# Aerospace Materials Formulae

AE1108 - Aerospace Materials & Structures

v2.0 - 28/01/2022

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# Introduction

This document contains a curated list of formulas for the AE1108-I Aerospace Materials' course based on Dr.S.J.García Espallargas' document on Brightspace, where the formulas from the Ashby book are listed in 3 categories:

1. "Must Know": These are the only formulas to be learnt by heart. The rest will be given.
2. "Others": These are the formulas whose meaning and the relation between the parameters must be understood, and be able to derive it if necessary from the "Must Know" equations. What I understand, from experience, is that if needed they will be given on the exam, but they will not tell you what each parameter represents.
3. "Know Existence": These are the formulas you must be aware of and be able to interpret. What I understand, from experience, is that if needed they will be given on the exam, and will let you know what each parameter represents.

For simplicity's sake, only the first two categories have been transcribed, being the most important. Further, if a formula is trivial, or is redundant because it is repeated, it will not be added.
If there are no formulas in a certain category, that category has not been added.
If a formula has been added for completeness but is NOT ON THE LIST, it will be indicated with an asterisk (*).

If the variables of an equation are not understood by the context, then their meaning can be checked in the Glossary. The terms are ordered in order or appearance.

# Chapter 4: Elastic Stiffness & Weight

## Must Know

**Normal Stress (4.1)**

$$\sigma = \frac{F}{A_0}$$

**Shear Stress (4.2)**

$$\tau = \frac{F_s}{A_0}$$

**Volumetric Stress ***

$$p = \frac{F}{A_0}$$

**Normal Strain (4.3)**

$$\varepsilon = \frac{\Delta L}{L_0}$$

**Shear Strain (4.4)**

$$\gamma = \frac{w}{L_0}$$

**Vol. Strain (4.5)**

$$\Delta = \frac{\Delta V}{V_0}$$

**Elastic Modulus (4.6)**

$$\sigma = E \cdot \varepsilon$$

**Shear Modulus (4.7)**

$$\tau = G \cdot \gamma$$

**Bulk Modulus ***

$$p = K \cdot \Delta$$

**Poisson's Ratio (4.9)**

$$\nu = -\frac{\varepsilon_t}{\varepsilon}$$

For METALS:
For POLYMERS: $\nu = \frac{1}{3}$
For CERAMICS:
For ELASTOMERS: $\nu = \frac{1}{2}$

**E-K Relation (4.10/4.10a/4.10b)**

$$K = \frac{E}{3(1 - 2\nu)}$$

For METALS:
For POLYMERS: $K \approx E$
For CERAMICS:
For ELASTOMERS: $K >> E$

**E-G Relation (4.10/4.10a/4.10b)**

$$G = \frac{E}{2(1+\nu)}$$

For METALS:
For POLYMERS: $G \approx \frac{3}{8}E$
For CERAMICS:
For ELASTOMERS: $G \approx \frac{1}{3}E$

**Upper Hybrid Modulus (4.19)**

$$\widetilde{E_U} = \widetilde{E_\parallel} = f_r \cdot E_r + (1 - f_r) \cdot E_m$$

**Lower Hybrid Modulus (4.20)**

$$\widetilde{E_L} = \widetilde{E_\perp} = \frac{E_m \cdot E_r}{f_r \cdot E_m + (1 - f_r) \cdot E_r}$$

**Hybrid Density (4.18)**

$$\tilde{\rho} = f_r \cdot \rho_r + (1 - f_r) \cdot \rho_m$$

**Thermal Expansion Coefficient (4.17)**

$$\alpha = \frac{\varepsilon_T}{\Delta T}$$

## Others

**Hooke's Law (4.13)**

**Direction 1**

$$\varepsilon_1 = \frac{1}{E} \cdot (\sigma_1 - \nu \cdot \sigma_2 - \nu \cdot \sigma_3)$$

**Direction 2**

$$\varepsilon_2 = \frac{1}{E} \cdot (-\nu \cdot \sigma_1 + \sigma_2 - \nu \cdot \sigma_3)$$

