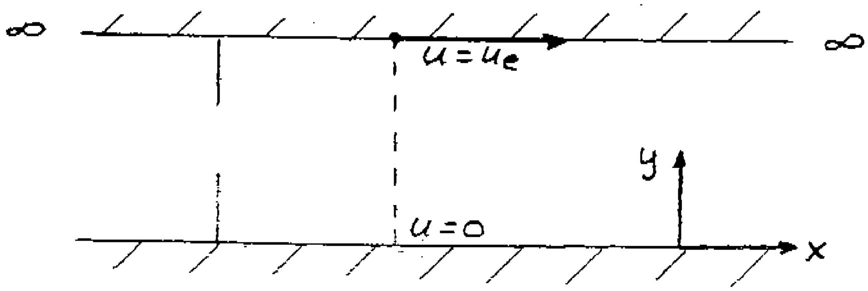
CHAPTER 15

15.1



(a) Since the plates are infinite in length, u = u(y) only. Also, v = 0, i.e., the flow is in the x-direction only. The governing equation is Eq. (15.18a), which reduces to the following u = u(y), v = 0 and p = const.

$$0 = \frac{d}{dy}\left(\mu \frac{du}{dy}\right)$$

Integrating:

$$\mu \frac{du}{dy} = \text{const} = c_1$$

$$\mu u = c_1 y + c_2$$

At y = 0, u = 0: $c_2 = 0$

At y = h, $u = u_e$: $\mu u_e = c_1 h$

$$c_1 = \frac{\mu u_e}{h}$$

Thus:

$$\mu u = \frac{\mu u_e}{h} y, \text{ or } \boxed{u = u_e \left(\frac{y}{h}\right)}$$

The velocity variation is linear between the plates.

(b) $\frac{du}{dy} = \frac{u_e}{h}$

$$\tau = \mu \frac{du}{dy} = \mu \frac{u_e}{h}$$

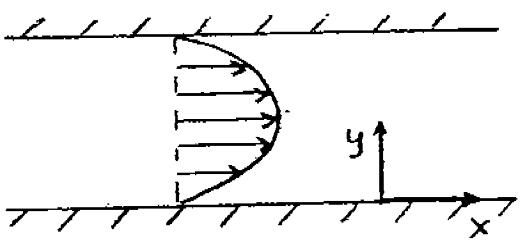
$$\frac{\mu}{\mu_o} = \left(\frac{T}{T_o}\right)^{3/2} \frac{T_o + 110}{T + 110} = \left(\frac{320}{288.16}\right)^{3/2} \frac{288.16 + 110}{320 + 110} = 1.084$$

$$\mu = 1.084\ \mu_o = 1.084\ (1.7894 \times 10^{-5}) = 1.94 \times 10^{-5}\ \frac{kg}{m\ sec}$$

$$\tau = (1.94 \times 10^{-5}) \left(\frac{30}{0.01}\right) = \boxed{5.82 \times 10^{-2}\ N/m^2}$$

The shear stress is constant, and hence is the same on the top and bottom walls.

15.2



$$u = u(y),\ v = 0,\ p = p(x)$$

$$0 = \frac{dp}{dx} + \frac{d}{dy}\left(\mu \frac{du}{dy}\right)$$

$$\mu \frac{du}{dy} = -\left(\frac{dp}{dx}\right) y + c_1$$

$$\mu u = -\left(\frac{dp}{dx}\right) \frac{y^2}{2} + c_1 y + c_2$$

$$u = -\left(\frac{dp}{dx}\right) \frac{y^2}{2\mu} + \frac{c_1}{\mu} y + \frac{c_2}{\mu}$$

At $y = 0$, $u = 0$. Thus $c_2 = 0$

At y = h, u = 0. Thus,

$$0 = -\left(\frac{dp}{dx}\right)\frac{h^2}{2\mu} + \frac{c_1}{\mu}h \qquad\qquad c_1 = \left(\frac{dp}{dx}\right)\frac{h}{2}$$

$$u = -\left(\frac{dp}{dx}\right)\frac{y^2}{2\mu} + \left(\frac{dp}{dx}\right)\frac{h}{2\mu}y$$

$$\boxed{u = \frac{1}{2\mu}\left(\frac{dp}{dx}\right)(h\,y - y^2)}$$

The velocity profile is parabolic.

$$\frac{du}{dy} = -\left(\frac{dp}{dx}\right)\frac{y}{\mu} + \left(\frac{dp}{dx}\right)\frac{h}{2\mu}$$

On the bottom plate, y = 0: $\tau = \mu\,\frac{du}{dy}$

$$\tau = [-\left(\frac{dp}{dx}\right)\frac{0}{\mu} + \left(\frac{dp}{dx}\right)\frac{h}{2\mu}]\,\mu = \frac{h}{2}\left(\frac{dp}{dx}\right)$$

On the top plate, y = h: $\tau = \mu\,(-\frac{du}{dy})$ since dy is negative, i.e., the distance away from the top plate is in the downward (negative direction)

$$\tau = \mu\,[+\left(\frac{dp}{dx}\right)\frac{h}{\mu} - \left(\frac{dp}{dx}\right)\frac{h}{2\mu}]$$

$$\tau = \frac{h}{2}\left(\frac{dp}{dx}\right)$$

For both the top and bottom walls,

$$\tau = \frac{h}{2}\left(\frac{dp}{dx}\right)$$

Shear stress varies linearly with the magnitude of the pressure gradient.

Note: Due to the content of chapters 16, 17, and 18, no homework problems are required.