to the gain-adjusted system. The peak time and the steady-state error requirements have been met, although the phase margin is less than that proposed and the percent overshoot is 2.6% larger than proposed. Finally, if the performance is not acceptable, a redesign is necessary.

TABLE 11.3 Characteristic of the lead-compensated system of Example 11.3

Parameter	Proposed specification	Actual gain-compensated value	Actual lead-compensated value
K_v	40	40	40
Phase margin	48.1°	34°	45.5°
Phase-margin frequency	—	29.6 rad/s	39 rad/s
Closed-loop bandwidth	46.6 rad/s	50 rad/s	68.8 rad/s
Percent overshoot	20	37	22.6
Peak time	0.1 second	0.1 second	0.075 second

MATLAB

ML

Students who are using MATLAB should now run ch11p3 in Appendix B. You will learn how to use MATLAB to design a lead compensator. You will enter the desired percent overshoot, peak time, and K_v. MATLAB then designs a lead compensator using Bode plots, evaluates K_v, and generates a closed-loop step response. This exercise solves Example 11.3 using MATLAB.

Skill-Assessment Exercise 11.3

WileyPLUS

WPCS

Control Solutions

PROBLEM: Design a lead compensator for the system in Skill-Assessment Exercise 11.1 to meet the following specifications: $\%OS = 20\%$, $T_s = 0.2$ s and $K_v = 50$.

ANSWER: $G_{\text{lead}}(s) = \dfrac{2.27(s+33.2)}{(s+75.4)}$; $G(s) = \dfrac{300{,}000}{s(s+50)(s+120)}$

The complete solution is at www.wiley.com/college/nise.

TryIt 11.3

Use MATLAB, the Control System Toolbox, and the following statements to solve Skill-Assessment Exercise 11.3.

```
pos=20
Ts=0.2
z=(-log(pos/100))/(sqrt(pi^2+log(pos/100)^2))
Pm=atan(2*z/(sqrt(-2*z^2+sqrt(1+4*z^4))))*(180/pi)
Wbw=(4/(Ts*z))*sqrt((1-2*z^2)+sqrt(4*z^4-4*z^2+2))
K=50*50*120
G=zpk([],[0,-50,-120],K)
sisotool(G,1)
```

When the **SISO Design for SISO Design Task Window** appears:

1. Right-click on the Bode plot area and select **Grid.**
2. Note the phase margin and bandwidth shown in the MATLAB **Command Window.**
3. On the **SISO Design for SISO Design Task Window toolbar,** click on the red pole.
4. Place the pole of the compensator by clicking on the gain plot at a frequency that is to the right of the desired bandwidth found in Step 2.
5. On the **SISO Design for SISO Design Task Window toolbar,** click on the red zero.
6. Place the zero of the compensator by clicking on the gain plot to the left of the desired bandwidth.
7. Reshape the Bode plots: alternately grab the pole and the zero with the mouse and alternately move them along the phase plot until the phase plot show a P.M. equal to that found in Step 2 and a phase-margin frequency close to the bandwidth found in Step 2.
8. Right-click in the Bode plot area and select **Edit Compensator** . . .
9. Read the lead compensator in the **Control and Estimation Tools Manager Window.**

Keep in mind that the previous examples were designs for third-order systems and must be simulated to ensure the desired transient results. In the next section, we look at lag-lead compensation to improve steady-state error and transient response.

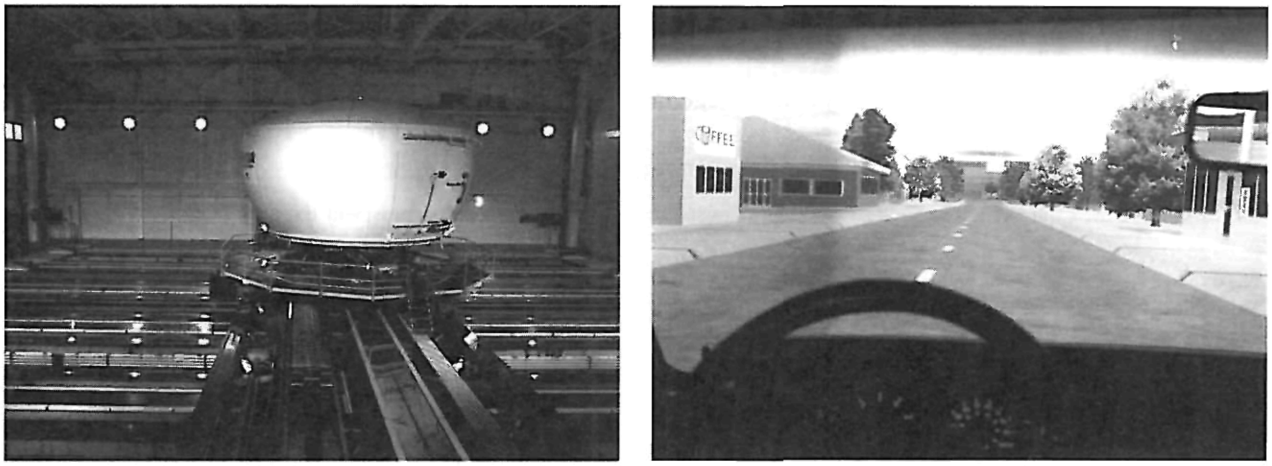
11.5 Lag-Lead Compensation

In Section 9.4, using root locus, we designed lag-lead compensation to improve the transient response and steady-state error. Figure 11.10 is an example of a system to which lag-lead compensation can be applied. In this section we repeat the design, using frequency response techniques. One method is to design the lag compensation to lower the high-frequency gain, stabilize the system, and improve the steady-state error and then design a lead compensator to meet the phase-margin requirements. Let us look at another method.

Section 9.6 describes a passive lag-lead network that can be used in place of separate lag and lead networks. It may be more economical to use a single, passive network that performs both tasks, since the buffer amplifier that separates the lag network from the lead network may be eliminated. In this section, we emphasize lag-lead design, using a single, passive lag-lead network.

The transfer function of a single, passive lag-lead network is

$$G_c(s) = G_{\text{Lead}}(s)G_{\text{Lag}}(s) = \left(\frac{s+\dfrac{1}{T_1}}{s+\dfrac{\gamma}{T_1}}\right)\left(\frac{s+\dfrac{1}{T_2}}{s+\dfrac{1}{\gamma T_2}}\right) \qquad (11.15)$$



(a) (b)

FIGURE 11.10 **a.** The National Advanced Driving Simulator at the University of Iowa; **b.** test driving the simulator with its realistic graphics (Katharina Bosse/laif/Redux Pictures.)

where $\gamma > 1$. The first term in parentheses produces the lead compensation, and the second term in parentheses produces the lag compensation. The constraint that we must follow here is that the single value γ replaces the quantity α for the lag network in Eq. (11.2) and the quantity β for the lead network in Eq. (11.6). For our design, α and β must be reciprocals of each other. An example of the frequency response of the passive lag-lead is shown in Figure 11.11.

We are now ready to enumerate a design procedure.

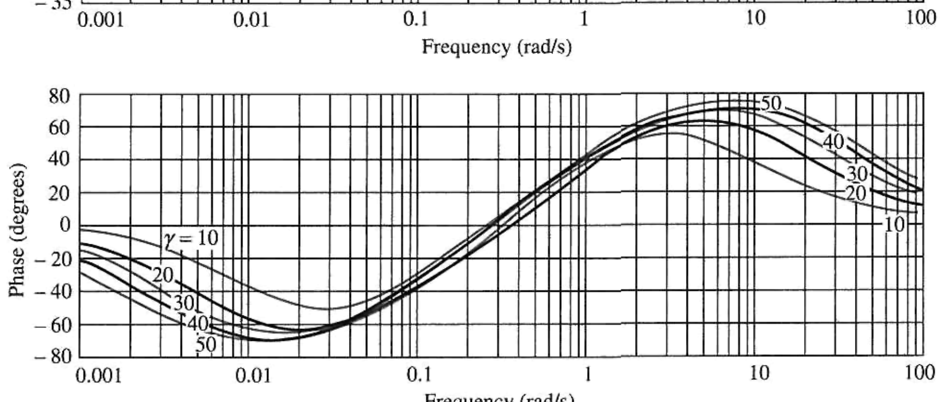


FIGURE 11.11 Sample frequency response curves for a lag-lead compensator, $G_c(s) = [(s+1)(s+0.1)]/\left[(s+\gamma)\left(s+\frac{0.1}{\gamma}\right)\right]$