**Direction 3**

$$\varepsilon_3 = \frac{1}{E} \cdot (-\nu \cdot \sigma_1 - \nu \cdot \sigma_2 + \sigma_3)$$

**Stiffness (4.25)**

$$S = \frac{F}{\delta}$$

**Atomic View of Stifness (4.26)**

$$S = E \cdot a_0$$

# Chapter 5: Stiffness-Limited Design

## Must Know

**Tie-Rod Elongation (5.1)**

$$\delta = \frac{L_0 \cdot F}{A \cdot E}$$

**Tie-Rod Stiffness (5.2)**

$$S = \frac{A \cdot E}{L_0}$$

**Beam Bending Flexural Rigidity (5.3)**

$$\frac{\sigma}{y_m} = \frac{M}{I} = E \cdot k$$

**Beam Bending Stiffness (5.5)**

$$S = \frac{c_1 \cdot E \cdot I}{L^3}$$

**Column/Plate Bucking Force (5.9)**

$$F_{crit} = \frac{n^2 \cdot \pi^2 \cdot E \cdot I}{L^2}$$

**Mass (5.13)**

$$m = \rho \cdot A \cdot L_0$$

**Stiffness Material Indices for Minimal Weight (5.16/5.22/5.27)**

**Tie-Rod**

$$M_t = \frac{E}{\rho}$$

**Plate Buckling**

$$M_p = \frac{E^{1/3}}{\rho}$$

**Beam Bending**

$$M_b = \frac{E^{1/2}}{\rho}$$

**Shape Factor (5.28)**

$$\phi = \frac{I}{I_{square}}$$

# Chapter 6: Beyond Elasticity

## Must Know

**Plastic Strain (6.1)**

$$\varepsilon_{pl} = \varepsilon_{tot} - \frac{\sigma}{E}$$

**Indentation Hardness (6.3)**

$$H = \frac{F}{A}$$

**Total Hardening (6.22)**

$$\tau_y = \tau_i + \tau_{ss} + \tau_{ppt} + \tau_{wh} + \tau_{gb}$$

## Others

**True Stress (6.5)**

$$\sigma_t = \frac{F}{A}$$

**True/Nominal Stress (6.9)**

$$\sigma_t = \sigma_n \cdot (1 + \varepsilon_n)$$

**True/Nominal Strain (6.10)**

$$\varepsilon_t = ln(1 + \varepsilon_n)$$

**Plastic Work**

$$W_{pl} = \int_0^{\varepsilon_f} \sigma d\varepsilon_{pl}$$

**Dislocation Shear Force (6.11)**

$$\tau = \alpha \cdot \frac{G \cdot b}{L}$$

## Additional Dislocation Shear Resistance by:

### Solid Solution Hardening (6.18)

$$\tau_{ss} = \alpha \cdot G \cdot \sqrt{c}$$

### Work Hardening (6.20)

$$\tau_{wh} = \frac{\alpha \cdot G}{L} = \alpha \cdot G \cdot b \cdot \sqrt{\rho_d}$$

### Particle Precipitation Hardening (6.19)

$$\tau_{ppt} = \frac{2 \cdot T}{b \cdot L} = \frac{G \cdot b}{L}$$

### Grain Boundary Hardening (6.21)

$$\tau_{gb} = \frac{k_p}{\sqrt{D}}$$

# Chapter 7: Strength-Limited Design

## Must Know

**Beam Yield Stress (7.2)**

$$\sigma_{max,y} = \frac{M \cdot y_m}{I} = \frac{M}{Z_e}$$

**Elastic Modulus Section (7.2)**

$$Z_e = \frac{I}{y_m}$$

**Beam Failure Moment (7.3)**

$$M_f = Z_p \cdot \sigma_y$$

**Plastic Section Modulus (7.3)**

$$Z_p = 2 \cdot \int_0^{h/2} (b \cdot y) dy$$

## Others

**Stress Concentration Factor (7.12)**

$$K_{SC} = \frac{\sigma_{max}}{\sigma_{nom}} = 1 + \alpha \cdot (\frac{c}{\rho_{sc}})^{1/2}$$

For TENSION: $\alpha = 2$
For BUCKLING: $\alpha = 0.5$
For BENDING: $\alpha = 0.5$

