CHAPTER 10

10.1 From Table A.1, for $A_e/A^* = 2.193$, $\boxed{M_e = 2.3}$

$$\frac{p_{o_e}}{p_e} = 12.5, \quad \frac{T_{o_e}}{T_e} = 2.058.$$

For isentropic flow, T_o = constant and p_o = constant. Hence,

$$p_{o_e} = p_o = \boxed{5 \text{ atm}}, \text{ and } T_{o_e} = T_o = \boxed{520°R}$$

$$p_e = \frac{p_e}{p_{o_e}} p_o = \left(\frac{1}{12.5}\right) (5 \text{ atm}) = \boxed{0.4 \text{ atm}}$$

$$T_e = \frac{T_e}{T_{o_e}} T_o = \left(\frac{1}{2.058}\right) (520) = \boxed{252.7°R}$$

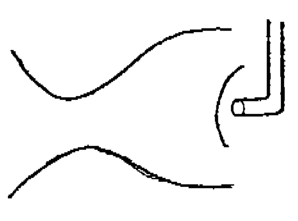
$$\rho_e = \frac{p_e}{RT_e} = \frac{(0.4)(2116)}{(1716)(252.7)} = \boxed{0.00195 \text{ slug/ft}^3}$$

$$a_e = \sqrt{\gamma R T_e} = \sqrt{(1.4)(1716)(252.7)} = 779.2 \text{ ft/sec}$$

$$u_e = M_e \, a_e = (2.3)(779.2) = \boxed{1792 \text{ ft/sec}}$$

10.2 $\dfrac{p_o}{p_e} = \dfrac{1}{0.3143} = 3.182$. From Table A.1, we see that $\boxed{M_e = 1.4}$, and $A_e/A^* = \boxed{1.115}$.

10.3 Ahead of the normal shock in front of the Pitot tube,



$$p_{o_1} = p_o = 2.02 \times 10^5 \text{ N/m}^2$$

$$\frac{p_{o_2}}{p_{o_1}} = \frac{8.92 \times 10^4}{2.02 \times 10^5} = 0.4416$$

From Table A.2: $M_e = 2.65$

From Table A.1: $A_e/A^* = \boxed{3.036}$

10.4 $\dot{m} = \rho^* u^* A^*$; $\rho_o = \dfrac{p_o}{RT_o} = \dfrac{(5)(2116)}{(1716)(520)} = 0.01186 \dfrac{\text{slug}}{\text{ft}^3}$

$$\rho^* = \frac{\rho^*}{\rho_o}\,\rho_o = (0.634)(0.01186) = 0.007519 \text{ slug/ft}^3$$

$$T^* = \frac{T^*}{T_o}\,T_o = (0.833)(520) = 433.2°\text{R}$$

$$u^* = a^* = \sqrt{(1.4)(1716)(433.2)} = 1020 \text{ ft/sec}$$

$$\dot{m} = \rho^* u^* A^* = (0.007519)(1020)\left(\frac{4}{144}\right) = \boxed{0.213 \ \frac{\text{slug}}{\text{sec}}}$$

10.5 $\dot{m} = \rho^* u^* A^*$

$$u^* = \sqrt{\gamma R T^*} \text{ and } \rho^* = \frac{p^*}{RT^*}$$

Hence,

$$\dot{m} = \frac{p^*}{RT^*}\,A^*\,\sqrt{\gamma R T^*} = \frac{p^* A^*}{RT^*}\,\sqrt{\gamma}$$

Since, $M^* = 1$, then

$$\frac{T_o}{T^*} = 1 + \frac{\gamma - 1}{2}\,M^{*2} = \frac{\gamma + 1}{2}$$

$$\frac{p_o}{p^*} = \left(\frac{\gamma + 1}{2}\right)^{\gamma/(\gamma - 1)}$$

Thus,

$$\dot{m} = \sqrt{\frac{\gamma}{R}}\ A^* \left(\frac{\gamma+1}{2}\right)^{-/(\gamma+1)/2(\gamma-1)} \frac{p_o}{\sqrt{T_o}}$$

or,

$$\boxed{\dot{m} = \frac{p_o A^*}{\sqrt{T_o}} \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{(\gamma+!)/(\gamma-1)}}}$$