Design Procedure

1. Using a second-order approximation, find the closed-loop bandwidth required to meet the settling time, peak time, or rise time requirement (see Eqs. (10.55) and (10.56)).
2. Set the gain, K, to the value required by the steady-state error specification.
3. Plot the Bode magnitude and phase diagrams for this value of gain.
4. Using a second-order approximation, calculate the phase margin to meet the damping ratio or percent overshoot requirement, using Eq. (10.73).
5. Select a new phase-margin frequency near ω_{BW}.
6. At the new phase-margin frequency, determine the additional amount of phase lead required to meet the phase-margin requirement. Add a small contribution that will be required after the addition of the lag compensator.
7. Design the lag compensator by selecting the higher break frequency one decade below the new phase-margin frequency. The design of the lag compensator is not critical, and any design for the proper phase margin will be relegated to the lead compensator. The lag compensator simply provides stabilization of the system with the gain required for the steady-state error specification. Find the value of γ from the lead compensator's requirements. Using the phase required from the lead compensator, the phase response curve of Figure 11.8 can be used to find the value of $\gamma = 1/\beta$. This value, along with the previously found lag's upper break frequency, allows us to find the lag's lower break frequency.
8. Design the lead compensator. Using the value of γ from the lag compensator design and the value assumed for the new phase-margin frequency, find the lower and upper break frequency for the lead compensator, using Eq. (11.9) and solving for T.
9. Check the bandwidth to be sure the speed requirement in Step 1 has been met.
10. Redesign if phase-margin or transient specifications are not met, as shown by analysis or simulation.

Let us demonstrate the procedure with an example.

Example 11.4

Lag-Lead Compensation Design

PROBLEM: Given a unity feedback system where $G(s) = K/[s(s+1)(s+4)]$, design a passive lag-lead compensator using Bode diagrams to yield a 13.25% overshoot, a peak time of 2 seconds, and $K_v = 12$.

SOLUTION: We will follow the steps previously mentioned in this section for lag-lead design.

1. The bandwidth required for a 2-seconds peak time is 2.29 rad/s.
2. In order to meet the steady-state error requirement, $K_v = 12$, the value of K is 48.
3. The Bode plots for the uncompensated system with $K = 48$ are shown in Figure 11.12. We can see that the system is unstable.
4. The required phase margin to yield a 13.25% overshoot is 55°.

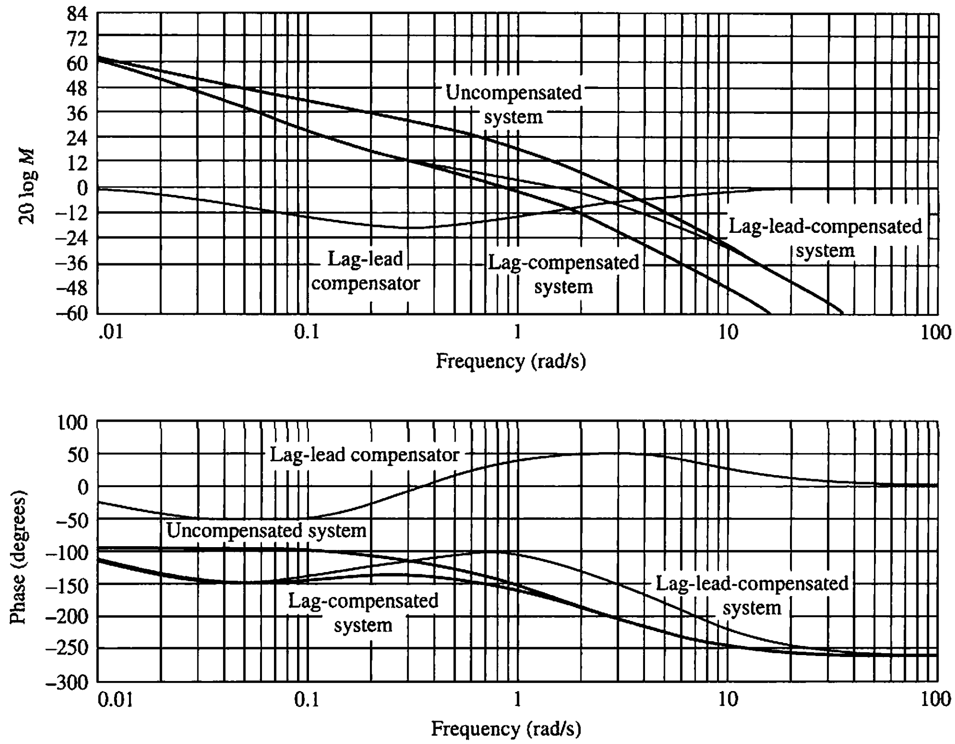


FIGURE 11.12 Bode plots for lag-lead compensation in Example 11.4

5. Let us select $\omega = 1.8$ rad/s as the new phase-margin frequency.
6. At this frequency, the uncompensated phase is $-176°$ and would require, if we add a $-5°$ contribution from the lag compensator, a $56°$ contribution from the lead portion of the compensator.
7. The design of the lag compensator is next. The lag compensator allows us to keep the gain of 48 required for $K_v = 12$ and not have to lower the gain to stabilize the system. As long as the lag compensator stabilizes the system, the design parameters are not critical since the phase margin will be designed with the lead compensator. Thus, choose the lag compensator so that its phase response will have minimal effect at the new phase-margin frequency. Let us choose the lag compensator's higher break frequency to be 1 decade below the new phase-margin frequency, at 0.18 rad/s. Since we need to add 56° of phase shift with the lead compensator at $\omega = 1.8$ rad/s, we estimate from Figure 11.8 that, if $\gamma = 10.6$ (since $\gamma = 1/\beta$, $\beta = 0.094$), we can obtain about 56° of phase shift from the lead compensator. Thus with $\gamma = 10.6$ and a new phase-margin frequency of $\omega = 1.8$ rad/s, the transfer function of the lag compensator is

$$G_{\text{lag}}(s) = \frac{1}{\gamma}\frac{\left(s + \dfrac{1}{T_2}\right)}{\left(s + \dfrac{1}{\gamma T_2}\right)} = \frac{1}{10.6}\frac{(s + 0.183)}{(s + 0.0172)} \tag{11.16}$$

where the gain term, $1/\gamma$, keeps the dc gain of the lag compensator at 0 dB. The lag-compensated system's open-loop transfer function is

$$G_{\text{lag-comp}}(s) = \frac{4.53(s+0.183)}{s(s+1)(s+4)(s+0.0172)} \tag{11.17}$$

8. Now we design the lead compensator. At $\omega = 1.8$, the lag-compensated system has a phase angle of 180°. Using the values of $\omega_{\max} = 1.8$ and $\beta = 0.094$, Eq. (11.9) yields the lower break, $1/T_1 = 0.56$ rad/s. The higher break is then $1/\beta T_1 = 5.96$ rad/s. The lead compensator is

$$G_{\text{lead}}(s) = \gamma \frac{\left(s + \frac{1}{T_1}\right)}{\left(s + \frac{\gamma}{T_1}\right)} = 10.6\frac{(s+0.56)}{(s+5.96)} \tag{11.18}$$

The lag-lead-compensated system's open-loop transfer function is

$$G_{\text{lag-lead-comp}}(s) = \frac{48(s+0.183)(s+0.56)}{s(s+1)(s+4)(s+0.0172)(s+5.96)} \tag{11.19}$$

9. Now check the bandwidth. The closed-loop bandwidth is equal to that frequency where the open-loop magnitude response is approximately −7 dB. From Figure 11.12, the magnitude is −7 dB at approximately 3 rad/s. This bandwidth exceeds that required to meet the peak time requirement.

The design is now checked with a simulation to obtain actual performance values. Table 11.4 summarizes the system's characteristics. The peak time requirement is also met. Again, if the requirements were not met, a redesign would be necessary.

