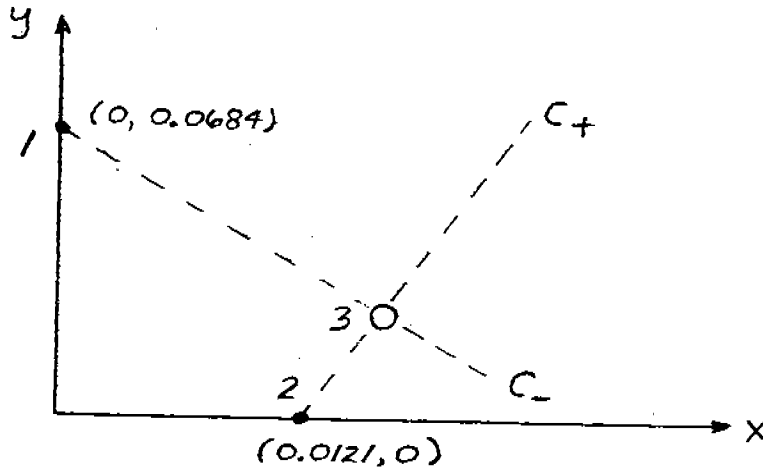


CHAPTER 13

13.1



At point 1:

$$a_1 = \sqrt{\gamma RT_1} = \sqrt{(1.4)(287)(288)} = 340 \text{ m/sec}$$

$$V_1 = \sqrt{u_1^2 + v_1^2} = \sqrt{(639)^2 + (232.6)^2} = 680 \text{ m/sec}$$

$$M_1 = \frac{V_1}{a_1} = \frac{680}{340} = 2$$

$$\theta_1 = \text{Tan}^{-1} \frac{v_1}{u_1} = \text{Tan}^{-1} \left( \frac{232.6}{639} \right) = 20^\circ$$

$$v_1 = (M_1) = 26.38^\circ$$

$$K_- = \theta + v = 20 + 26.38 = 46.38^\circ$$

At point 2:

$$a_2 = \sqrt{\gamma RT_2} = \sqrt{(1.4)(287)(288)} = 340 \text{ m/sec}$$

$$V_2 = 680 \text{ m/sec}$$

$$M_2 = \frac{V_1}{a_1} = \frac{680}{340} = 2$$

$$\theta_2 = 0^\circ$$

$$v_2 = 26.38^\circ$$

$$K_+ = \theta - v = -26.38^\circ$$

At point 3:

$$\theta_3 = \frac{1}{2} [(K_-)_1 + (K_+)_2] = \frac{1}{2} (46.38 - 26.38) = 10^\circ$$

$$v_3 = \frac{1}{2} [(K_-)_1 + (K_+)_2] = \frac{1}{2} (46.38 - 26.38) = 36.38^\circ$$

$$M_3 = 2.4$$

To obtain the other flow variables at point 3, note that:

$$\frac{P_{o_1}}{P_1} = 7.824 \text{ and } \frac{P_{o_3}}{P_3} = 14.62$$

$$P_3 = \frac{P_3}{P_{o_3}} \frac{P_{o_3}}{P_{o_1}} \frac{P_{o_1}}{P_1} P_1 = \left( \frac{1}{14.62} \right) (1)(7.824)(1 \text{ atm}) = \boxed{0.535 \text{ atm}}$$

$$\frac{T_{o_1}}{T_1} = 1.8 \text{ and } \frac{T_{o_3}}{T_3} = 2.152$$

$$T_3 = \frac{T_3}{T_{o_3}} \frac{T_{o_3}}{T_{o_1}} \frac{T_{o_1}}{T_1} T_1 = \left( \frac{1}{2.152} \right) (1)(1.8)(288) = \boxed{240.9^\circ\text{K}}$$

$$a_3 = \sqrt{\gamma R T_3} = \sqrt{(1.4)(287)(240.9)} = 211.1 \text{ m/sec}$$

$$V_3 = M_3 a_3 = 2.4 (211.1) = 506.6 \text{ m/sec}$$

$$u_3 = V_3 \cos \theta_3 = 506.6 \cos 10^\circ = \boxed{497.3 \text{ m/sec}}$$

$$v_3 = V_3 \sin \theta_3 = 506.6 \sin 10^\circ = \boxed{88.3 \text{ m/sec}}$$

To locate point 3:

Along the  $C_+$  Characteristic:

$$\theta_{ave} = \frac{1}{2} (\theta_2 + \theta_3) = \frac{1}{2} (0 + 10) = 5^\circ$$

$$\mu_{ave} = \frac{1}{2} (\mu_2 + \mu_3) = \frac{1}{2} (30^\circ + 24.62^\circ) = 27.31^\circ$$

$$\frac{dy}{dx} = \text{Tan} (\theta_{ave} + \mu_{ave}) = \text{Tan} (5^\circ + 27.31^\circ) = 0.6324$$

Thus:

$$y = 0.6324 x - 0.00765 \quad (1)$$

Along the C. characteristic:

$$\theta_{ave} = \frac{1}{2} (\theta_1 + \theta_3) = \frac{1}{2} (20^\circ + 10^\circ) = 15^\circ$$

$$\mu_{ave} = \frac{1}{2} (\mu_1 + \mu_3) = \frac{1}{2} (30 + 24.62) = 27.31^\circ$$

$$\frac{dy}{dx} = \text{Tan} (\theta_{ave} - \mu_{ave}) = \text{Tan} (15^\circ - 27.31^\circ) = -0.2182$$

$$y = -0.2182 x + 0.0684 \quad (2)$$

Point 3 lies at the intersection of Eqs. (1) and (2)

$$y = 0.6324 x - 0.00765$$

$$y = -0.2182 x + 0.0684$$

Solving simultaneously:  $x = 0.0894$

$$y = 0.0489$$

Thus:  $(x_3, y_3) = (0.0894, 0.0489)$

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