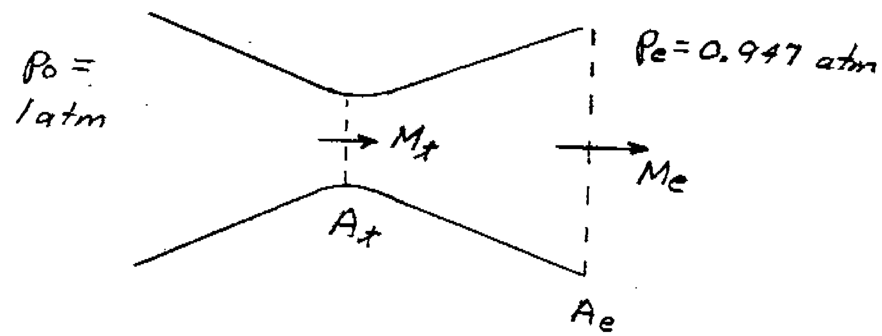
10.6 $p_o = 5$ atm $= 5(2116) = 10580$ lb/ft^2

$A^* = 4/144 = 0.02778$ ft^2

$$\dot{m} = \frac{(105809)(0.02778)}{\sqrt{520}} \sqrt{\frac{(1.4)}{(1716)}\left(\frac{2}{2.4}\right)^6} = \boxed{0.213\ \frac{\text{slug}}{\text{sec}}}$$

which is the same as obtained in Problem 10.4

10.7



First, check to see if the flow is sonic at the throat.

$$\frac{p_o}{p_e} = \frac{1}{0.947} = 1.056$$

From Table A., for $\frac{p_o}{p_e} = 1.056$: $M_e = 0.28$ and $A_e/A^* = 2.166$

Since $\frac{A_e}{A_t} = 1.616 < \frac{A_e}{A^*} = 2.166$, then $A_t > A^*$. The throat size is larger than that for sonic flow, hence the throat Mach number, M_t, is subsonic.

$$\frac{A_t}{A^*} = \frac{A_t}{A_e}\frac{A_e}{A^*} = \frac{1}{1.616}(2.166) = 1.34$$

From Table A.1, for $\frac{A_t}{A^*} = 1.34$; $\boxed{M_t = 0.5}$, $\frac{p_o}{p_t} = 1.186$

$$p_t = \frac{p_t}{p_o}\frac{p_o}{p_e}p_e = \left(\frac{1}{1.186}\right)(1.056)(0.947) = \boxed{0.843 \text{ atm}}$$

10.8 Note: The equation for $\dot{m}$ given in Problem 10.5 can not be used here because the flow is not choked, i.e., the throat Mach number is not sonic.

$$\dot{m} = \rho_e A_e u_e$$

From Table A.1, for $\frac{p_o}{p_e} = 1.056$: $M_e = 0.28$, $\frac{T_o}{T_e} = 1.016$

$$T_e = T_o/1.016 = 288/1.016 = 283.5°K$$

$$\rho_e = \frac{p_e}{RT_e} = \frac{(0.947)(1.01 \times 10^5)}{(287)(283.5)} = 1.176 \text{ kg/m}^3$$

$$a_e = \sqrt{\gamma R T_e} = \sqrt{(1.4)(287)(283.5)} = 337.5 \text{ m/sec}$$

$$u_e = M_e a_e = (0.28)(337.5) = 94.5 \text{ m/sec}$$

$$A_e = A_t \left(\frac{A_e}{A_t}\right) = (0.3)(1.616) = 0.4848 \text{ m}^2$$

$$\dot{m} = \rho_e A_e u_e = (1.176)(0.4848)(94.5) = \boxed{53.88 \text{ kg/sec}}$$

10.9 (a) $\frac{p_o}{p_e} = \frac{1}{0.94} = 1.064.$

From Table A.1: $M_e = 0.3$ and $A_e/A^* = 2.035$. $\frac{A_t}{A^*} \frac{A_t}{A_e} \frac{A_e}{A^*} = \left(\frac{1}{1.53}\right)(2.035) = 1.33.$

Since $A_t > A^*$, then the flow is completely subsonic. No shock wave exists. Hence, from Table A.1, for $\frac{p_o}{p_e} = 1.064$, $\boxed{M_e = 0.3}$.