TABLE 11.4 Characteristics of gain-compensated system of Example 11.4

Parameter	Proposed specification	Actual value
K_v	12	12
Phase margin	55°	59.3°
Phase-margin frequency	—	1.63 rad/s
Closed-loop bandwidth	2.29 rad/s	3 rad/s
Percent overshoot	13.25	10.2
Peak time	2.0 seconds	1.61 seconds

Students who are using MATLAB should now run ch11p4 in Appendix B. You will learn how to use MATLAB to design a lag-lead compensator. You will enter the desired percent overshoot, peak time, and K_v. MATLAB then designs a lag-lead compensator using Bode plots, evaluates K_v, and generates a closed-loop step response. This exercise solves Example 11.4 using MATLAB.

MATLAB ML

For a final example, we include the design of a lag-lead compensator using a Nichols chart. Recall from Chapter 10 that the Nichols chart contains a presentation of both the open-loop frequency response and the closed-loop frequency response. The axes of the Nichols chart are the open-loop magnitude and phase (y and x axis, respectively). The open-loop frequency response is plotted using the coordinates of the Nichols chart at each frequency. The open-loop plot is overlaying a grid that yields the closed-loop magnitude and phase. Thus, we have a presentation of both the open- and closed-loop responses. Thus, a design can be implemented that reshapes the Nichols plot to meet both open- and closed-loop frequency specifications.

From a Nichols chart, we can see simultaneously the following frequency response specifications that are used to design a desired time response: (1) phase margin, (2) gain margin, (3) closed-loop bandwidth, and (4) closed-loop peak amplitude.

In the following example, we first specify the following: (1) maximum allowable percent overshoot, (2) maximum allowable peak time, and (3) minimum allowable static error constant. We first design the lead compensator to meet the transient requirements followed by the lag compensator design to meet the steady-state error requirement. Although calculations could be made by hand, we will use MATLAB and SISOTOOL to make and shape the Nichols plot.

Let us first outline the steps that we will take in the example:

1. Calculate the damping ratio from the percent overshoot requirement using Eq. (4.39)
2. Calculate the peak amplitude, M_p, of the closed-loop response using Eq. (10.52) and the damping ratio found in (1).
3. Calculate the minimum closed-loop bandwidth to meet the peak time requirement using Eq. (10.56), with peak time and the damping ratio from (1).
4. Plot the open-loop response on the Nichols chart.
5. Raise the open-loop gain until the open-loop plot is tangent to the required closed-loop magnitude curve, yielding the proper M_p.
6. Place the lead zero at this point of tangency and the lead pole at a higher frequency. Zeros and poles are added in SISOTOOL by clicking either one on the tool bar and then clicking the position on the open-loop frequency response curve where you desire to add the zero or pole.
7. Adjust the positions of the lead zero and pole until the open-loop frequency response plot is tangent to the same M_p curve, but at the approximate frequency found in (3). This yields the proper closed-loop peak and proper bandwidth to yield the desired percent overshoot and peak time, respectively.
8. Evaluate the open-loop transfer function, which is the product of the plant and the lead compensator, and determine the static error constant.
9. If the static error constant is lower than required, a lag compensator must now be designed. Determine how much improvement in the static error constant is required.
10. Recalling that the lag pole is at a frequency below that of the lag zero, place a lag pole and zero at frequencies below the lead compensator and adjust to yield the desired improvement in static error constant. As an example, recall from Eq. (9.5) that the improvement in static error constant for a Type 1 system is equal to the ratio of the lag zero value divided by the lag pole value. Readjust the gain if necessary.

Example 11.5

MATLAB ML

Gui Tool GUIT

Lag-Lead Design Using the Nichols Chart, MATLAB, and SISOTOOL

PROBLEM: Design a lag-lead compensator for the plant, $G(s) = \dfrac{K}{s(s+5)(s+10)}$, to meet the following requirements: (1) a maximum of 20% overshoot, (2) a peak time of no more than 0.5 seconds, (3) a static error constant of no less than 6.

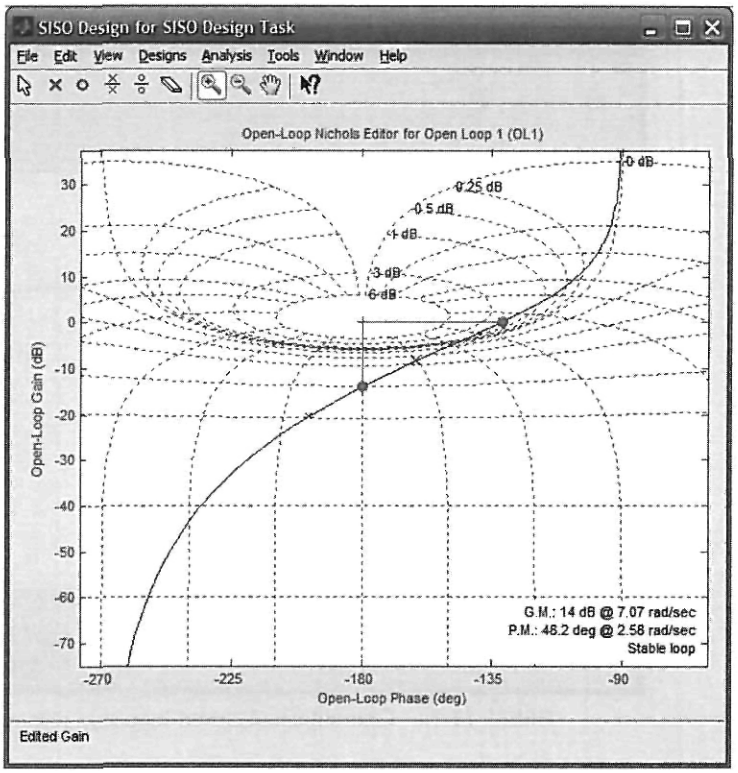


FIGURE 11.13 Nichols chart after gain adjustment

SOLUTION: We follow the steps enumerated immediately above,

1. Using Eq. (4.39), $\zeta = 0.456$ for 20% overshoot.
2. Using Eq. (10.52), $M_p = 1.23 = 1.81$ dB for $\zeta = 0.456$.
3. Using Eq. (10.56), $\omega_{BW} = 9.3$ r/s for $\zeta = 0.456$ and $T_p = 0.5$.
4. Plot the open-loop frequency response curve on the Nichols chart for $K = 1$.
5. Raise the open-loop frequency response curve until it is tangent to the closed-loop peak of 1.81 dB curve as shown in Figure 11.13. The frequency at the tangent point is approximately 3 r/s, which can be found by letting your mouse rest on the point of tangency. On the menu bar, select **Designs/Edit Compensator** . . . and find the gain added to the plant. Thus, the plant is now $G(s) = \dfrac{150}{s(s+5)(s+10)}$. The gain-adjusted closed-loop step response is shown in Figure 11.14. Notice that the peak time is about 1 second and must be decreased.
6. Place the lead zero at this point of tangency and the lead pole at a higher frequency.
7. Adjust the positions of the lead zero and pole until the open-loop frequency response plot is tangent to the same M_p curve, but at the approximate frequency found in **3**.
8. Checking **Designs/Edit Compensator** . . . shows $G(s)G_{\text{lead}}(s) = \dfrac{1286(s+1.4)}{s(s+5)(s+10)(s+12)}$, which yields a $K_v = 3$.

