CHAPTER 4

4.1 $q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(0.002377)(50)^2 = 2.97 \text{ lb/ft}^2$

$c_\ell = 0.64$ and $c_{m_{c/4}} = -0.036$

$L' = q_\infty s\ c_\ell = (2.97)(2)(1)(0.64) = \boxed{3.80 \text{ lb per unit span}}$

$M'_{c/4} = q_\infty S\ c\ c_{m_{c/4}} = (2.97)(2)(1)(2)(-0.36) = \boxed{-0.428 \text{ ft/lb per unit span}}$

4.2 $q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(1.23)(50)^2 = 1538 \text{ N/m}^2$

$$c_\ell = \frac{L'}{q_\infty S} = \frac{1353}{(1538)(2)} = 0.44$$

From Fig. 4.5,

$\boxed{\alpha = 2^\circ}$

4.3 $\Gamma = \oint_c \vec{V} \cdot \vec{ds}$

$$\frac{D\Gamma}{Dt} = \oint_c \frac{D\vec{V}}{Dt} \cdot \vec{ds} + \oint_c \vec{V} \cdot \vec{ds}$$

$$\frac{D\vec{ds}}{Dt} = d\vec{V}$$

Hence, the second term in Eq. (1) becomes

$$\oint_c \vec{V} \cdot d\vec{V} = \oint_c d\left(\frac{V^2}{2}\right) = 0$$

From the momentum equation,

$$\frac{D\vec{V}}{Dt} = -\frac{1}{\rho}\nabla p \text{ (neglecting body forces)}$$

Hence, the first term in Eq. (1) becomes

$$\oint_c \frac{D\vec{V}}{Dt} \cdot \vec{ds} = -\oint_c \frac{1}{\rho}\nabla p \cdot \vec{ds} = -\oint_c \frac{dp}{\rho}$$

When ρ = const, or ρ = ρ(p), then

$-\oint_c \frac{dp}{\rho} = 0$. Hence, from Eq. (3)

$$\oint_c \frac{D\vec{V}}{Dt} \cdot \vec{ds} = 0 \qquad (4)$$

Substituting Eqs. (2) and (4) into (1), we obtain

$$\boxed{\frac{D\Gamma}{Dt} = 0}$$

Note: See Karamcheti, Ideal-Fluid Aerodynamics, for more details (pp. 239-242).

4.4 $M'_{LE} = -\rho_\infty V_\infty \int_0^c \xi\,\gamma(\xi)\,d\xi$

$$= -\rho_\infty V_\infty \int_0^\pi \frac{c}{2}(1 - \cos\theta)\,\theta\,(\gamma)\frac{c}{2}\sin\theta\,d\theta$$

$$= -\rho_\infty V_\infty \frac{c^2}{4} 2\,\alpha\,V_\infty \int_0^\pi (1 - \cos^2\theta)\,d\theta$$

$$= -\rho_\infty V_\infty \frac{c^2}{2}\alpha\left[\frac{\pi}{2}\right] = -\left(\frac{1}{2}\rho_\infty V_\infty^2\right) c^2 \frac{\pi\alpha}{2}$$

$$= -q_\infty c^2 \frac{\pi\alpha}{2}$$ This is Eq. (4.36).

4.5 $c_\ell = 2\pi\alpha$ where α is in radians. Hence

$$c_\ell = 2\pi \left(\frac{1.5}{57.3}\right) = \boxed{0.164}$$

$$c_{m_{c/4}} = -\ c_\ell / r = \boxed{-0.041}$$

4.6 (a)

For $0 \le \frac{x}{c} \le 0.4$: $\left(\frac{dz}{dx}\right)_1 = 0.2 - 0.5 \left(\frac{x}{c}\right)$

For $0.4 \le \frac{x}{c} \le 1$: $\left(\frac{dz}{dx}\right)_2 = 0.0888 - 0.2222 \left(\frac{x}{c}\right)$