(b) $\frac{p_o}{p_e} = \frac{1}{0.886} = 1.129.$

From Table A.1, for $\frac{p_o}{p_e} = 1.129$: $M_e = 0.42$ and $\frac{A_e}{A^*} = 1.539.$

$$\frac{A_t}{A^*} = \frac{A_t}{A_e} \frac{A_e}{A^*} = \left(\frac{1}{1.53}\right)(1.529) = 0.999 \approx 1.0.$$

Hence, $A_t = A^*$, and the flow is precisely sonic at the throat. It is subsonic everywhere else. Hence, from the above $\boxed{M_e = 0.42}$.

(c) From the above results, clearly when p_e is reduced below 0.866 atm, sonic flow will occur at the throat, and the nozzle will be choked. Since $p_e = 0.75$ atm is far above the supersonic exit pressure, we suspect that a normal shock wave exists within the nozzle. Note that, if we run the same calculation as in parts (a) and (b) above, we find:

$$\frac{p_o}{p_e} = \left(\frac{1}{0.75}\right) = 1.333.$$

From Table A.1, for $\frac{p_o}{p_e} = 1.333$, we have

$$\frac{A_e}{A^*} = 1.127$$

$\frac{A_t}{A^*} = \frac{A_t}{A_e}\frac{A_e}{A^*} = \left(\frac{1}{1.53}\right)(1.127) = 0.7366$. Since it is impossible for $A_t < A^*$, then clearly the flow can not be completely isentropic. There must be a shock wave inside the nozzle, with a consequent change in both p_o and A^* across the shock. Hence, the above calculation is meaningless. Instead, set up the following trial-and-error process as follows:

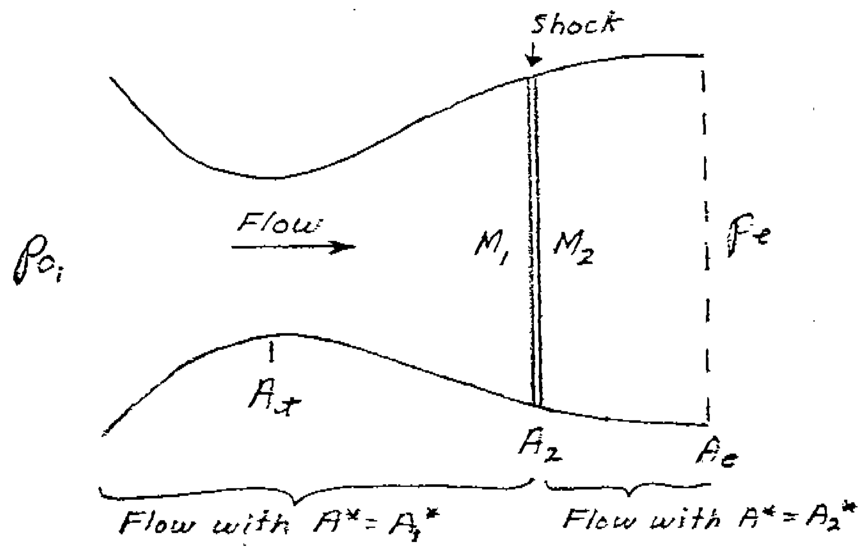
Assume a normal shock exists inside the nozzle, say at a location where $A_2/A_t =$ 1.024. Let:

A_1^* = sonic throat area for the flow ahead of the shock.

A_2^* = sonic throat area for the flow behind the shock.

p_{o_1} = total pressure for the flow ahead of shock.

p_{o_2} = total pressure for the flow behind shock.



Note that $p_{o_2} < p_{o_1}$ $A_2^* > A_1^*$

which comes from the shock wave theory discussed in the text.