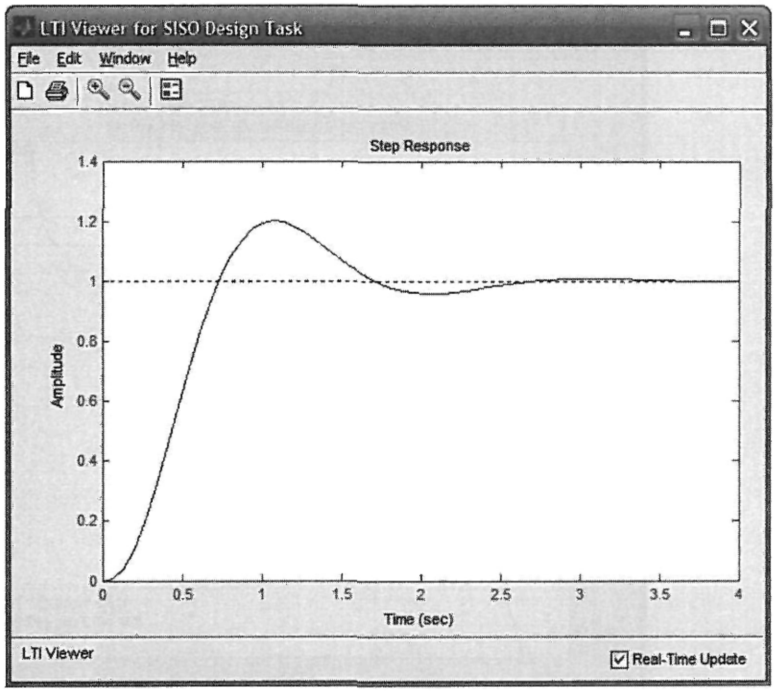


FIGURE 11.14 Gain-adjusted closed-loop step response

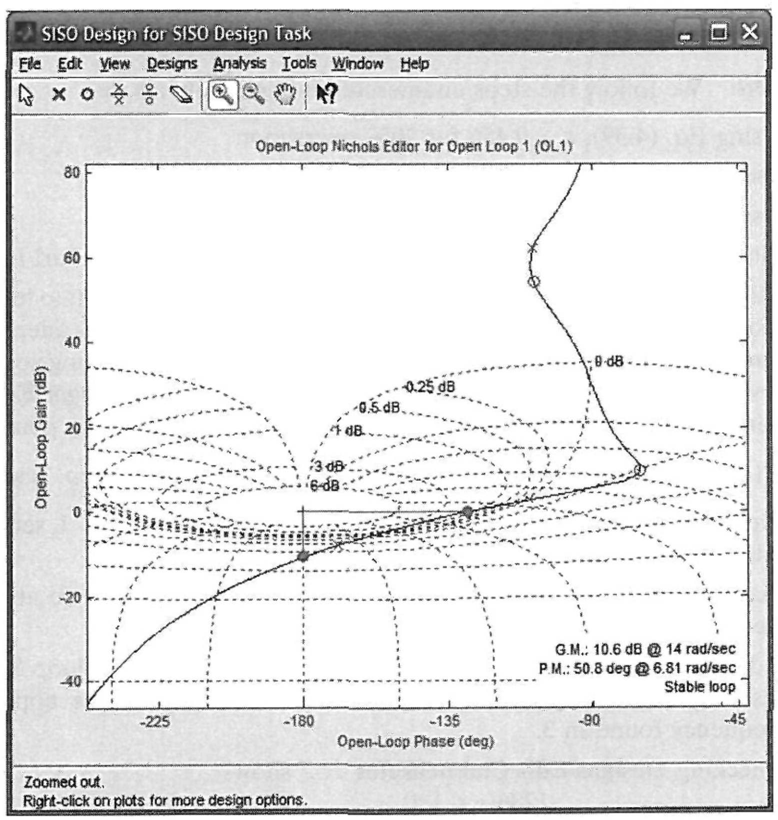


FIGURE 11.15 Nichols chart after lag-lead compensation

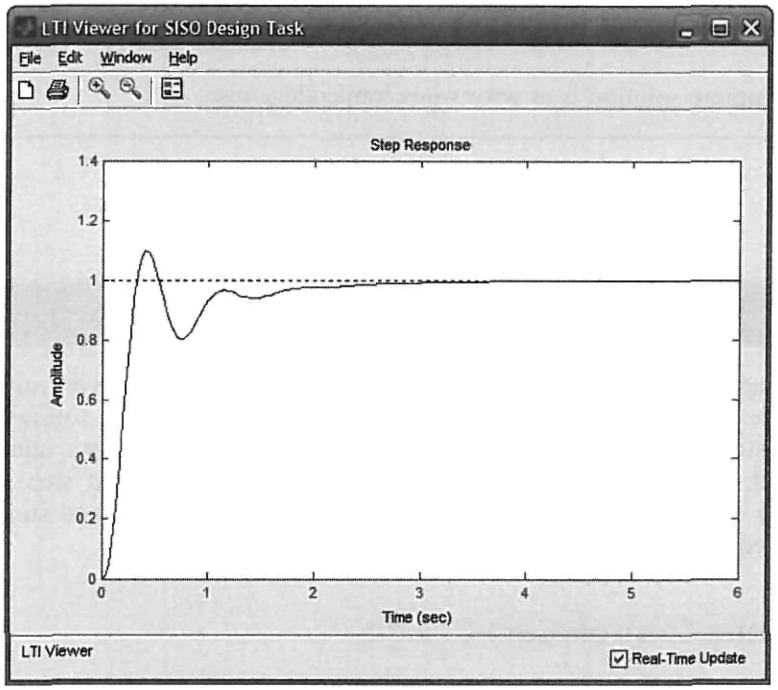


FIGURE 11.16 Lag-lead compensated closed-loop step response

9. We now add lag compensation to improve the static error constant by at least 2.
10. Now add a lag pole at −0.004 and a lag zero at −0.008. Readjust the gain to yield the same tangency as after the insertion of the lead. The final forward path is found to be $G(s)G_{\text{lead}}(s)G_{\text{lag}}(s) = \dfrac{1381(s+1.4)(s+0.008)}{s(s+5)(s+10)(s+12)(s+0.004)}$. The final Nichols chart is shown in Figure 11.15 and the compensated time response is shown in Figure 11.16. Notice that the time response has the expected slow climb to the final value that is typical of lag compensation. If your design requirements require a faster climb to the final response, then redesign the system with a larger bandwidth or attempt a design only with lead compensation. A problem at the end of the chapter provides the opportunity for practice.

Skill-Assessment Exercise 11.4

PROBLEM: Design a lag-lead compensator for a unity feedback system with the forward-path transfer function

$$G(s) = \frac{K}{s(s+8)(s+30)}$$

to meet the following specifications: $\%OS = 10\%$, $T_p = 0.6\,\text{s}$, and $K_v = 10$. Use frequency response techniques.

ANSWER: $G_{\text{lag}}(s) = 0.456\dfrac{(s+0.602)}{(s+0.275)}$; $G_{\text{lead}}(s) = 2.19\dfrac{(s+4.07)}{(s+8.93)}$; $K = 2400$.

The complete solution is at www.wiley.com/college/nise.

Case Studies

Our ongoing antenna azimuth position control system serves now as an example to summarize the major objectives of the chapter. The following cases demonstrate the use of frequency response methods to (1) design a value of gain to meet a percent overshoot requirement for the closed-loop step response and (2) design cascade compensation to meet both transient and steady-state error requirements.

Antenna Control: Gain Design

Design D

PROBLEM: Given the antenna azimuth position control system shown on the front endpapers, Configuration 1, use frequency response techniques to do the following:

a. Find the preamplifier gain required for a closed-loop response of 20% overshoot for a step input.
b. Estimate the settling time.

SOLUTION: The block diagram for the control system is shown on the inside front cover (Configuration 1). The loop gain, after block diagram reduction, is

$$G(s) = \frac{6.63K}{s(s+1.71)(s+100)} = \frac{0.0388K}{s\left(\frac{s}{1.71}+1\right)\left(\frac{s}{100}+1\right)} \tag{11.20}$$

Letting $K = 1$, the magnitude and phase frequency response plots are shown in Figure 10.61.

a. To find K to yield a 20% overshoot, we first make a second-order approximation and assume that the second-order transient response equations relating percent overshoot, damping ratio, and phase margin are true for this system. Thus, a 20% overshoot implies a damping ratio of 0.456. Using Eq. (10.73), this damping ratio implies a phase margin of 48.1°. The phase angle should therefore be $(-180° + 48.1°) = -131.9°$. The phase angle is $-131.9°$ at $\omega = 1.49$ rad/s, where the gain is -34.1 dB. Thus $K = 34.1$ dB $= 50.7$ for a 20% overshoot. Since the system is third-order, the second-order approximation should be checked. A computer simulation shows a 20% overshoot for the step response.
b. Adjusting the magnitude plot of Figure 10.61 for $K = 50.7$, we find -7 dB at $\omega = 2.5$ rad/s, which yields a closed-loop bandwidth of 2.5 rad/s. Using Eq. (10.55) with $\zeta = 0.456$ and $\omega_{BW} = 2.5$, we find $T_s = 4.63$ seconds. A computer simulation shows a settling time of approximately 5 seconds.

