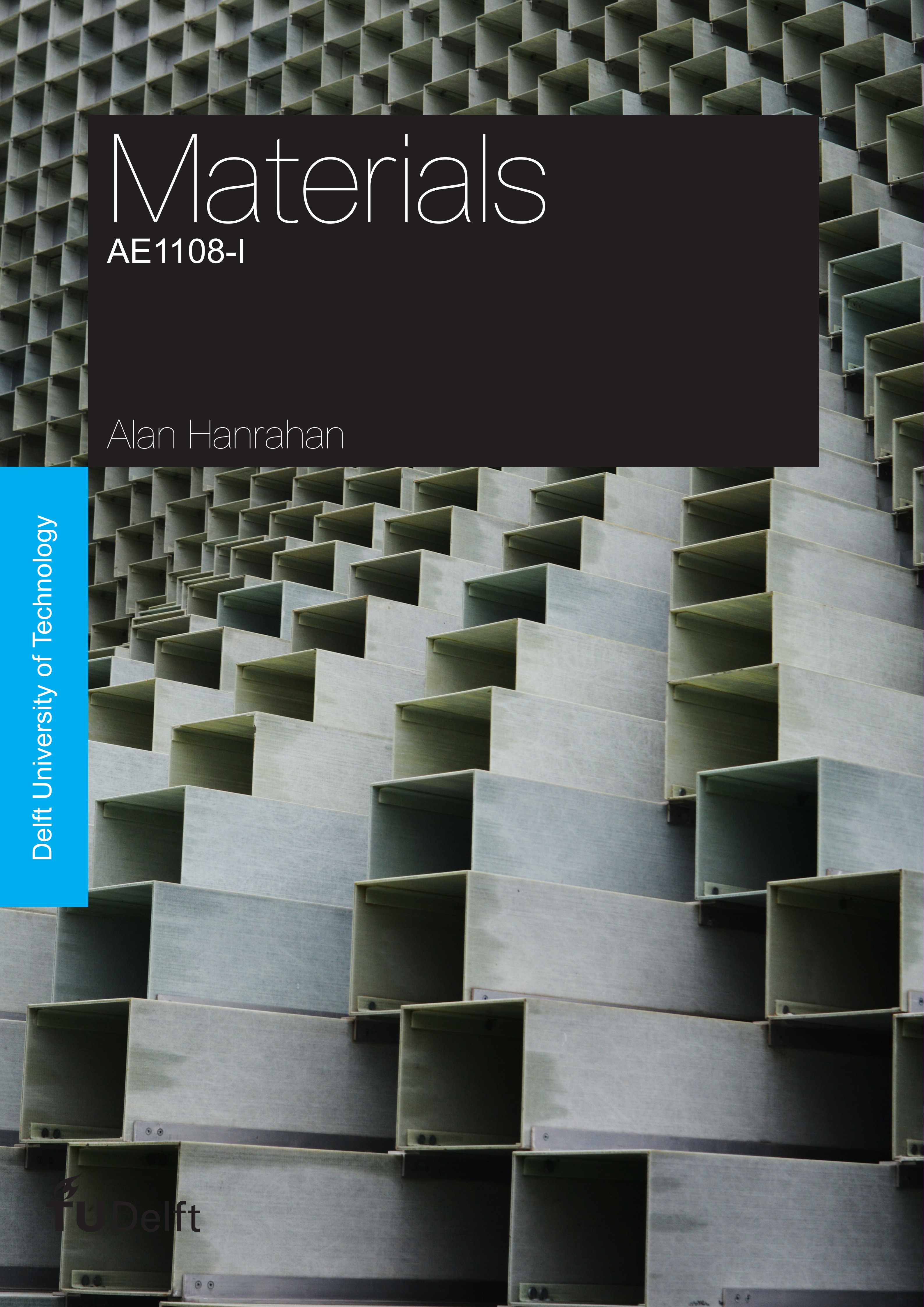

Materials

AE1108-I

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Preface

This summary of the materials course is much shorter than any others that I've written because this is just the equations from each chapter, The content itself can be found everywhere but for the purposes of the exam, you will need to know all of these equations. Best of luck. As always you can find the most up to date version of this summary on my website: alanrh.com