Key equation:

$$p_e = \frac{p_e}{p_{o_2}} \frac{p_{o_2}}{p_{o_1}} p_{o_1} \tag{1}$$

To find the values of the ratios in Eq. (1):

From Table A.1 for $A_2/A_1^* = 1.204$: $M_1 = 1.54$

From Table A.2 for $M_1 = 1.54$: $M_2 = 0.6874$, $\frac{p_{o_2}}{p_{o_1}} = 0.9166$

From Table A.1, for $M_2 = 0.6874$: $\frac{A_2}{A^*} = 1.1018$

$$\frac{A_e}{A_2^*} = \frac{A_e}{A_t} \frac{A_t}{A_2} \frac{A_2}{A_2^*} = (1.53)\left(\frac{1}{1.204}\right)(1.1018) = 1.4$$

From Table A.1, for $\frac{A_e}{A_2^*} = 1.4$: $M_e = 0.47$, $\frac{p_{o_2}}{p_e} = 1.163$

Returning to Eq. (1):

$$p_e = \frac{p_e}{p_{o_2}} \frac{p_{o_2}}{p_{o_1}} p_{o_1} = \left(\frac{1}{1.163}\right)(0.9166)(1 \text{ atm}) = 0.788 \text{ atm.}$$

This is slightly higher than the given $p_e = 0.75$. Hence, move the shock wave slightly downstream.

<u>Assume</u> $A_2/A_t = 1.301$

From Table A.1: $M_1 = 1.66$

From Table A.1, for $M_1 = 1.66$: $\frac{p_{o_2}}{p_{o_1}} = 0.872$, $M_2 = 0.6512$

From Table A.1, for $M_2 = 0.6512$: $\frac{A_2}{A_2^*} = 1.1356$

$$\frac{A_e}{A_2^*} = \frac{A_e}{A_t} \frac{A_t}{A_2} \frac{A_2}{A_2^*} = (1.53)\left(\frac{1}{1.301}\right)(1.1356) = 1.335$$

From Table A.1, for $\frac{A_e}{A_2{*}} = 1.335$: $M_e = 0.50$, $\frac{p_{o_2}}{p_e} = 1.1862$

From Eq. (1):

$$p_e = \frac{p_e}{p_{o_2}}\frac{p_{o_2}}{p_{o_1}}p_{o_1} = \left(\frac{1}{1.1862}\right)(0.872)(1\ \text{atm}) = 0.735\ \text{atm}.$$

Interpolate: $\frac{A_2}{A_t} = 1.301 - (1.301 - 1.204)\,\frac{0.75 - 0.735}{0.788 - 0.735} = 1.274$

Thus, <u>Assume</u> $A_2/A_t = 1.274$

From Table A.1: $M_1 = 1.63$

From Table A.2: $M_2 = 0.6596$, $\frac{p_{o_2}}{p_{o_1}} = 0.8838$

From Table A.1: $\frac{A_2}{A_2{*}} = 1.1265$

$$\frac{A_e}{A_2{*}} = \frac{A_e}{A_t}\frac{A_t}{A_2}\frac{A_2}{A_2{*}} = (1.53)\left(\frac{1}{1.274}\right)(1.1265) = 1.353$$

From Table A.1: $M_e = 0.49$, $\frac{p_{o_2}}{p_e} = 1.178$

$$p_e = \frac{p_e}{p_{o_2}}\frac{p_{o_2}}{p_{o_1}}p_{o_1} = \left(\frac{1}{1.178}\right)(0.8838)(1\ \text{atm}) = 0.75\ \text{atm}$$

Hence, p_e calculated agrees with p_e given. Thus,

$$\boxed{M_e = 0.49}$$

(d) $\frac{p_{o_1}}{p_e} = \frac{1\ \text{atm}}{0.154\ \text{atm}} = 6.49$

From Table A.1: $\frac{A_e}{A{*}} = 1.53$, which is precisely the given area ratio of the nozzle. Hence, for this case, we have a completely isentropic expansion, where,

$\boxed{M_e = 1.88}$

10.10 From the θ-β-M diagram, for $\theta = 20°$ and $\beta = 41.8°$, we have $M_1 = 2.6$. From Table A.1,

$$\frac{A_e}{A^*} = \boxed{2.896}$$

10.11 From Table A.1, for $\frac{A_e}{A^*} = 6.79$, $M_e = 3.5$

From Table A.2, for $M_e = 3.5$: $\frac{p_{o_2}}{p_{o_1}} = 0.2129$

$$p_{o_1} = \frac{p_{o_1}}{p_{o_2}}\, p_{o_2} = \left(\frac{1}{0.2129}\right)(1.448) = \boxed{6.8 \text{ atm}}$$