CHALLENGE: We now give you a problem to test your knowledge of this chapter's objectives. You are given the antenna azimuth position control system shown on the inside front cover (Configuration 3). Using frequency response methods do the following:

a. Find the value of K to yield 25% overshoot for a step input.

b. `Repeat Part a using MATLAB.`

MATLAB ML

Design D

Antenna Control: Cascade Compensation Design

PROBLEM: Given the antenna azimuth position control system block diagram shown on the front endpapers, Configuration 1, use frequency response techniques and design cascade compensation for a closed-loop response of 20% overshoot for a step input, a fivefold improvement in steady-state error over the gain-compensated system operating at 20% overshoot, and a settling time of 3.5 seconds.

SOLUTION: Following the lag-lead design procedure, we first determine the value of gain, K, required to meet the steady-state error requirement.

1. Using Eq. (10.55) with $\zeta = 0.456$, and $T_s = 3.5$ seconds, the required bandwidth is 3.3 rad/s.
2. From the preceding case study, the gain-compensated system's open-loop transfer function was, for $K = 50.7$,

$$G(s)H(s) = \frac{6.63K}{s(s+1.71)(s+100)} = \frac{336.14}{s(s+1.71)(s+100)} \tag{11.21}$$

   This function yields $K_v = 1.97$. If $K = 254$, then $K_v = 9.85$, a fivefold improvement.
3. The frequency response curves of Figure 10.61, which are plotted for $K = 1$, will be used for the solution.
4. Using a second-order approximation, a 20% overshoot requires a phase margin of 48.1°.
5. Select $\omega = 3$ rad/s to be the new phase-margin frequency.
6. The phase angle at the selected phase-margin frequency is $-152°$. This is a phase margin of 28°. Allowing for a 5° contribution from the lag compensator, the lead compensator must contribute $(48.1° - 28° + 5°) = 25.1°$.
7. The design of the lag compensator now follows. Choose the lag compensator upper break one decade below the new phase-margin frequency, or 0.3 rad/s. Figure 11.8 says that we can obtain 25.1° phase shift from the lead if $\beta = 0.4$ or $\gamma = 1/\beta = 2.5$. Thus, the lower break for the lag is at $1/(\gamma T) = 0.3/2.5 = 0.12$ rad/s.
   Hence,

$$G_{\text{lag}}(s) = 0.4\frac{(s+0.3)}{(s+0.12)} \tag{11.22}$$

8. Finally, design the lead compensator. Using Eq. (11.9), we have

$$T = \frac{1}{\omega_{\text{max}}\sqrt{\beta}} = \frac{1}{3\sqrt{0.4}} = 0.527 \tag{11.23}$$

   Therefore the lead compensator lower break frequency is $1/T = 1.9$ rad/s, and the upper break frequency is $1/(\beta T) = 4.75$ rad/s. Thus, the lag-lead-compensated forward path is

$$G_{\text{lag-lead-comp}}(s) = \frac{(6.63)(254)(s+0.3)(s+1.9)}{s(s+1.71)(s+100)(s+0.12)(s+4.75)} \tag{11.24}$$

9. A plot of the open-loop frequency response for the lag-lead-compensated system shows −7 dB at 5.3 rad/s. Thus, the bandwidth meets the design requirements for settling time. A simulation of the compensated system shows a 20% overshoot and a settling time of approximately 3.2 seconds, compared to a 20% overshoot for the uncompensated system and a settling time of approximately 5 seconds. K_v for the compensated system is 9.85 compared to the uncompensated system value of 1.97.

CHALLENGE: We now give you a problem to test your knowledge of this chapter's objectives. *You are given the antenna azimuth position control system shown on* the front endpapers (Configuration 3). Using frequency response methods, do the following:

a. Design a lag-lead compensator to yield a 15% overshoot and $K_v = 20$. In order to speed up the system, the compensated system's phase-margin frequency will be set to 4.6 times the phase-margin frequency of the uncompensated system.

MATLAB
ML

b. Repeat Part **a** using MATLAB.

Summary

This chapter covered the design of feedback control systems using frequency response techniques. We learned how to design by gain adjustment as well as cascaded lag, lead, and lag-lead compensation. Time response characteristics were related to the phase margin, phase-margin frequency, and bandwidth.

Design by gain adjustment consisted of adjusting the gain to meet a phase-margin specification. We located the phase-margin frequency and adjusted the gain to 0 dB.

A lag compensator is basically a low-pass filter. The low-frequency gain can be raised to improve the steady-state error, and the high-frequency gain is reduced to yield stability. Lag compensation consists of setting the gain to meet the steady-state error requirement and then reducing the high-frequency gain to create stability and meet the phase-margin requirement for the transient response.

A lead compensator is basically a high-pass filter. The lead compensator increases the high-frequency gain while keeping the low-frequency gain the same. Thus, the steady-state error can be designed first. At the same time, the lead compensator increases the phase angle at high frequencies. The effect is to produce a faster, stable system since the uncompensated phase margin now occurs at a higher frequency.

A lag-lead compensator combines the advantages of both the lag and the lead compensator. First, the lag compensator is designed to yield the proper steady-state error with improved stability. Next, the lead compensator is designed to speed up the transient response. If a single network is used as the lag-lead, additional design considerations are applied so that the ratio of the lag zero to the lag pole is the same as the ratio of the lead pole to the lead zero.

In the next chapter, we return to state space and develop methods to design desired transient and steady-state error characteristics.

Review Questions

1. What major advantage does compensator design by frequency response have over root locus design?
2. How is gain adjustment related to the transient response on the Bode diagrams?
3. Briefly explain how a lag network allows the low-frequency gain to be increased to improve steady-state error without having the system become unstable.
4. From the Bode plot perspective, briefly explain how the lag network does not appreciably affect the speed of the transient response.
5. Why is the phase margin increased above that desired when designing a lag compensator?
6. Compare the following for uncompensated and lag-compensated systems designed to yield the same transient response: low-frequency gain, phase-margin frequency, gain curve value around the phase-margin frequency, and phase curve values around the phase-margin frequency.
7. From the Bode diagram viewpoint, briefly explain how a lead network increases the speed of the transient response.
8. Based upon your answer to Question 7, explain why lead networks do not cause instability.
9. Why is a correction factor added to the phase margin required to meet the transient response?
10. When designing a lag-lead network, what difference is there in the design of the lag portion as compared to a separate lag compensator?

Problems

1. Design the value of gain, K, for a gain margin of 10 dB in the unity feedback system of Figure P11.1 if [Section: 11.2]

   a. $G(s) = \dfrac{K}{(s+4)(s+10)(s+15)}$

   b. $G(s) = \dfrac{K}{s(s+4)(s+10)}$

   c. $G(s) = \dfrac{K(s+2)}{s(s+4)(s+6)(s+10)}$

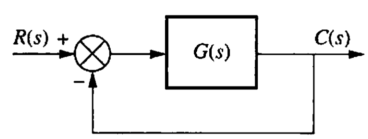


FIGURE P11.1

2. For each of the systems in Problem 1, design the gain, K, for a phase margin of 40°. [Section: 11.2]
3. Given the unity feedback system of Figure P11.1, use frequency response methods to determine the value of gain, K, to yield a step response with a 20% overshoot if [Section: 11.2]

a. $G(s) = \dfrac{K}{s(s+8)(s+15)}$

b. $G(s) = \dfrac{K(s+4)}{s(s+8)(s+10)(s+15)}$

c. $G(s) = \dfrac{K(s+2)(s+7)}{s(s+6)(s+8)(s+10)(s+15)}$

4. Given the unity feedback system of Figure P11.1 with

$$G(s) = \frac{K(s+20)(s+25)}{s(s+6)(s+9)(s+14)}$$

do the following: [Section: 11.2]

a. Use frequency response methods to determine the value of gain, K, to yield a step response with a 15% overshoot. Make any required second-order approximations.

b. Use MATLAB or any other computer program to test your second-order approximation by simulating the system for your designed value of K. MATLAB ML

5. The unity feedback system of Figure P11.1 with WileyPLUS WPCS Control Solutions

$$G(s) = \frac{K}{s(s+7)}$$

is operating with 15% overshoot. Using frequency response techniques, design a compensator to yield $K_v = 50$ with the phase-margin frequency and phase margin remaining approximately the same as in the uncompensated system. [Section: 11.3]