Alan Hanrahan
Delft, January 19, 2021

Contents

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Elastic Stiffness, and Weight

This a list of all of the equations you should know for the exam. Yes, there are a lot of them. This might not even be the list of the equations you need to know, but for the 2021 exam these are what you need to know:

$$\sigma = \frac{F}{A} \tag{1.1}$$

Stress is force per unit area.

$$\tau = \frac{F_s}{A} \tag{1.2}$$

Shear force.

$$\varepsilon = \frac{\delta L}{L_0} \tag{1.3}$$

Strain is percentage change in length

$$\tan\gamma = \frac{w}{L_0} \approx \gamma \tag{1.4}$$

Shear strain, w being the distance sheared

$$\Delta = \frac{\delta V}{V} \tag{1.5}$$

$$\sigma = E\varepsilon \tag{1.6}$$

$$\tau = G\gamma \tag{1.7}$$

$$p = k\Delta \tag{1.8}$$

$$\nu = -\frac{\varepsilon_t}{\varepsilon} \tag{1.9}$$

Poisson's ratio of transverse to longitudinal strain

$$G = \frac{E}{2(1+\nu)} \tag{1.10}$$

$$K = \frac{E}{3(1-2\nu)} \tag{1.11}$$

$$\varepsilon_1 = \frac{\sigma_1}{E},\ \varepsilon_2 = \varepsilon_3 = -\nu\varepsilon_1 = \frac{\nu\sigma}{E} \tag{1.12}$$

$$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2 - \nu\sigma_3) \tag{1.13}$$

$$\frac{\sigma_1}{\varepsilon_1} = \frac{E}{(1-\nu^2)} \tag{1.14}$$

For a cube confined by two walls.

$$dW = \frac{F \cdot dL}{A \cdot L} = \sigma d\varepsilon \tag{1.15}$$

$$W = \int_0^{\sigma^*} \sigma d\varepsilon = \int_0^{\sigma^*} \frac{\sigma d\sigma}{E} = \frac{1}{2}\frac{(\sigma^*)^2}{E} \tag{1.16}$$

The work done per unit volume as the stress is raised from zero to a final value σ^* is the area under the stress–strain curve.

$$\varepsilon_T = \alpha\Delta T \tag{1.17}$$

Thermal strain is a linear relationship

$$S = \frac{F}{\delta} \tag{1.18}$$

$$\sigma = \frac{S}{a_0}\varepsilon \tag{1.19}$$

$$E = \frac{S}{a_0} \tag{1.20}$$

$$\widetilde{\rho} = f\rho_A + (1-f)\,\rho_B \tag{1.21}$$

$$\widetilde{\rho} = f\rho_r + (1-f)\,\rho_m \tag{1.22}$$

$$\widetilde{E_U} = fE_r + (1-f)\,E_m \tag{1.23}$$

$$\widetilde{E_L} = \frac{E_m E_r}{fE_m + (1-f)E_r} \tag{1.24}$$

$$\frac{\widetilde{\rho}}{\rho_s} = 3\left(\frac{t}{L}\right)^2 \tag{1.25}$$

$$\frac{\widetilde{E}}{E_s} = \left(\frac{\widetilde{\rho}}{\rho_s}\right)^2 \tag{1.26}$$