10.12 From Table A.1, for $M_e = 2.8$: $\frac{p_{o_e}}{p_e} = 27.14$, $\frac{T_{o_e}}{T_e} = 2.568$

At standard sea level: $p = 2116 \text{ lb/ft}^2$, $T = 519°R$

$$p_o = \frac{p_{o_e}}{p_e}\, p_e = (27.14)(2116) = \boxed{57{,}430 \text{ lb/ft}^2 = 27.14 \text{ atm}}$$

$$T_o = \frac{T_{o_e}}{T_e}\, T_e = (2.568)(519) = \boxed{1333°R}$$

$$\rho_o = \frac{p_o}{RT_o} = \frac{57430}{(1716)(1333)} = 0.251 \text{ slug/ft}^3$$

$$\rho^* = (0.6339)(0.0251) = 0.0159 \text{ slug/ft}^3$$

$$T^* = 0.833\ (1333) = 1110°R$$

$$a^* = \sqrt{\gamma RT^*} = \sqrt{(1.4)(1716)(1110)} = 1633 \text{ ft/sec} = u^*$$

$$\dot{m} = \rho^* u^* A_1^*$$

$$A_1^* = \frac{\dot{m}}{\rho^* u^*} = \frac{1}{(0.0159)(1633)} = \boxed{0.0385 \text{ ft}^3}$$

From Table A.1: $A_e/A^* = 3.5$

$$A_e = \frac{A_e}{A^*} A^* = (3.5)(0.0385) = \boxed{0.1348 \text{ ft}^2}$$

From Eq. (10.38) in text: $\frac{A_{t_2}}{A_{t_1}} = \frac{A_2^*}{A_1^*} = \frac{p_{o_1}}{p_{o_2}}$

From Table A.2: for $M_e = 2.8$: $\frac{p_{o_2}}{p_{o_1}} = 0.3895$

$$A_{t_2} = A_{t_1}\left(\frac{p_{o_1}}{p_{o_2}}\right) = A_1^*\left(\frac{p_{o_1}}{p_{o_2}}\right) = (0.0385)\left(\frac{1}{0.3895}\right) = \boxed{0.0988 \text{ ft}^2}$$

10.13 $\dot{m} = \rho^* u^* A^*$ (1)

Also, $R = \mathcal{R}/M = \frac{8314}{16} = 519.6 \frac{\text{joule}}{\text{kg K}}$

$$\rho^* = \frac{\rho^*}{\rho_o}\rho_o = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \frac{p_o}{RT_o} = \left(\frac{2}{2.2}\right)^{\frac{1}{0.2}} \frac{p_o}{(519.6)(3600)} = 3.319 \times 10^{-7}\, p_o$$

$$T^* = \frac{T^*}{T_o} T_o = \left(\frac{2}{\gamma+1}\right)(3600) = 3273 \text{ K}$$

$$u^* = a^* = \sqrt{\gamma R T^*} = \sqrt{(1.2)(519.6)(3273)} = 1428.6 \text{ m/sec}$$

Hence, from Eq. (1), with $\dot{m} = 287.2 \frac{\text{kg}}{\text{sec}}$,

$$287.2 = (3.319 \times 10^{-7}\, p_o)(1428.6)(0.2)$$

or,

$$p_o = \frac{287.2}{(3.319x10^{-7})(1428.6)(0.2)} = 3.029 \times 10^6 \ \frac{N}{m^2}$$

or,

$$p_o = \frac{3.029 \times 10^6}{1.01 \times 10^5} = \boxed{30 \text{ atm}}$$

10.14 We assume the flow velocity is low at the diffuser exit; hence the total pressure at the exit is 1 atm. From Appendix B, for M = 3, $\frac{p_{o_2}}{p_{o_1}} = 0.3283$.

$$\eta_D = \frac{p_B / p_o}{p_{o_2} / p_{o_1}} = 1.2$$

$$\frac{p_B}{p_o} = 1.2 \ \frac{p_{o_2}}{p_{o_1}} = 1.2\ (0.3283) = 0.394$$

$$p_o = \frac{p_B}{0.394} = \frac{1}{0.394} = \boxed{2.54 \text{ atm}}$$