6. Given the unity feedback system of Figure P11.1 with

$$G(s) = \frac{K(s+10)(s+11)}{s(s+3)(s+6)(s+9)}$$

do the following: [Section: 11.3]

a. Use frequency response methods to design a lag compensator to yield $K_v = 1000$ and 15% overshoot for the step response. Make any required second-order approximations.

b. Use MATLAB or any other computer program to test your second-order approximation by simulating the system for your designed value of K and lag compensator. MATLAB ML

7. The unity feedback system shown in Figure P11.1 with

$$G(s) = \frac{K}{(s+2)(s+5)(s+7)}$$

is operating with 15% overshoot. Using frequency response methods, design a compensator to yield a five-fold improvement in steady-state error without appreciably changing the transient response. [Section: 11.3]

8. Design a lag compensator so that the system of Figure P11.1 where

$$G(s) = \frac{K(s+4)}{(s+2)(s+6)(s+8)}$$

operates with a 45° phase margin and a static error constant of 100. [Section: 11.3]

9. Design a PI controller for the system of Figure 11.2 that will yield zero steady-state error for a ramp input and a 9.48% overshoot for a step input. [Section: 11.3]

10. For the system of Problem 6, do the following: [Section: 11.3]

a. Use frequency response methods to find the gain, K, required to yield about 15% overshoot. Make any required second-order approximations.

b. Use frequency response methods to design a PI compensator to yield zero steady-state error for a ramp input without appreciably changing the transient response characteristics designed in Part **a**.

c. Use MATLAB or any other computer program to test your second-order approximation by simulating the system for your designed value of K and PI compensator. MATLAB ML

11. Write a MATLAB program that will design a PI controller assuming a second-order approximation as follows: MATLAB ML

a. Allow the user to input from the keyboard the desired percent overshoot

b. Design a PI controller and gain to yield zero steady-state error for a closed-loop step response as well as meet the percent overshoot specification

c. Display the compensated closed-loop step response

Test your program on

$$G(s) = \frac{K}{(s+5)(s+10)}$$

and 25% overshoot.

12. Design a compensator for the unity feedback system of Figure P11.1 with WileyPLUS WPCS Control Solutions

$$G(s) = \frac{K}{s(s+3)(s+15)(s+20)}$$

to yield a $K_v = 4$ and a phase margin of 40°. [Section: 11.4]

13. Consider the unity feedback system of Figure P11.1 with

$$G(s) = \frac{K}{s(s+5)(s+20)}$$

The uncompensated system has about 55% overshoot and a peak time of 0.5 second when $K_v = 10$. Do the following: [Section: 11.4]

a. Use frequency response methods to design a lead compensator to reduce the percent overshoot to 10%, while keeping the peak time and steady-state error about the same or less. Make any required second-order approximations.

b. Use MATLAB or any other computer program to test your second-order approximation by simulating the system for your designed value of K. MATLAB ML

14. The unity feedback system of Figure P11.1 with

$$G(s) = \frac{K(s+5)}{(s+2)(s+6)(s+10)}$$

is operating with 20% overshoot. [Section: 11.4]

a. Find the settling time.

b. Find K_p.

c. Find the phase margin and the phase-margin frequency.

d. Using frequency response techniques, design a compensator that will yield a threefold improvement in K_p and a twofold reduction in settling time while keeping the overshoot at 20%.

15. Repeat the design of Example 11.3 in the text using a PD controller. [Section: 11.4] WileyPLUS WPCS Control Solutions

16. Repeat Problem 13 using a PD compensator. [Section: 11.4]

17. Write a MATLAB program that will design a lead compensator assuming second-order approximations as follows: MATLAB ML

a. Allow the user to input from the keyboard the desired percent overshoot, peak time, and gain required to meet a steady-state error specification

b. Display the gain-compensated Bode plot

c. Calculate the required phase margin and bandwidth

d. Display the pole, zero, and gain of the lead compensator

e. Display the compensated Bode plot

f. Output the step response of the lead-compensated system to test your second-order approximation

Test your program on a unity feedback system where

$$G(s) = \frac{K(s+1)}{s(s+2)(s+6)}$$

and the following specifications are to be met: percent overshoot = 10%, peak time = 0.1 second, and $K_v = 30$.

18. Repeat Problem 17 for a PD controller. MATLAB ML

19. Use frequency response methods to design a lag-lead compensator for a unity feedback system where [Section: 11.4]

$$G(s) = \frac{K(s+7)}{s(s+5)(s+15)}$$

and the following specifications are to be met: percent overshoot = 15%, settling time = 0.1 second, and $K_v = 1000$.

20. Write a MATLAB program that will design a lag-lead compensator assuming second-order approximations as follows: [Section: 11.5] MATLAB ML

a. Allow the user to input from the keyboard the desired percent overshoot, settling time, and gain required to meet a steady-state error specification

b. Display the gain-compensated Bode plot

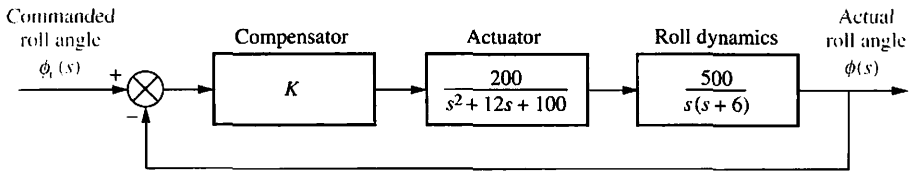


FIGURE P11.2 Towed-vehicle roll control

c. Calculate the required phase margin and bandwidth

d. Display the poles, zeros, and the gain of the lag-lead compensator

e. Display the lag-lead-compensated Bode plot

f. Display the step response of the lag-lead compensated system to test your second-order approximation

Use your program to do Problem 19.

21. Given a unity feedback system with

$$G(s) = \frac{K}{s(s+2)(s+5)}$$

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design a PID controller to yield zero steady-state error for a ramp input, as well as a 20% overshoot, and a peak time less than 2 seconds for a step input. Use only frequency response methods. [Section: 11.5]

22. A unity feedback system has

$$G(s) = \frac{K}{s(s+3)(s+6)}$$

MATLAB ML

If this system has an associated 0.5 second delay, use MATLAB to design the value of K for 20% overshoot. Make any necessary second-order approximations, but test your assumptions by simulating your design. The delay can be represented by cascading the MATLAB function padé (T, n) with $G(s)$, where T is the delay in seconds and n is the order of the Pade approximation (use 5). Write the program to do the following:

a. Accept your value of percent overshoot from the keyboard

b. Display the Bode plot for $K = 1$

c. Calculate the required phase margin and find the phase-margin frequency and the magnitude at the phase-margin frequency

d. Calculate and display the value of K

DESIGN PROBLEMS

23. Aircraft are sometimes used to tow other vehicles. A roll control system for such an aircraft was discussed in Problem 58 in Chapter 6. If Figure P11.2 represents the roll control system, use only frequency response techniques to do the following (*Cochran, 1992*):

a. Find the value of gain, K, to yield a closed-loop step response with 10% overshoot.

b. Estimate peak time and settling time using the gain-compensated frequency response.

c. Use MATLAB to simulate your system. Compare the results of the simulation with the requirements in Part **a** and your estimation of performance in Part **b**.

MATLAB ML

24. The model for a specific linearized TCP/IP computer network queue working under a random early detection (RED) algorithm has been modeled using the block diagram of Figure P11.1, where $G(s) = M(s)P(s)$, with

$$M(s) = \frac{0.005L}{s+0.005}$$

and

$$P(s) = \frac{140{,}625e^{-0.1s}}{(s+2.67)(s+10)}$$

Also, L is a parameter to be varied (*Hollot, 2001*).

a. Adjust L to obtain a 15% overshoot in the transient response for step inputs.

b. Verify Part **a** with a Simulink unit step response simulation.