**Permanent Deformation Resistance Material Indices for Minimal Weight ***

**Tie-Rod**

$$M_t = \frac{\sigma_y}{\rho}$$

**Plate**

$$M_p = \frac{\sigma_y{}^{1/2}}{\rho}$$

**Beam**

$$M_b = \frac{\sigma_y{}^{2/3}}{\rho}$$

**Spring**

$$M_s = \frac{\sigma_y{}^2}{E}$$

# Chapter 8: Fracture & Fracture Toughness

## Must Know

**Mode 1 Stress Intensity (8.3)**

$$K_1 = Y \cdot \sigma\sqrt{\pi \cdot c}$$

**Fracture Toughness (8.3/8.10)**

$$K_{1c} = \sqrt{E \cdot G_c} = Y \cdot \sigma^* \sqrt{\pi \cdot c}$$

**Critical Crack Length (8.13)**

$$c_{crit} = \frac{{K_{1c}}^2}{Y^2 \cdot \pi \cdot \sigma_y^2}$$

## Others

**Energy Release Rate (8.5)**

$$G \geq 2\gamma$$

**Surface Energy (p.215)**

$$\gamma = \frac{1}{3} \cdot H_c \cdot r_0$$

**Stored Elastic Energy (8.6)**

$$U_V = \frac{\sigma^2}{2E}$$

**Crack Released Energy (8.7)**

$$U(c) = \frac{\sigma^2}{2E} \cdot \frac{\pi \cdot c^2}{2}$$

**Toughness (8.9)/(8.10)**

$$G_c = \frac{{K_{1c}}^2}{E}$$

**Plastic Region Radius (8.11)**

$$r_y = 2 \cdot (\frac{\sigma \cdot c}{2 \cdot {\sigma_y}^2})$$

# Chapters 9/10: Cyclic Loading & Fatigue Failure Design

## Must Know

**Stress Ampl. (9.1)**

$$\sigma_A = \frac{\Delta\sigma}{2}$$

**Mean Stress (9.2)**

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

**R-Value (9.3)**

$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

## Others

**Basquin's Law (9.4)**

$$\Delta\sigma \cdot {N_f}^b = C_1 \quad [\sigma_A < \sigma_y]$$

**Coffin's Law (9.6)**

$$\Delta\varepsilon_{pl} \cdot {N_f}^c = C_2 \quad [\sigma_A > \sigma_y]$$

**Goodman's Rule (9.7)**

$$\Delta\sigma_{\sigma_m} = \Delta\sigma_{\sigma_0} \cdot (1 - \frac{\sigma_m}{\sigma_{ts}})$$

**Miner's Rule (9.8)**

$$\sum_{i=1}^{n} \frac{N_i}{N_{f,i}} = 1$$

**Stress Intensity Range (9.9)**

$$\Delta K = \Delta\sigma\sqrt{\pi \cdot c}$$

**Paris Law (9.10)**

$$\frac{dc}{dN} = A \cdot \Delta K^m$$

# Chapter 12: Materials & Heat

## Must Know

**Units**

**Thermal Expansion Coefficient**

$$\alpha[K^{-1}]$$

**Specific Heat Capacity**

$$C_p[J \cdot kg^{-1} \cdot K^{-1}]$$

**Thermal Conductivity**

$$\lambda[W \cdot m^{-1} \cdot K^{-1}]$$

**Heat Flux**

$$q[W \cdot m^{-2}]$$

**Thermal Diffusivity**

$$a[m^2 \cdot s^{-1}]$$

## Others

**Thermal Exp. Coefficient (12.1)**

$$\alpha = \frac{\Delta L}{L_0 \cdot \Delta T}$$

**Heat Flux (12.2)**

$$q = -\lambda \cdot \frac{\Delta T}{\Delta x}$$

**Thermal Diffusivity (12.3)**

$$a = \frac{\lambda}{\rho \cdot C_p}$$

**Thermal Conductivity (12.10)**

$$\lambda = \frac{\rho \cdot C_p}{3} \cdot l_m \cdot C_0$$

### Volumetric Heat Capacity (12.4/12.6/12.7)

$$\rho \cdot C_p = \frac{3k_b}{\Omega} \quad [2 \cdot 10^6 < \rho \cdot C_p < 3 \cdot 10^6]$$