Since $x = \frac{c}{2}\ (1 - \cos\theta)$, then

$$\left(\frac{dz}{dx}\right)_1 = -0.05 + 0.25\cos\theta, \text{ for } 0 \le \theta \le 1.3694$$

$$\left(\frac{dz}{dx}\right)_2 = -0.0223 + 0.1111\cos\theta, \text{ for } 1.3694 \le \theta \le \pi$$

$$\alpha_{L=0} = -\frac{1}{\pi}\int_0^\pi \frac{dz}{dx}\,(\cos\theta - 1)\,d\theta$$

$$= \frac{1}{\pi}\int_0^{1.3694} (-0.05 + 0.25\cos\theta)(\cos\theta - 1)\,d\theta - \frac{1}{\pi}\int_{1.3694}^{\pi}$$

$$\cdot\,(-0.0223 + 0.1111\cos\theta)(\cos\theta - 1)\,d\theta$$

$$= -\frac{1}{\pi}\int_0^{1.3694} (0.05 - 0.3\cos\theta + 0.25\cos^2\theta)\,d\theta$$

$$-\frac{1}{\pi}\int_{1.3694}^{\pi} (0.0223 - 0.13334\cos\theta + 0.1111\cos^2\theta)\,d\theta$$

$$= -\frac{1}{\pi}\left[0.05\theta - 0.3\sin\theta + 0.25\left(\frac{\theta}{2} + \frac{1}{4}\sin 2\theta\right)\right]_0^{1.3694}$$

$$- \frac{1}{\pi} \left[0.0223\,\theta - 0.1334\sin\theta + 0.111\left(\frac{\theta}{2} + \frac{1}{4}\sin 2\theta\right)\right]_{1.3694}^{\pi}$$

$$= -\frac{1}{\pi}[0.06847 - 0.2939 + 0.1712 + 0.0245] - \frac{1}{\pi}[0.0701 + 0.1745]$$

$$+ \frac{1}{\pi}[0.0305 - 0.1307 + 0.0761 + 0.0109]$$

$$= \frac{-0.2281}{\pi} = -0.0726 \text{ rad} = \boxed{-4.16^{\circ}}$$

(b)

$c_\ell = 2\pi(\alpha + \alpha_{L=0})$ where α is in radians

$$c_\ell = \frac{2\pi}{57.3}[3 - (-4.16)] = \boxed{0.782}$$

4.7 $$A_1 = \frac{2}{\pi}\int_0^{\pi} \frac{dz}{dx}\cos\theta\, d\theta$$

$$= \frac{2}{\pi}\int_0^{1.3694} (0.05 + 0.25\cos\theta)\cos\theta\, d\theta$$

$$+ \frac{2}{\pi}\int_{1.3694}^{\pi} (-0.0223 + 0.1111\cos\theta)\cos\theta\, d\theta$$

$$= \frac{2}{\pi}\int_0^{1.3694} (-0.05\cos\theta + 0.25\cos^2\theta)\, d\theta +$$

$$\frac{2}{\pi}\int_{1.3694}^{\pi} (-0.0223\cos\theta + 0.1111\cos^2\theta)\, d\theta$$

$$= \frac{2}{\pi}\left[-0.05\sin\theta + 0.25\left(\frac{\theta}{2} + \frac{1}{4}\sin 2\theta\right)\right]_0^{1.3694}$$

$$+ \frac{2}{\pi}\left[(-0.0233)\sin\theta + 0.1111\left(\frac{\theta}{2} + \frac{1}{4}\sin 2\theta\right)\right]_{1.3694}^{\pi}$$

$$= \frac{2}{\pi} [-0.04899 + 0.25\,(0.6847 + 0.09800) + 0.1745$$

$$+ 0.02185 - 0.1111\,(0.6847 + 0.09800)]$$

$$A_1 = (0.2561)\,\frac{2}{\pi} = 0.1630$$

$$A_2 = \frac{2}{\pi} \int_0^{\pi} \frac{dz}{dx} \cos 2\theta \, d\theta$$

$$= \frac{2}{\pi} \int_0^{1.3694} (-0.05 + 0.25 \cos\theta) \cos 2\theta d\theta + \frac{2}{\pi} \int_{1.3694}^{\pi} (-0.0223$$

$$+ 0.1111 \cos\theta) \cos\theta \, d\theta$$

$$= \frac{2}{\pi} [\frac{1}{2}(-0.05) \sin 2\theta + 0.25\,(\frac{\sin\theta}{2} + \frac{\sin 3\theta}{6})]_0^{1.3694}$$