Simulink SL

25. An electric ventricular assist device (EVAD) that helps pump blood concurrently to a defective natural heart in sick patients can be shown to have a transfer function

$$G(s) = \frac{P_{ao}(s)}{E_m(s)} = \frac{1361}{s^2+69s+70.85}$$

The input, $E_m(s)$, is the motor's armature voltage, and the output is $P_{ao}(s)$, the aortic blood pressure (*Tasch, 1990*). The EVAD will be controlled in the closed-loop configuration shown in Figure P11.1.

a. Design a phase lag compensator to achieve a tenfold improvement in the steady-state error to step inputs without appreciably affecting the transient response of the uncompensated system.

b. `Use MATLAB to simulate the uncompensated and compensated systems for a unit step input.` MATLAB ML

26. A Tower Trainer 60 Unmanned Aerial Vehicle has a transfer function

$$P(s) = \frac{h(s)}{\delta_e(s)} = \frac{-34.16s^3 - 144.4s^2 + 7047s + 557.2}{s^5 + 13.18s^4 + 95.93s^3 + 14.61s^2 + 31.94s}$$

where $\delta_e(s)$ is the elevator angle and $h(s)$ is the change in altitude (*Barkana, 2005*).

a. Assuming the airplane is controlled in the closed-loop configuration of Figure P11.1 with $G(s) = KP(s)$, find the value of K that will result in a 30° phase margin.

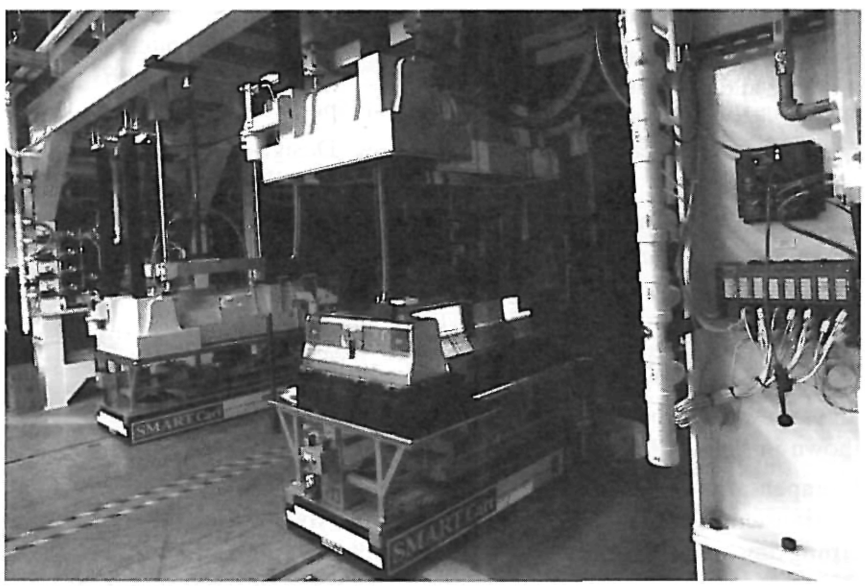
b. For the value of K calculated in Part **a**, obtain the corresponding gain margin.

c. Obtain estimates for the system's $\%OS$ and settling times T_s for step inputs.

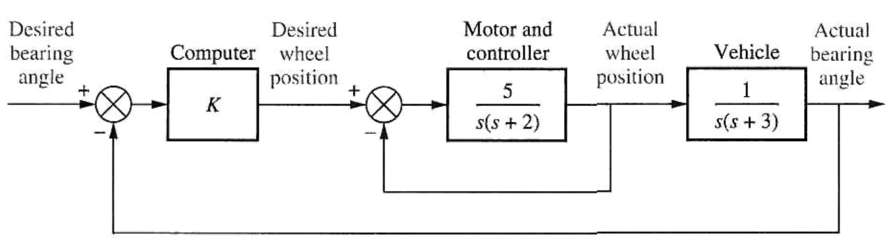
d. `Simulate the step response of the system using MATLAB.` MATLAB ML

e. Explain the simulation results and discuss any inaccuracies in the estimates obtained in Part **c**.

27. Self-guided vehicles, such as that shown in Figure P11.3(a), are used in factories to transport products from station to station. One method of construction



(a)



(b)

FIGURE P11.3 **a.** Automated guided carts in the final assembly area of lithium-ion batteries for Chevrolet Volt™ electric vehicles (Rebecca Cook/Rueters/©Corbis); **b.** simplified block diagram of a guided cart

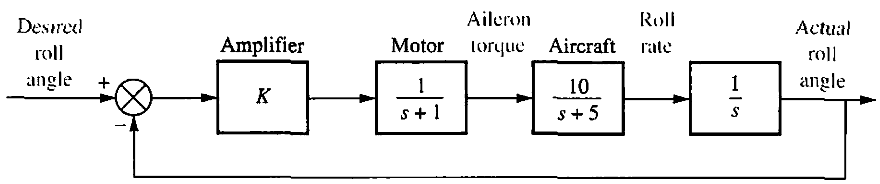


FIGURE P11.4

is to embed a wire in the floor to provide guidance. Another method is to use an onboard computer and a laser scanning device. Bar-coded reflective devices at known locations allow the system to determine the vehicle's angular position. This system allows the vehicle to travel anywhere, including between buildings (*Stefanides, 1987*). Figure P11.3(*b*) shows a simplified block diagram of the vehicle's bearing control system. For 11% overshoot, K is set equal to 2. Design a lag compensator using frequency response techniques to improve the steady-state error by a factor of 30 over that of the uncompensated system.

28. An aircraft roll control system is shown in Figure P11.4. The torque on the aileron generates a roll rate. The resulting roll angle is then controlled through a feedback system as shown. Design a lead compensator for a 60° phase margin and $K_v = 5$.

29. The transfer function from applied force to arm displacement for the arm of a hard disk drive has been identified as

$$G(s) = \frac{X(s)}{F(s)} = \frac{3.3333 \times 10^4}{s^2}$$

The position of the arm will be controlled using the feedback loop shown in Figure P11.1 (*Yan, 2003*).

a. Design a lead compensator to achieve closed-loop stability with a transient response of 16% overshoot and a settling time of 2 msec for a step input.

b. `Verify your design through MATLAB simulations.` MATLAB ML

30. A pitch axis attitude control system utilizing a momentum wheel was the subject of Problem 61 in Chapter 8. In that problem, the compensator is shown as a PI compensator. We want to replace the PI compensator with a lag-lead compensator to improve both transient and steady-state error performance. The block diagram for the pitch axis attitude control is shown in Figure P11.5, where $\theta_c(s)$ is a commanded pitch angle and $\theta(s)$ is the actual pitch angle of the spacecraft. If τ = 23 seconds and I_z = 963 1 in-lb-s^2, do the following (*Piper, 1992*): WileyPLUS WPCS Control Solutions

a. Design a lag-lead compensator and find $G_c(s)$ and K to yield a system with the following performance specifications: percent overshoot = 20%, settling time = 10 seconds, K_v = 200. Make any required second-order approximations.

b. `Use MATLAB or any other computer program to test your second-order approximation by simulating the system for your designed value of K and lag-lead compensator.` MATLAB ML

31. For the heat exchange system described in Problem 36, Chapter 9 (*Smith, 2002*):

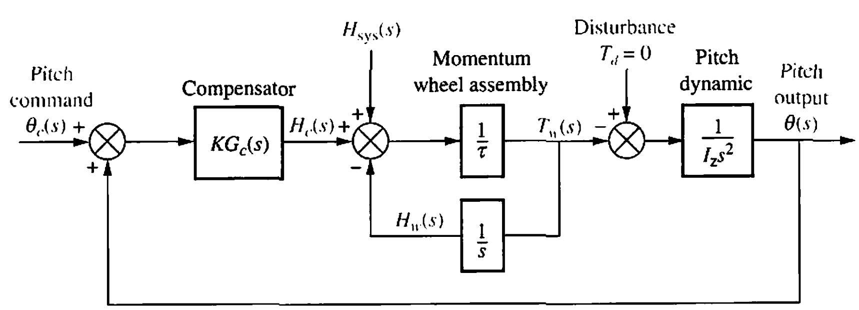


FIGURE P11.5

a. Design a passive lag-lead compensator to achieve 5% steady-state error with a transient response of 10% overshoot and a settling time of 60 seconds for step inputs.

b. `Use MATLAB to simulate and verify your design.` MATLAB ML

32. Active front steering is used in front-steering four-wheel cars to control the yaw rate of the vehicle as a function of changes in wheel-steering commands. For a certain car, and under certain conditions, it has been shown that the transfer function from steering wheel angle to yaw rate is given by (*Zhang, 2008*):

$$P(s) = \frac{28.4s + 119.7}{s^2 + 7.15s + 14.7}$$

The system is controlled in a unity-feedback configuration.