### Thermal Shock Resistance (12.14)

$$\sigma = \frac{E}{1-\nu} \cdot \alpha \cdot \Delta T$$

# Chapter 13: Diffusion & Creep

## Others

**Property Approximation (13.1)**

$$P_n \approx P_0 \cdot (1 + \beta \cdot \frac{T_0}{T_n})$$

**Viscous Flow (13.2)**

$$\dot{\varepsilon} = \frac{\sigma}{3\eta}$$

**Steady-State Creep (13.3/13.6)**

$$\dot{\varepsilon_{ss}} = C' \cdot \sigma^n \cdot e^{-\frac{Q_c}{R \cdot T}} = \dot{\varepsilon}_0 \cdot (\frac{\sigma}{\sigma_0})^n \cdot e^{-\frac{Q_c}{R \cdot T}}$$

**Fick's Law (13.11)**

$$J = -D \cdot \frac{\Delta c}{\Delta x}$$

**Diffusion Coefficient (13.12)**

$$D = D_0 \cdot e^{-(\frac{Q_D}{R \cdot T})}$$

# Chapter 14: Durability

## Must Know

**Oxidation Reaction (14.1)**

$$x \cdot M + y \cdot O \leftrightarrow M_xO_y + \Delta E$$

**Oxidation Weight Gain (14.2/14.3)**

$$\Delta m = k_l \cdot t \quad \text{or} \quad \Delta m^2 = k_p \cdot t$$

**Oxidation Half-Reactions (p.384)**

**Metal**

$$M = M^{2+} + 2e^-$$

**Oxygen**

$$O + 2e^- = O^{2-}$$

**Water Dissociation (14.7)**

$$H_2O \leftrightarrow H^+ + OH^-$$

**Law of Mass Action (14.8)**

$$[H^+] \cdot [OH^-] = 10^{-14}$$

**Hydrolisis Reaction (14.18)**

$$H_2O + O + 2e^- \leftrightarrow 2OH^-$$

**Acidity (14.9)**

$$pH = -log[H^+]$$

## Others

**Nerst Equation (14.18)**

$$E = E_0 + \frac{0.059}{z} \cdot log(C_{ion})$$

# Glossary

## Chapter 4

| Symbol | Meaning | Symbol | Meaning |
|---|---|---|---|
| $\sigma$ | Normal Stress | $\varepsilon_t$ | Transversal Strain |
| $F$ | Normal Force | $\widetilde{E_U}$ | Upper-Bound Hybrid Elastic Modulus |
| $A_0$ | Initial Cross-Sectional Area | $\widetilde{E_\parallel}$ | Hybrid Elastic Modulus Parallel to Reinforcement |
| $\tau$ | Shear Stress | $f_r$ | Volume of Reinforcement |
| $F_s$ | Shear Force | $E_r$ | Reinforcement Elastic Modulus |
| $p$ | Pressure | $E_m$ | Matrix Elastic Modulus |
| $\varepsilon$ | Normal Strain | $\widetilde{E_L}$ | Lower-Bound Hybrid Elastic Modulus |
| $\Delta L$ | Elongation | $\widetilde{E_\perp}$ | Hybrid Elastic Modulus Perpendicular to Reinforcement |
| $L_0$ | Initial Length | $r\tilde{h}o$ | Hybrid Density |
| $\gamma$ | Shear Strain | $\rho_r$ | Reinforcement Density |
| $w$ | Shear Displacement | $\rho_m$ | Matrix Density |
| $\Delta$ | Volumetric Strain | $\alpha$ | Thermal Expansion Coefficient |
| $\Delta V$ | Change in Volume | $\varepsilon_T$ | Thermal Strain |
| $V_0$ | Initial Volume | $\Delta T$ | Change in Temperature |
| $E$ | Young's (ELastic) Modulus | $S$ | Stiffness |
| $G$ | Shear Modulus | $\delta$ | Displacement/Deflection |
| $K$ | Bulk Modulus | $a_0$ | Atomic Radius |
| $\nu$ | Poisson's Ratio | | |

