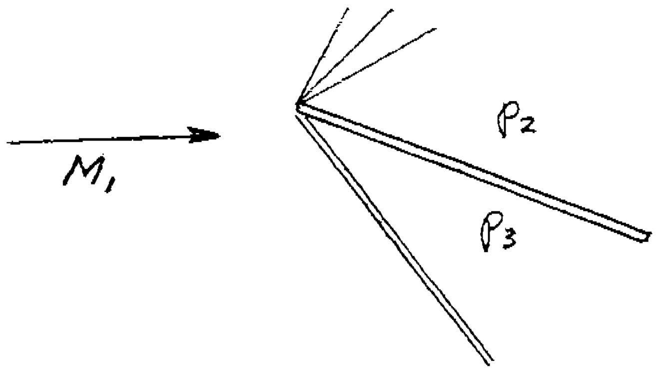
CHAPTER 14

14.1



$$c_\ell = \left(C_{p_3} - C_{p_2}\right)\cos\alpha$$

$$c_d = \left(C_{p_3} - C_{p_2}\right)\sin\alpha$$

(a) Using straight Newtonian theory:

$$C_p = 2\sin^2\alpha$$

For $\alpha = 5°$:

$$C_{p_3} = 2\sin^2 5° = 0.0152$$

$$C_{p_2} = 0$$

$$c_\ell = 0.0152\cos 5° = \boxed{0.0151}$$

$$c_d = 0.0152\sin 5° = \boxed{0.00132}$$

For $\alpha = 15°$:

$$C_{p_3} = 2\sin^2 15° = 0.1340,\ C_{p_2} = 0$$

$$c_\ell = 0.1340\cos 15° = \boxed{0.129}$$

$$c_d = 0.1340\sin 15° = \boxed{0.0347}$$

For $\alpha = 30°$:

$$C_{p_3} = 2 \sin^2 30° = 0.5$$

$$c_\ell = 0.5 \cos 30° = \boxed{0.433}$$

$$c_d = 0.5 \sin 35° = \boxed{0.25}$$

(b) Using modified Newtonian:

$$C_p = C_{p_{max}} \sin^2 \alpha$$

$$C_{p_{max}} = \frac{p_o - p_\infty}{q_\infty} = \frac{p_o - p_\infty}{\frac{\gamma}{2} M_\infty^2 p_\infty} = \frac{2}{\gamma M_\infty^2}\left(\frac{p_o}{p_\infty} - 1\right)$$

$$C_{p_{max}} = \frac{2}{(1.4)(2.6)^2} (9.181 - 1) = 1.729$$

For $\alpha = 15°$

$$C_{p_3} = 1.729 \sin^2 5° = 0.0131$$

$$c_\ell = 0.0131 \cos 5° = \boxed{0.0131}$$

$$c_d = 0.0131 \sin 5° = \boxed{0.00114}$$

For $\alpha = 15°$

$$C_{p_3} = 1.729 \sin^2 15° = 0.1158$$

$$c_\ell = 0.1158 \cos 15° = \boxed{0.1119}$$

$$c_d = 0.1158 \sin 15° = \boxed{0.030}$$

For $\alpha = 30°$

$$C_{p_3} = 1.729 \sin^2 30° = 0.4323$$

$$c_\ell = 0.4323 \cos 30° = \boxed{0.374}$$

$$c_d = 0.4323 \sin 30° = \boxed{0.216}$$

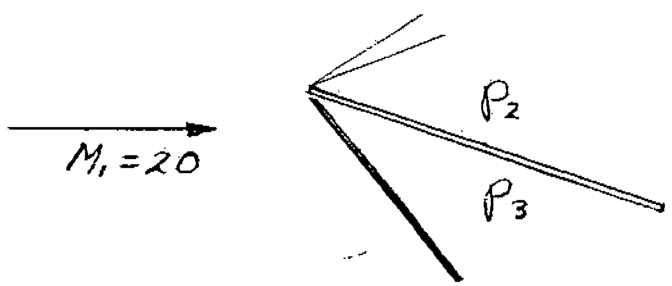
Comparison:

α	Exact c_ℓ (Prob. 9.13)	Newtonian c_ℓ	% error	Mod. Newtonian c_ℓ	% error
5°	0.148	0.0151	90	0.0131	91
15°	0.452	0.129	71	0.1119	75.2
30°	1.19	0.433	63.6	0.374	68.6

α	Exact c_d (Prob. 9.13)	Newtonian c_ℓ	% error	Mod. Newtonian c_d	% error
5°	0.0129	0.00132	90	0.00114	91
15°	0.121	0.0347	71	0.03	75.2
30°	0.687	0.25	63.6	0.216	68.6

Conclusion: Newtonian theory gives terrible results for a flat plate a moderate α at low Supersonic Mach numbers.

14.2



From Newtonian theory:

$$C_p = 2\sin^2\alpha = 2\sin^2 20° = 0.234$$

$$c_\ell = 0.234 \ \cos\alpha = \boxed{0.220}$$

$$c_d = 0.234 \sin\alpha = \boxed{0.08}$$

From shock-expansion theory:

<u>On the top surface</u>: $\nu_2 = \nu_1 + \theta = 116.2 + 20 = 136.20$

This is beyond the maximum expansion angle. Hence, a "void" exists on the top surface, i.e., $p_2 = 0$.