a. Use the Nichols chart and follow the procedure of Example 11.5 to design a lag-lead compensator such that the system has zero steady-state error for a step input. The bandwidth of the closed-loop system must be $\omega_B = 10$ rad/sec. Let the open-loop magnitude response peak be less than 1 dB and the steady-state error constant $K_v = 20$.

b. Relax the bandwidth requirement to $\omega_B \geq 10$ rad/sec. Design the system for a steady-state error of zero for a step input. Let the open-loop magnitude response peak be less than 1 dB and $K_v = 20$ using only a lead compensator.

c. `Simulate the step response of both designs using MATLAB.` MATLAB ML

PROGRESSIVE ANALYSIS AND DESIGN PROBLEMS

33. **High-speed rail pantograph.** Problem 21 in Chapter 1 discusses active control of a pantograph mechanism for high-speed rail systems. In Problem 79(a), Chapter 5, you found the block diagram for the active pantograph control system. In Chapter 8, Problem 72, you designed the gain to yield a closed-loop step response with 38% overshoot. A plot of the step response should have shown a settling time greater than 0.5 second as well as a high-frequency oscillation superimposed over the step response. In Chapter 9, Problem 55, we reduced the settling time to about 0.3 second, reduced the step response steady-state error to zero, and eliminated the high-frequency oscillations using a notch filter (*O'Connor, 1997*). Using the equivalent forward transfer function found in Chapter 5 cascaded with the notch filter specified in Chapter 9, design, using frequency response techniques, a lag-lead compensator to meet the following specifications:

a. At least 35° phase margin

b. A maximum of 10% steady-state error for the closed-loop step response

c. At least 35 rad/s bandwidth

34. **Control of HIV/AIDS.** In Chapter 6, the model for an HIV/AIDS patient treated with RTIs was linearized and shown to be

$$P(s) = \frac{Y(s)}{U_1(s)} = \frac{-520s - 10.3844}{s^3 + 2.6817s^2 + 0.11s + 0.0126} = \frac{-520(s + 0.02)}{(s + 2.2644)(s^2 + 0.04s + 0.0048)}$$

It is assumed here that the patient will be treated and monitored using the closed-loop configuration shown in Figure P11.1 Since the plant has a negative dc gain, assume for simplicity that $G(s) = G_c(s)P(s)$ and $G_c(0) < 0$. Assume also that the specifications for the design are (1) zero steady-state error for step inputs, (2) overdamped time-domain response, and (3) settling time $T_s \approx 100$ days (*Craig, 2004*).

a. The overdamped specification requires a $\Phi_M \approx 90°$. Find the corresponding bandwidth required to satisfy the settling time requirement.

b. The zero steady-state error specification implies that the open-loop transfer function must be augmented to Type 1. The −0.02 zero of the plant adds too much phase lead at low frequencies, and the complex conjugate poles, if left uncompensated within the loop, result in undesired oscillations in the time domain. Thus, as an initial approach to compensation for this system we can try

$$G_c(s) = \frac{-K(s^2 + 0.04s + 0.0048)}{s(s + 0.02)}$$

For $K = 1$, make a Bode plot of the resulting system. Obtain the value of K necessary to achieve the design demands. Check for closed-loop stability.

c. `Simulate the unit step response of the system using MATLAB. Adjust K to achieve the desired response.` MATLAB ML

35. **Hybrid vehicle.** In Part **b** of Problem 10.55 we used a proportional-plus-integral (PI) speed controller that resulted in an overshoot of 20% and a settling time, $T_s = 3.92$ seconds (*Preitl, 2007*).

a. Now assume that the system specifications require a steady-state error of zero for a step input, a ramp input steady-state error $\leq 2\%$, a $\%OS \leq 4.32\%$, and a settling time ≤ 4 seconds. One way to achieve these requirements is to cancel the PI-controller's zero, Z_I, with the real pole of the uncompensated system closest to the origin (located at -0.0163). Assuming exact cancellation is possible, the plant and controller transfer function becomes

$$G(s) = \frac{K(s+0.6)}{s(s+0.5858)}$$

Design the system to meet the requirements. You may use the following steps:

i. Set the gain, K, to the value required by the steady-state error specifications. Plot the Bode magnitude and phase diagrams.

ii. Calculate the required phase margin to meet the damping ratio or equivalently the $\%OS$ requirement, using Eq. (10.73). If the phase margin found from the Bode plot obtained in Step **i** is greater than the required value, simulate the system to check whether the settling time is less than 4 seconds and whether the requirement of a $\%OS \leq 4.32\%$ has been met. Redesign if the simulation shows that the $\%OS$ and/or the steady-state error requirements have not been met. If all requirements are met, you have completed the design.

b. In most cases, perfect pole-zero cancellation is not possible. Assume that you want to check what happens if the PI-controller's zero changes by $\pm$ 20%, e.g., if Z_I moves to:

Case 1: -0.01304

or to

Case 2: -0.01956.

The plant and controller transfer function in these cases will be, respectively:

$$\text{Case 1: } G(s) = \frac{K(s+0.6)(s+0.01304)}{s(s+0.0163)(s+0.5858)}$$

$$\text{Case 2: } G(s) = \frac{K(s+0.6)(s+0.01956)}{s(s+0.0163)(s+0.5858)}$$

Set K in each case to the value required by the steady-state error specifications and plot the Bode magnitude and phase diagrams. Simulate the closed-loop step response for each of the three locations of Z_I: pole/zero cancellation, Case 1, and Case 2, given in the problem.

Do the responses obtained resemble a second-order overdamped, critically damped, or underdamped response? Is there a need to add a derivative mode?

Cyber Exploration Laboratory

Experiment 11.1

Objectives To design a PID controller using MATLAB's SISO Design Tool. To see the effect of a PI and a PD controller on the magnitude and phase responses at each step of the design of a PID controller.

Minimum Required Software Packages MATLAB, and the Control System Toolbox

Prelab

1. What is the phase margin required for 12% overshoot?
2. What is the bandwidth required for 12% overshoot and a peak time of 2 seconds?

3. Given a unity feedback system with $G(s) = \frac{K}{s(s+1)(s+4)}$, what is the gain, K, required to yield the phase margin found in Prelab 1? What is the phase-margin frequency?
4. Design a PI controller to yield a phase margin 5° more than that found in Prelab 1.
5. Complete the design of a PID controller for the system of Prelab 3.

Lab

1. Using MATLAB's SISO Design Tool, set up the system of Prelab 3 and display the open-loop Bode plots and the closed-loop step response.
2. Drag the Bode magnitude plot in a vertical direction until the phase margin found in Prelab 1 is obtained. Record the gain K, the phase margin, the phase-margin frequency, the percent overshoot, and the peak time. Move the magnitude curve up and down and note the effect upon the phase curve, the phase margin, and the phase-margin frequency.
3. Design the PI controller by adding a pole at the origin and a zero one decade below the phase-margin frequency found in Lab 2. Readjust the gain to yield a phase margin 5° higher than that found in Prelab 1. Record the gain K, the phase margin, the phase-margin frequency, the percent overshoot, and the peak time. Move the zero back and forth in the vicinity of its current location and note the effect on the magnitude and phase curve. Move the magnitude curve up and down and note its effect on the phase curve, the phase margin, and the phase-margin frequency.
4. Design the PD portion of the PID controller by first adjusting the magnitude curve to yield a phase-margin frequency slightly below the bandwidth calculated in Prelab 2. Add a zero to the system and move it until you obtain the phase margin calculated in Prelab 1. Move the zero and note its effect. Move the magnitude curve and note its effect.

Postlab

1. Compare the Prelab PID design with that obtained via the SISO Design Tool. In particular, compare the gain K, the phase margin, the phase-margin frequency, the percent overshoot, and the peak time.
2. For the uncompensated system, describe the effect of changing gain on the phase curve, the phase margin, and the phase-margin frequency.
3. For the PI-compensated system, describe the effect of changing gain on the phase curve, the phase margin, and the phase-margin frequency. Repeat for changes in the zero location.
4. For the PID-compensated system, describe the effect of changing gain on the phase curve, the phase margin, and the phase-margin frequency. Repeat for changes in the PD zero location.

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