# Chapter 5

| Symbol | Meaning | Symbol | Meaning |
|---|---|---|---|
| $\delta$ | Displacement/Deflection | $c_1$ | Configuration-Specific Bending Constant |
| $L_0$ | Initial Length | $L$ | Length |
| $F$ | Force Applied | $F_{crit}$ | Critical Buckling Force |
| $A$ | Cross-Sectional Area | $n$ | Configuration-Specific Buckling Constant |
| $E$ | Young's (Elastic) Modulus | $m$ | Mass |
| $S$ | Stiffness | $\rho$ | Density |
| $\sigma$ | Stress | $M_t$ | Tie-Rod Material Index |
| $y_m$ | Vertical Distance to Neutral Axis | $M_p$ | Plate Material Index |
| $M$ | Bending Moment | $M_b$ | Beam Material Index |
| $I$ | Second Moment of Area | $\phi$ | Shape Factor |
| $k$ | Beam Bending Curvature | $I_{square}$ | Equivalent Square Cross-Section 2nd MM of Area |

# Chapter 6

| Symbol | Meaning | Symbol | Meaning |
|---|---|---|---|
| $\varepsilon_{pl}$ | Plastic Strain | $\tau_{wh}$ | Work Hardening DSR |
| $\varepsilon_{tot}$ | Total Strain | $\tau_{gb}$ | Grain Boundary Hardening DSR |
| $\sigma$ | Stress | $\sigma_t$ | True Stress |
| $E$ | Young's (Elastic) Modulus | $\sigma_n$ | Nominal Stress |
| $H$ | Indentation Hardness | $\varepsilon_t$ | True Strain |
| $F$ | Force Applied | $\varepsilon_n$ | Nominal Strain |
| $A$ | Indentation Area | $W_{pl}$ | Work due to Plastic Deoformation |
| $\tau$ | Dislocation Shear Resistance (DSR) | $\varepsilon_f$ | Final Strain |
| $\tau_y$ | Total DSR | $\alpha$ | Obstacle Strength Constant |
| $\tau_i$ | Initial DSR | $G$, $b$ | Material-Specific Constants |
| $\tau_{ss}$ | Solid Solution Hardening DSR | $L$ | Average Obstacle Spacing |
| $\tau_{ppl}$ | Particle Precipitation Hardening DSR | $\rho_d$ | Dislocation Density |

# Chapter 7

| Symbol | Meaning | Symbol | Meaning |
|---|---|---|---|
| $\sigma_{max,y}$ | Beam Yield Stress | $\sigma_{nom}$ | Nominal Stress |
| $M$ | Bending Moment | $\alpha$ | Loading Mode Specific Constant |
| $y_m$ | Vertical Distance to Neutral Axis | $c$ | Characteristic Dimension |
| $I$ | Second Moment of Area | $\rho_{sc}$ | Minimum Radius of Feature |
| $Z_e$ | Elastic Modulus Section | $\rho$ | Density |
| $M_f$ | Beam Failure Bending Moment | $M_t$ | Tie-Rod Material Index |
| $Z_p$ | Plastic Modulus Section | $M_p$ | Plate Material Index |
| $\sigma_y$ | Yield Stress | $M_b$ | Beam Material Index |
| $K_{SC}$ | Stress Concentration Factor | $M_s$ | Spring Material Index |
| $\sigma_{max}$ | Maximum Local Stress | $E$ | Young's (Elastic) Modulus |

# Chapter 8

| Symbol | Meaning | Symbol | Meaning |
|---|---|---|---|
| $K_1$ | Mode 1 Stress Intensity Factor | $G_c$ | Toughness |
| $Y$ | Geometry-Specific Contant | $c_{crit}$ | Critical Crack Length |
| $\sigma$ | Stress | $G$ | Energy Release Rate |
| $c$ | Crack Length | $\gamma$ | Surface Energy |
| $K_{1c}$ | Fracture Toughness | $U_V$ | Stored Elastic Energy |
| $\sigma^*$ | Failure Stress | $U(c)$ | Crack propagation Energy Release |
| $E$ | Young's (Elastic) Modulus | | |