<u>On the bottom surface</u>: From the θ-β-M diagram,

$$\beta = 24.9°$$

$$M_{n_1} = M_1 \sin\beta = 20 \sin 24.9° = 8.4$$

$$\frac{p_3}{p_1} = 82.15$$

From Prob. 9.13:

$$c_\ell = \frac{2}{\gamma\, M_\infty^2}\left(\frac{p_3}{p_1} - \frac{p_2}{p_1}\right)\cos\alpha$$

and

$$c_d = c_\ell \frac{\sin\alpha}{\cos\alpha}$$

$$c_\ell = \frac{2}{(1.4)(20)^2}(82.15 - 0)\cos 20° = 0.2757$$

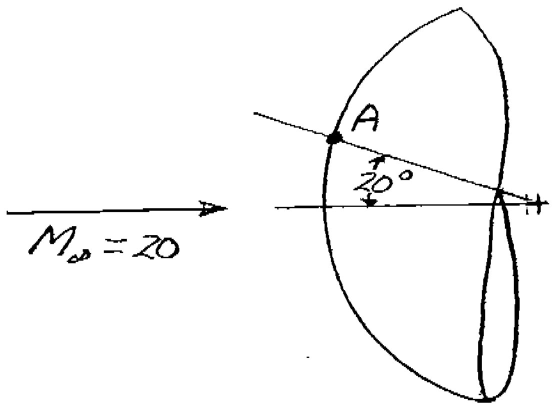
$$c_d = 0.2757 \ \mathrm{Tan}\ 20° = 0.100$$

For c_ℓ: $\%\ \text{error} = \dfrac{0.2757 - 0.220}{0.2757} = 20\%$

For c_d: $\%\ \text{error} = \dfrac{0.100 - 0.08}{0.10} = 20\%$

<u>Note:</u> Newtonian theory works much better for blunt bodies, i.e., for large values of θ.

14.3



(a) Use Eq. (14.7) to estimate the pressure at point A. We first need to obtain $C_{p,max}$, which is a function of $p_{o,2}/p_\infty$. From Appendix B for $M_\infty = 20$, $p_{o,2}/p_\infty = 0.5155 \times 10^3$. Hence,

$$C_{p,max} = \frac{2}{\gamma\ M_\infty^2}\left(\frac{p_{o,2}}{p_\infty} - 1\right) = \frac{2}{(1.4)(20)^2}\ (515.3 - 1) = 1.837$$

From Eq. (14.7), at point A on the surface

$$C_{p_A} = C_{p,max} \sin^2\theta = (1.837) \sin^2 20° = 0.2149$$

Since

$$C_{p_A} = \frac{2}{\gamma\ M_\infty^2}\left(\frac{p_A}{p_\infty} - 1\right)$$

then,

$$\frac{p_A}{p_\infty} = \frac{\gamma\ M_\infty^2 C_{p_A}}{2} + 1 = \frac{(1.4)(20)^2(0.2149)}{2} + 1 = 61.17$$

Hence,

$$P_A = 61.17\ (3.06) = \boxed{187.2\ lb/ft^2}$$

(b) The stagnation temperature is found from Eq. (8.40)

$$\frac{T_o}{T_\infty} = 1 + \frac{\gamma - 1}{2}\ M_\infty{}^2 = 1 + 0.2\ (20)^2 = 81$$

Assuming an isentropic flow from the stagnation point to point A,

$$\frac{p_A}{p_{o,2}} = \frac{p_A / p_\infty}{p_{o,2} / p_\infty} = \left(\frac{T_A}{T_o}\right)^{\frac{\gamma}{\gamma-1}}$$

or,

$$\frac{T_A}{T_o} = \left(\frac{61.17}{515.5}\right)^{\frac{\gamma-1}{\gamma}} = (0.1187)^{0.2857} = 0.5439$$

$$T_A = \frac{T_A}{T_o}\left(\frac{T_o}{T_\infty}\right) T_\infty = (0.5439)(81)(500) = \boxed{22{,}028°R}$$

(Please note. Relative to our discussion in Problems 8.17 and 8.18, we know this estimate of T_A to be too large because we are not taking into account the effect of chemically reacting flow.)

(c) At point A, for an isentropic flow, $p_{o,A} = p_{o,2}$

$$\frac{p_{o,A}}{p_A} = \left(1 + \frac{\gamma-1}{2} M_A^2\right)^{\frac{\gamma}{\gamma-1}} = \frac{p_{o,2} / p_\infty}{p_A / p_\infty} = \frac{515.5}{61.17} = 8.427$$

$$1 + \frac{\gamma-1}{2} M_A^2 = (8.427)^{\frac{\gamma-1}{\gamma}} = (8.427)^{0.2857} = 1.8385$$

$$M_A^2 = (1.8385 - 1)\ \frac{2}{\gamma-1} = (0.8385)(5) = 4.1925$$

$$\boxed{M_A = 2.05}$$

(d) $a_A = \sqrt{\gamma R T_A} = \sqrt{(1.4)(1716)(22{,}028)} = 7275$ ft/sec

$$V_A = a_A M_A = (7275)(2.05) = \boxed{1.49 \times 10^4 \text{ ft/sec}}$$

Note: Once again, this estimate of V_A is too high because T_A, hence a_A, is too high.

Also note: The purpose of this problem is to illustrate that, from the Newtonian sine-squared law for pressure variations, the other flow field quantities can also be obtained.
