Flow types caused by viscous effects

1 Definitions

δ = Boundary layer thickness (m)

τ_w = Shear stress at the surface ($Pa = N/m^2$)

V = Air flow velocity (m/s)

μ = Absolute viscosity coefficient (or short: viscosity) (For normal air ($T = 288K$): $1.7894 \times 10^{-5} kg/(ms)$)

Re_x = The Reynolds number at a position x (dimensionless)

ρ = Air density (kg/m^3)

x = The distance from the leading edge of the air foil (m)

c_{f_x} = Skin friction coefficient (dimensionless)

q = Dynamic pressure ($Pa = N/m^2$)

D_f = Total skin friction drag (N)

b = Wing span (m)

S = Wing surface (m^2)

C_f = Total skin friction drag coefficient (dimensionless)

Re_{cr} = The critical Reynolds number - Reynolds number where transition occurs (dimensionless)

x_{cr} = The critical value - Distance between the leading edge and the transition point (m)

2 Boundary layers and surface shear stress

Involving viscous effects, the airflow doesn't move smoothly over the air foil. Instead, a boundary layer appears with thickness δ. In this boundary layer, at a distance of y from the air foil, the flow velocity is V_y. V_0 is 0, and V_δ is equal to the flow speed calculated for frictionless flow, assuming the wing has the shape of the airfoil plus its boundary layer.

A very important number in aerodynamics is the Reynolds number. The Reynolds number at a distance x from the leading edge of the airfoil can be calculated using this formula:

$$Re_x = \frac{V_\infty \rho_\infty x}{\mu_\infty} \tag{2.1}$$

Important for drag calculations, is the shear stress caused by the flow. The shear stress at the surface can be calculated using the slope of the V_y-curve at $y = 0$:

$$\tau_w = \mu \left(\frac{dV}{dy} \right)_{y=0} \tag{2.2}$$

For laminar flows, the ratio $\frac{dV}{dy}$ is smaller than for turbulent flows, so laminar flows also have a lower surface shear stress compared to turbulent flows. But rather than dealing with shear stress, aerodynamicists find it often easier to work with the dimensionless skin friction coefficient, which is per definition equal to the following:

$$c_{f_x} = \frac{\tau_w}{\frac{1}{2}\rho_\infty V_\infty^2} = \frac{\tau_w}{q_\infty} \tag{2.3}$$

For laminar flows, experiments have shown that the following two formulas apply:

$$\delta = \frac{5.2x}{\sqrt{Re_x}} \tag{2.4}$$

$$c_{f_x} = \frac{0.664}{\sqrt{Re_x}} \tag{2.5}$$

And for turbulent flows, the following applies:

$$\delta = \frac{0.37x}{Re_x^{0.2}} \tag{2.6}$$

$$c_{f_x} = \frac{0.0592}{Re_x^{0.2}} \tag{2.7}$$

3 Skin friction drag

Note: The following formulas only apply for airfoils with the shape of a flat plate. It also only applies for incompressible flows. Otherwise a correction factor, depending on the Mach number, is required.

Using the surface shear stress, the total skin friction drag can be calculated. To find this, the surface shear stress should be integrated over the entire wing area. In formula this is:

$$D_f = \int_0^b \left(\int_0^L c_{f_x} q_\infty dx \right) dw$$

Solving this for laminar flow gives:

$$D_f = \frac{1.328 q_\infty S}{\sqrt{\frac{\rho_\infty V_\infty L}{\mu}}}$$

Note that the distances are now based on the total length of the airfoil. To simplify this equation, the total skin friction drag coefficient C_f (not to be mistaken with the local skin friction coefficient c_{f_x}) is introduced:

$$C_f = \frac{D_f}{q_\infty S} \tag{3.1}$$

Using this definition with the total skin friction drag gives:

$$C_f = \frac{1.328}{\sqrt{Re_L}} \tag{3.2}$$

And doing all the steps analog for turbulent flow results in:

$$C_f = \frac{0.074}{Re_L^{0.2}} \tag{3.3}$$

Research has shown that the transition from laminar to turbulent flow occurs, when the Reynolds number rises above a critical value: the critical Reynolds number Re_{cr}. The point at which that occurs, is the transition point, and the distance from that point to the leading edge is the critical value x_{cr}. The following relation then applies:

$$Re_{cr} = \frac{\rho_\infty V_\infty x_{cr}}{\mu_\infty} \tag{3.4}$$