# Chapters 9 & 10

| Symbol | Meaning | Symbol | Meaning |
|---|---|---|---|
| $\sigma_A$ | Stress Amplitude | $\Delta\sigma_{pl}$ | Plastic Strain Increment |
| $\Delta\sigma$ | Stress Increment | $c$, $C_2$ | Coffin's Law Specific Constants |
| $\sigma_m$ | Mean Stress | $\Delta\sigma_{\sigma_0}$ | Stress Increment at 0 Mean Stress |
| $\sigma_{max}$ | Maximum Stress | $\Delta\sigma_{\sigma_m}$ | Equivalent Stress Increment |
| $\sigma_{min}$ | Minimum Stress | $\sigma_{ts}$ | Ultimate Tensile Stress |
| $R$ | R-Value | $N$ | Fatigue Cycles |
| $\sigma_y$ | Yield Stress | $\Delta K$ | Stress Intensity Range |
| $N_f$ | Fatigue Cycles until Failure | $\frac{dc}{dN}$ | Crack Length Growth per Fatigue Cycle |
| $b$, $C_1$ | Basquin's Law Specific Constants | $A$, $m$ | Paris Law Specific Constants |

# Chapter 12

| Symbol | Meaning | Symbol | Meaning |
|---|---|---|---|
| $\alpha$ | Thermal Expansion Coefficient | $\Delta T$ | Change in Temperature |
| $C_p$ | Specific Heat Capacity | $\frac{\Delta T}{\Delta x}$ | Uniform Temperature Gradient |
| $\lambda$ | Thermal Conductivity | $\rho$ | Density |
| $q$ | Heat Flux | $l_m$ | Mean Free Path |
| $a$ | Thermal Diffusivity | $C_0$ | Speed of Sound |
| $\Delta L$ | Elongation | $k_b$ | Boltzmann Constant |
| $L_0$ | Initial Length | $\Omega$ | Atom Volume |

# Chapter 13

| Symbol | Meaning | Symbol | Meaning |
|---|---|---|---|
| $P_n$ | New Property Value | $\dot{\varepsilon}_0$ | Initial Creep |
| $P_0$ | Initial Property Value | $\sigma_0$ | Initial Stress |
| $\beta$ | Specific Proportionality Constant | $Q_c$ | Creep Activation Energy |
| $T_0$ | Initial Temperature | $R$ | Gas Constant |
| $T_n$ | New Initial Temperature | $T$ | Temperature |
| $\dot{\varepsilon}$ | Creep | $J$ | Net Flux of Diffused Atoms per Second |
| $\sigma$ | Stress | $D$ | Diffusion Coefficient |
| $\eta$ | Viscosity | $\frac{\Delta c}{\Delta x}$ | Atom Concentration Gradient |
| $\dot{\varepsilon}_{ss}$ | Steady-State Creep | $D_0$ | Diffusion Contant |
| $B$, $n$ | Steady-State Creep Specific Constants | $Q_D$ | Activation Energy per Mole |

# Chapter 14

| Symbol | Meaning | Symbol | Meaning |
|---|---|---|---|
| $x$ | Moles of Metal | $H^+$ | Hydrogen Cation (Proton) |
| $M$ | Any Metal | $OH^-$ | Hydroxyl Group |
| $y$ | Moles of Oxygen | $[H^+]$ | Proton Concentration |
| $O$ | Oxygen | $[OH^-]$ | Hydroxyl Concentration |
| $M_xO_y$ | Resulting Metal Oxide | $e^-$ | Free Electron |
| $\Delta E$ | Energy | $pH$ | Hydrogen Potential |
| $\Delta m$ | Mass Increase | $E$ | Corrected Reduction Potential |
| $t$ | Time | $E_0$ | Standard Reduction Potential |
| $k_t$ | Linear Kinetic Constant | $z$ | Ion Valence |
| $k_p$ | Parabolic Kinetic Constant | $C_{ion}$ | Ion Concentration |
| $H_2O$ | Water | | |