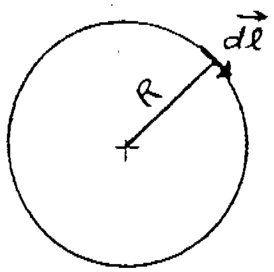
CHAPTER 5

5.1

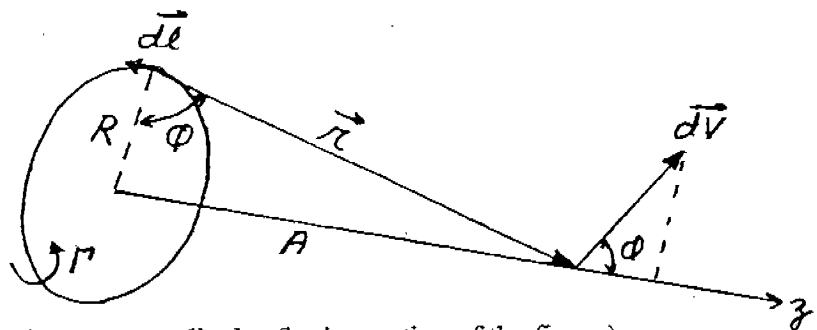


$\vec{d\ell} \times \vec{r} = (R\ d\ell)\ \vec{e}$

where $\vec{e}$ is a unit vector perpendicular to the plane of the loop, directed <u>into</u> the page.

$$\vec{V} = \frac{\Gamma}{4\pi}\int \frac{\vec{d\ell} \times \vec{r}}{\left|\vec{r}\right|^{3}} = \frac{\Gamma}{4\pi}\int_{0}^{2\pi R} \frac{(R\ d\ell)\vec{e}}{R^3} = \frac{\Gamma}{4\pi R^2}(2\pi R)\ \vec{e} = \boxed{\frac{\Gamma}{2R}\vec{e}}$$

5.2



Since $\vec{d\ell}$ and $\vec{r}$ are always perpendicular (by inspection of the figure),

$$\left|d\vec{V}\right| = \left|\frac{\Gamma}{4\pi}\frac{\vec{d\ell} \times \vec{r}}{\left|\vec{r}\right|^{3}}\right| = \frac{\Gamma}{4\pi}\frac{d\ell}{r^2}$$

By symmetry, the resultant velocity due to the entire loop must be along the x-axis. Hence,

$$\left|\vec{V}\right| = \int \left|d\vec{V}\right| \cos\theta = \left(\frac{\Gamma}{4\pi}\int_{0}^{2\pi R}\frac{d\ell}{r^2}\right)\cos\theta =$$

$$\frac{\Gamma}{4\pi}\frac{1}{(A^2+R^2)}(2\pi R)\cos\theta =$$

$$\frac{\Gamma}{2}\frac{R}{(A^2+R^2)}\frac{R}{\sqrt{R^2+A^2}} = \frac{\Gamma\ R^2}{2(A^2+\ R^2)^{3/2}}$$

5.3 $$a = \frac{a_o}{1+\frac{a_o}{\pi AR}(1+\tau)},$$ where $a_o = 0.1080$ per degree = 6.188 per radian

From Fig. 5.18: $\delta = \tau = 0.054$.

$$a = \frac{6.188}{1+\frac{6.188}{\pi(8)}(1+0.054)} = 4.91 \text{ per rad.}$$

$$= 0.0857 \text{ per degree}$$

$$C_L = a\,(\alpha - \alpha_{L=0}) = 0.0857\ [7-(-1.3) = \boxed{0.712}$$

$$C_{D_i} = \frac{C_L^2}{\pi a R}\ (1+\delta) = \frac{(0.712)^2}{\pi(8)}\ (1.054) = \boxed{0.0212}$$

5.4 $$AR = \frac{b^2}{S} = \frac{(32)^2}{170} = 6.02$$

At standard sea level, $\rho_\infty = 0.002377$ slug/ft^3

$$V_\infty = 120 \text{ mph}\left(\frac{88\ \text{ft/sec}}{60\text{mph}}\right) = 176 \text{ ft/sec}$$

$$q_\infty = \frac{1}{2}\ \rho_\infty\, V^2_\infty = \frac{1}{2}\ (0.002377)\,(176)^2 = 36.8 \text{ lb/ft}^2$$