$$f = \frac{v}{\lambda} \tag{1.27}$$

$$v_1 = \sqrt{\frac{E}{\rho}} \tag{1.28}$$

$$v_B = \sqrt{\frac{E(1-\nu)}{(1-\nu-2\nu^2)\rho}} \tag{1.29}$$

$$\frac{v_{||}}{v_{\perp}} = \sqrt{\frac{E_{||}}{E_{\perp}}} \tag{1.30}$$

$$Z = \sqrt{\rho E} \tag{1.31}$$

$$R = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2 \tag{1.32}$$

$$T = 1 - R = \frac{4Z_1 Z_2}{(Z_2 + Z_1)^2} \tag{1.33}$$

$$I \propto \sqrt{\frac{E}{\rho^3}} \tag{1.34}$$

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Stiffness-Limited Design

$$\delta = \frac{L_0 F}{AE} \tag{2.1}$$

$$S = \frac{F}{\delta} = \frac{AE}{L_0} \tag{2.2}$$

$$\kappa = \frac{d^2u}{dx^2} = \frac{1}{R} \tag{2.3}$$

$$\frac{\sigma}{\gamma} = \frac{M}{I} = E\kappa = E\frac{d^2u}{dx^2} \tag{2.4}$$

$$I = \int_{\text{section}} y^2 b(y)\ dy \tag{2.5}$$

$$S = \frac{F}{\delta} = \frac{C_1 EI}{L^3} \tag{2.6}$$

$$\frac{tau}{r} = \frac{T}{K} \tag{2.7}$$

$$J = \int_{\text{section}} 2\pi r^3\ dr \tag{2.8}$$

$$\frac{tau}{r} = \frac{T}{K} = \frac{G\theta}{L} \tag{2.9}$$

$$F_{\text{crit}} = \frac{n^2\pi^2 EI}{L^2} \tag{2.10}$$

Minimising weight, light, stiff, tie-rod

$$m = AL_0\rho$$

$$S^* = \frac{AE}{L_0}$$

$$m = S^* L_0^2 \left(\frac{\rho}{E}\right)$$

$$\Rightarrow \left(\frac{\rho}{E}\right)_{\text{min}}$$

$$M_t = \frac{E}{\rho} \tag{2.11}$$

Minimising weight, light, stiff, panel

$$m = AL\rho = bhL\rho$$

$$S^* = \frac{C_1 EI}{L^3}$$

$$I = \frac{bh^3}{12}$$

$$m = \left(\frac{12S^*}{C_1 b}\right)^{\frac{1}{3}} (bL^2)\left(\frac{\rho}{E^{\frac{1}{3}}}\right)$$

$$M_p = \frac{E^{\frac{1}{3}}}{\rho} \tag{2.12}$$

Minimising weight, light, stiff, beam

$$m = AL\rho = b^2 L\rho$$

$$S^* = \frac{C_1 EI}{L^3}$$

$$I = \frac{b^4}{12} = \frac{A^2}{12}$$

$$m = \left(\frac{12S^* L^3}{C_1}\right)^{\frac{1}{2}} (L)\left(\frac{\rho}{E^{\frac{1}{2}}}\right)$$

$$M_b = \frac{E^{\frac{1}{2}}}{\rho} \tag{2.13}$$

$$\Phi = \frac{I}{I_{\text{square}}} \tag{2.14}$$

$$\log(E) = 3\log(\rho) + \log(C) \tag{2.15}$$

3

Beyond Elasticity

$$\varepsilon_{pl} = \varepsilon_{tot} - \frac{\sigma}{E} \tag{3.1}$$

$$W_{pl} = \int_0^{\varepsilon_f} \sigma \; d\varepsilon_{pl} \tag{3.2}$$

$$H = \frac{F}{A} \tag{3.3}$$

$$\sigma_t = \frac{F}{A} \tag{3.4}$$

$$\text{Volume} = A_0 L_0 = AL \tag{3.5}$$

$$\sigma_n = \frac{F}{A_0} = \frac{F}{A}\left(\frac{L_0}{L}\right) \tag{3.6}$$

$$\varepsilon_n = \frac{\delta L}{L} = \left(\frac{L - L_0}{L)}\right) = \left(\frac{L}{L_0}\right) - 1 \tag{3.7}$$

$$\sigma_t = \sigma_n(1 + \varepsilon_n) \tag{3.8}$$

$$\varepsilon_t = \int_{L_0}^{L} \frac{dL}{L} = \ln\left(\frac{L}{L_0}\right) = \ln\,(1 + \varepsilon_n) \tag{3.9}$$