$$+ \frac{2}{\pi} [\frac{1}{2}\,(-0.0223) \sin 2\theta + 0.1111\,(\frac{\sin\theta}{2} + \frac{\sin 3\theta}{6})]_{1.3694}^{\pi}$$

$$= \frac{2}{\pi}[-0.009800 + 0.25\,(0.4899 - 0.1372) + 0.004371$$

$$-\quad 0.1111\,(0.4899 - 0.1372)]$$

$$= (0.0436)\,\frac{2}{\pi} - 0.0277$$

$$c_{m_{c/4}} = \frac{\pi}{4}\,(A_2 - A_1) = \frac{\pi}{4}\,(0.0277 - 0.1630) = \boxed{-0.1063}$$

$$\frac{x_{cp}}{c} = \frac{1}{4}\,[1 + \frac{\pi}{c_\ell}\,(A_1 - A_2)] = \frac{1}{4}\,[1 + \frac{\pi}{0.782}\,(0.1630 - 0.0277)] = \boxed{0.386}$$

4.8

	Experiment (Ref. 11)	Theory	% Difference
$\alpha_{L=0}$	-3.9°	-4.16°	6.25%
c_ℓ	0.76	0.782	2.8%
$c_{m_{c/4}}$	-0.095	-0.1063	10.6%

4.9 $M'_{LE} = -\rho_\infty V_\infty \int_0^c \xi\, \gamma(\xi)\, d\xi$

$$c_{m_{c/4}} = \frac{M'_{LE}}{\frac{1}{2}\rho_\infty V_\infty^2 c^2} = \frac{-2}{V_\infty c^2} \int_0^c \xi\, \gamma(\xi)\, d\xi \qquad (1)$$

$$\xi = \frac{c}{2}(1 - \cos\theta)$$

$$d\xi = \frac{c}{2} \sin\theta\, d\theta$$

$$\gamma(\theta) = 2 V_\infty \left[A_o \frac{(1+\cos\theta)}{\sin\theta} + \sum_{n=1}^{\infty} A_n \sin n\theta \right]$$

With the above, Eq. (1) becomes

$$c_{m_{c/4}} = - \int_0^\pi A_o (1 - \cos^2\theta)\, d\theta - \sum_{n=1}^{\infty} \int_0^\pi A_n (1 - \cos\theta) \sin\theta \sin n\theta\, d\theta \qquad (2)$$

Note the following definite integrals:

$$\int_0^\pi \cos^2\theta\, d\theta = \frac{\pi}{2}$$

$$\int_0^\pi \sin^2\theta\, d\theta = \frac{\pi}{2}$$

$$\int_0^\pi \cos\theta \sin^2\theta\, d\theta = 0$$

$$\int_0^{\pi} \sin\theta \sin n\theta \, d\theta = 0 \qquad \text{for } n = 2, 3, \ldots$$

$$\int_0^{\pi} \cos\theta \sin\theta \sin 2\theta \, d\theta = \frac{\pi}{4}$$

$$\int_0^{\pi} \cos\theta \sin\theta \sin n\theta \, d\theta = 0 \qquad \text{for } n = 3, 4, \ldots$$

Hence, Eq. (2) becomes:

$$c_{m_{c/4}} = -[\pi A_o - \frac{\pi}{2} A_o + \frac{\pi}{2} A_1 - \frac{\pi}{4} A_2]$$

$$\boxed{c_{m_{c/4}} = -\frac{\pi}{2} (A_o + A_1 - \frac{A_2}{2})}$$

4.10 The slope of the lift curve is

$$a_o = \frac{0.65 - (-0.39)}{4 - (-6)} = 0.104 \text{ per degree}$$

The slope of the moment coefficient curve is

$$m_o = \frac{-0.037 - (-0.045)}{4 - (-6)} = 8 \times 10^{-4} \text{ per degree}$$

From Eq. (4.71),

$$\bar{x}_{ac} = -\frac{m_o}{a_o} + 0.25 = -\frac{8 \times 10^{-4}}{0.104} + 0.25 = \boxed{0.242}$$