$a_o = 0.1033$ per degree

$= 5.92$ per rad

$$C_L = \frac{L}{q_\infty S} = \frac{W}{q_\infty S} = \frac{2450}{(36.8)(170)} = 0.3916$$

$$a = \frac{a_o}{1 + \frac{a_o}{\pi AR}(1+\tau)} = \frac{5.92}{1 + \frac{5.92}{\pi(6.02)}(1+0.12)} = 4.38 \text{ per rad}$$

$$= 0.0764 \text{ per deg}$$

$$C_L = a\,(\alpha - \alpha_{L=0})$$

$$\alpha = \frac{C_L}{a} + \alpha_{L=0} = \frac{0.3916}{0.0764} - 3^\circ = \boxed{2.12^\circ}$$

5.5 $$C_{D_i} = \frac{C_L^2}{\pi e AR} = \frac{(0.3916)^2}{\pi(.64)(6.02)} = 0.01267$$

$$D_i = q_\infty\, S\, C_{D_i} = (36.8)(170)(0.01267) = \boxed{79.3 \text{ lb}}$$

5.6 To be consistent, we will use Helmbold's equations for both the straight and swept wings.

(a) $a_o = 0.1$ per degree $= 0.1\ (57.3) = 5.73$ per radian

$$\frac{a_o}{\pi AR} = \frac{5.73}{\pi(6)} = 0.304$$

From Helmbold's equation for a straight wing, Eq. (5.81),

$$a = \frac{a_o}{\sqrt{1 + [a_o/(\pi AR)]^2} + a_o/(\pi AR)}$$

$$= \frac{5.73}{\sqrt{1 + (0.304)^2} + 0.304} = \frac{5.73}{1.349} = \boxed{4.247 \text{ per radian}}$$

(b) From Helmbold's equation for a swept wing, Eq. (5.82), where

$a_o \cos\Lambda = 5.73 \cos 45^\circ = 4.05$ per radian

and

$$\frac{a_o \cos\Lambda}{\pi AR} = \frac{4.05}{\pi(6)} = 0.215$$

we have

$$a = \frac{a_o \cos\Lambda}{\sqrt{1+[a_o \cos\Lambda/(\pi AR)]^2} + a_o \cos\Lambda/(\pi AR)}$$

$$= \frac{4.05}{\sqrt{1+(0.215)^2}+0.215} = \frac{4.05}{1.23785} = \boxed{3.27 \text{ per radian}}$$

Comparing the results of parts (a) and (b), we readily conclude that the effect of wing sweep is to reduce the lift slope. Moreover, the reduction is substantial.

5.7 Again, we use Helmbold's equations.

(a) $a_0 = 5.73$ per radian

$$\frac{a_o}{\pi AR} = \frac{5.73}{\pi(3)} = 0.608$$

$$a = \frac{a_o}{\sqrt{1+[a_o/(\pi AR)]^2} + a_o/(\pi AR)}$$

$$= \frac{5.73}{\sqrt{1+(0.608)^2}+0.608} = \frac{5.73}{1.778} = \boxed{3.222 \text{ per radian}}$$

(b) $a_0 \cos\Lambda = 4.05$

$$\frac{a_o \cos\Lambda}{\pi AR} = \frac{4.05}{\pi(3)} = 0.43$$

$$a = \frac{a_o \cos\Lambda}{\sqrt{1+[a_o \cos\Lambda/(\pi AR)]^2} + a_o \cos\Lambda/(\pi AR)}$$

$$= \frac{4.05}{\sqrt{1+(0.43)^2}+0.43} = \frac{4.05}{1.5185} = \boxed{2.667}$$

In Problem 5.6, with an aspect ratio of 6, we had

$$\frac{a_{swept}}{a_{straight}} = \frac{3.27}{4.247} = 0.77$$

The lift slope for the swept wing is only 77% of that for the straight wing when the aspect ratio of both wings is 6.

In Problem 5.7, with aspect ratio 3, we have

$$\frac{a_{swept}}{a_{straight}} = \frac{2.667}{3.222} = 0.83$$

The lift slope for the swept wing is 83% of that for the straight wing.

Conclusion: Wing sweep decreases the lift slope. Moreover, wing sweep affects the lift slope to a greater degree for higher aspect ratio wings than for lower aspect ratio wings. This makes some sense, because the lift slope for low aspect ratio wings is already considerably reduced just due to the aspect ratio effect.