$$\tau = \alpha \frac{Gb}{L} \tag{3.10}$$

$$\tau_{ss} = \alpha G c^{\frac{1}{2}} \tag{3.11}$$

$$\tau_{ppt} = \frac{2T}{bL} = \frac{Gb}{L} \tag{3.12}$$

$$\tau_{wh} = \alpha \frac{Gb}{L} = \alpha G b \sqrt{\rho_d} \tag{3.13}$$

$$\tau_{gb} = \frac{k_p}{\sqrt{D}} \tag{3.14}$$

$$\tau_y = \tau_i + \tau_{ss} + \tau_{ppt} + \tau_{wh} + \tau_{gb} \tag{3.15}$$

4

Strength-Limited Design

$$\frac{\sigma}{y} = \frac{M}{I} = E\kappa \tag{4.1}$$

$$\sigma_{max} = \frac{My_m}{I} = \frac{M}{Z_e} \tag{4.2}$$

$$M_f = \int_{\text{section}} b(y)|y|\sigma_y \; dy = Z_p\sigma_y \tag{4.3}$$

$$K_{sc} = \frac{\sigma_{max}}{\sigma_{nom}} = 1 + \alpha\left(\frac{c}{\rho_{sc}}\right)^{\frac{1}{2}} \tag{4.4}$$

Minimising weight; light, strong, tie rod

$$m = AL\rho$$

$$\frac{F}{A} \leq \sigma_y$$

$$m \geq FL\left(\frac{\rho}{\sigma_y}\right)$$

$$M_t = \frac{\sigma_y}{\rho} \tag{4.5}$$

Minimising weight; light, strong, panel

$$m = AL\rho = bhL\rho$$

$$M \leq Z_e\sigma_y = \frac{I}{y_m}\sigma_y = \frac{bh^2}{6}\sigma_y$$

$$m = bL\rho\left(\frac{6M}{b\sigma_y}\right)^{\frac{1}{2}}$$

$$M_p = \frac{\sigma_y^{\frac{1}{2}}}{\rho} \tag{4.6}$$

Minimising weight; light, strong, panel

$$m = AL\rho = b^2L\rho$$

$$F_f = C_2\frac{Z\sigma_f}{L}$$

$$m = \left(\frac{6\sqrt{\alpha}F_f}{C_2 L^2}\right)^{\frac{2}{3}} L^3 \left[\frac{\rho}{\sigma_y^{\frac{2}{3}}}\right]$$

$$M_b = \frac{\sigma_y^{\frac{2}{3}}}{\rho} \tag{4.7}$$

5

Fracture and Fracture Toughness

$$\sigma_{\text{local}} = \sigma\left(1 + Y\sqrt{\frac{\pi c}{2\pi r}}\right) \tag{5.1}$$

$$\sigma_{\text{local}} = Y\frac{\sigma\sqrt{\pi c}}{\sqrt{2\pi r}} \tag{5.2}$$

$$K_1 = Y\sigma\sqrt{\pi c} \tag{5.3}$$

$$K_{1c} = K_1 = Y\sigma^*\sqrt{\pi c} \approx \sigma^*\sqrt{\pi c} \tag{5.4}$$

$$G \geq 2\gamma \tag{5.5}$$

$$K_{1c} = \sqrt{EG_c} \tag{5.6}$$

$$r_y = 2\left(\frac{\sigma^2\pi c}{2\sigma_y^2}\right) = \frac{K_1^2}{\pi\sigma_y^2} \tag{5.7}$$

$$\sigma_f = \frac{K_{1c}}{\sqrt{\pi c}} \tag{5.8}$$

$$c_{\text{crit}} = \frac{K_{1c}^2}{\pi\sigma_y^2} \tag{5.9}$$

6

Cyclic Loading and Fatigue Failure

$$\sigma_a = \frac{\Delta\sigma}{2} = \frac{\sigma_{\max - \sigma_{\min}}}{2} \tag{6.1}$$

$$\sigma_m = \frac{\sigma_{\max + \sigma_{\min}}}{2} \tag{6.2}$$

$$R = \frac{\sigma_{\min}}{\sigma_{\max}} \tag{6.3}$$

$$\Delta\sigma N_f^b = C_1 \tag{6.4}$$

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} = \frac{C_1/E}{N_f^b} \tag{6.5}$$

$$\Delta\sigma_{\sigma_m} = \Delta\sigma_{\sigma_0}\left(1 - \frac{\sigma_m}{\sigma_{ts}}\right) \tag{6.6}$$

$$\sum_{i=1}^{n} \frac{N_i}{N_{f,i}} = 1 \tag{6.7}$$

$$\Delta K = K_{max} - K_{min} = \Delta\sigma\sqrt{\pi c} \tag{6.8}$$

$$\frac{dc}{dN} = A\Delta K^m \tag{6.9}$$

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Fracture- and Fatigue-Limited Design

$$M_1 = K_{1c} \tag{7.1}$$

$$U_e = \frac{1}{2}\sigma\varepsilon = \frac{1}{2}\frac{\sigma^2}{E}$$

$$U_e^{\text{max}} \propto \frac{Y^2}{2\pi c}\left(\frac{K_{1c}^2}{E}\right)$$

$$M_2 = \frac{K_{1c}^2}{E} = G_c \tag{7.2}$$

$$\varepsilon = \frac{C}{\sqrt{\pi c_{\text{max}}}}\frac{K_{1c}}{E}$$

$$M_3 = \frac{K_{1c}}{E} \tag{7.3}$$

8

Materials and Heat

$$\alpha = \frac{1}{L}\frac{dL}{dT} \tag{8.1}$$

$$q = -\lambda\frac{dT}{dx} = \lambda\frac{(T_1 - T_2}{x} \tag{8.2}$$

$$a = \frac{\lambda}{C_p} \tag{8.3}$$

$$\rho C_p \approx 3 \times 10^6 \left[Jm^{-3}K\right] \tag{8.4}$$

$$\alpha \approx \frac{1.6 \times 10^{-3}}{E} \tag{8.5}$$

$$\sigma = \frac{E}{(1 - \nu)}\alpha\Delta T \tag{8.6}$$

9

Diffusion and Creep

$$P \approx P_0 \left(1 + \beta \frac{T}{T_m}\right) \tag{9.1}$$

$$\dot{\varepsilon} = \frac{\sigma}{3\eta} \tag{9.2}$$

$$\dot{\varepsilon}_{ss} = B\sigma^n \tag{9.3}$$

$$\dot{\varepsilon}_{ss} = Ce^{-\frac{Q_c}{RT}} \tag{9.4}$$

$$\dot{\varepsilon}_{ss} = \dot{\varepsilon}_0 \left(\frac{\sigma}{\sigma_0}\right)^n e^{-\frac{Q_c}{RT}} \tag{9.5}$$

$$J = -D\frac{dc}{dx} \tag{9.6}$$

$$D = D_0 e^{-\frac{Q_d}{RT}} \tag{9.7}$$

$$\dot{\varepsilon}_{ss} = A\sigma^n e^{-\frac{Q_c}{RT}} \tag{9.8}$$

10

Durability

$$xM + yO = M_xO_y + \text{energy} \tag{10.1}$$

$$\frac{d(\Delta m)}{dt} \Rightarrow \Delta m = k_l t \tag{10.2}$$

$$\frac{d(\Delta m)}{dt} \propto \frac{1}{\Delta m} \Rightarrow \Delta m = k_p t \tag{10.3}$$

$$k_p \propto D_0 C_0 e^{-\frac{Q_d}{RT}} \tag{10.4}$$

$$pH = -\log_{10}[H^+] \tag{10.5}$$

$$Fe \rightarrow Fe^{2+} + 2e^- \tag{10.6}$$

$$Cu^{2+} + 2e^- \rightarrow Cu \tag{10.7}$$

$$H_2O + O + 2e^- \leftrightarrow 2OH^- \tag{10.8}$$