CHAPTER 12

12.1 Consider $\alpha = 5° = 0.0873$ rad.

$$c_\ell = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} = \frac{4(0.0873)}{\sqrt{(2.6)^2 - 1}} = \boxed{0.1455}$$

From exact theory (Prob. 9.13): $c_\ell = 0.148$

$$\% \text{ error} = \frac{0.148 - 0.1455}{0.148} \times 100 = 1.69\%$$

$$c_d = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}} = c_\ell\ \alpha = (0.1455)(0.0873) = \boxed{0.0127}$$

From exact theory (Prob. 9.13): $c_d = 0.0129$

$$\% \text{ error} = \frac{0.0129 - 0.0127}{0.0129} \times 100 = 1.53\%$$

(b) $\alpha = 15° = 0.2618$ rad

$$c_\ell = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} = \boxed{0.436}$$

From exact theory (Prob. 9.13): $c_\ell = 0.452$

$$\% \text{ error} = \frac{0.452 - 0.426}{0.452} \times 100 = 3.47\%$$

$$c_d = c_\ell\ \alpha = (0.436)(0.2618) = 0.114$$

From exact theory (Prob. 9.13): $c_d = 0.121$

$$\% \text{ error} = \frac{0.121 - 0.114}{0.121} \times 100 = 5.7\%$$

(c) $\alpha = 30° = 0.5236$ rad

$$c_\ell = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} = \frac{4(0.5236)}{\sqrt{(2.6)^2 - 1}} = \boxed{0.873}$$

From exact theory (Prob. 9.13): $c_\ell = 1.19$

$$\% \text{ error} = \frac{1.19 - 0.873}{1.19} \text{ x } 100 = 26.7\%$$

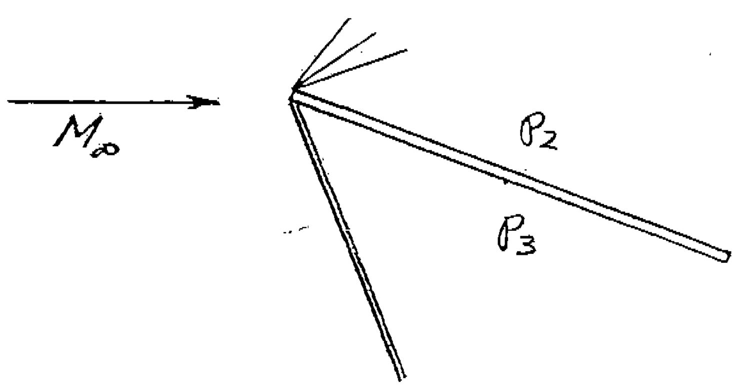
$$c_d = c_\ell\ \alpha = (0.873)(0.5236) = \boxed{0.457}$$

From exact theory (Prob. 9.13): $c_d = 0.687$

$$\% \text{ error} = \frac{0.687 - 0.457}{0.687} = 33.5\%$$

Conclusion: At low α, linear theory is reasonably accurate. However, its accuracy deteriorates rapidly at high α. This is no surprise; we do not expect linear theory to hold for large perturbations. It appears that linear theory is reasonable to at least 5°, and that it is acceptable as high as 15°. At 30° it is unacceptable. Keep in mind that the above comments pertain to the lift and wave drag coefficients only. They say nothing about the accuracy of the pressure distributions themselves.

12.2



(a) $C_p = \frac{p - p_\infty}{q_\infty} = \frac{p - p_\infty}{\frac{\gamma}{2} p_\infty M_\infty^2} = \frac{2}{\gamma \, M_\infty^2}\left(\frac{p}{p_\infty} - 1\right)$

$$\frac{p}{p_\infty} = \frac{\gamma \, M_\infty^2 \, C_p}{2} + 1$$

$$C_p = \pm \frac{2\theta}{\sqrt{M_\infty^2 - 1}} = \pm \frac{2\theta}{\sqrt{(2.6)^2 - 1}} = \pm \frac{2\theta}{2.4}$$

or,

$$C_p = \pm \, 0.8333\theta$$

$$\frac{p}{p_\infty} = \frac{\gamma \, M_\infty^2 \, C_p}{2} + 1 = \pm \frac{(1.4)(2.6)^2 (0.8333)}{2} + 1$$

$$\frac{p}{p_\infty} = \pm \, 3.943\theta + 1$$

Hence: Examining the physical picture: recalling $\alpha = 5° = 0.873$ rad.

$$\frac{p_2}{p_\infty} = -3.943 \, (.0873) + 1 = \boxed{0.6558}$$

From exact theory (Prob. 9.13): $\frac{p_2}{p_\infty} = 0.7022$

$$\% \text{ error} = \frac{0.7022 - 0.6558}{0.7022} \text{ x } 100 = 6.6\%$$

$$\frac{p_3}{p_\infty} + 3.943\theta + 1 = 3.943 \, (.0873) + 1 = \boxed{1.344}$$

From exact theory (Prob. 9.13): $\frac{p_3}{p_\infty} = 1.403$

$$\% \text{ error} = \frac{1.403 - 1.344}{1.403} \text{ x } 100 = 4.2\%$$

(b) For $\alpha = 15° = 0.2618$ rad:

$$\frac{p_2}{p_\infty} = -3.943\theta + 1 = -3.943\ (.2618) + 1 = -0.0322 \text{ (physically impossible)}$$

The result from exact theory (Prob. 9.13) is $\frac{p_2}{p_\infty} = 0.315$

$$\frac{p_3}{p_\infty} = 3.943\theta + 1 = 3.943\ (.2618) + 1 = \boxed{2.032}$$

From exact theory (Prob. 9.13): $\frac{p_3}{p_\infty} = 2.529$

$$\% \text{ error} = \frac{2.529 - 2.032}{2.529} \times 100 = 19.7\%$$

(c) For $\alpha = 30° = 0.5236$ rad

$$\frac{p_2}{p_\infty} = -3.943\theta + 1 = -3.943\ (0.5236) + 1 = -1.064 \text{ (physically impossible)}$$

The result from exact theory (Prob. 9.13) is $\frac{p_2}{p_\infty} = 0.0725$

$$\frac{p_3}{p_\infty} = 3.943\theta + 1 = 3.943\ (0.5236) + 1 = \boxed{3.065}$$

From exact theory (Prob. 9.13): $\frac{p_3}{p_\infty}$ 5.687

$$\% \text{ error} = \frac{5.687 - 3.065}{5.687} \times 100 = 46\%$$

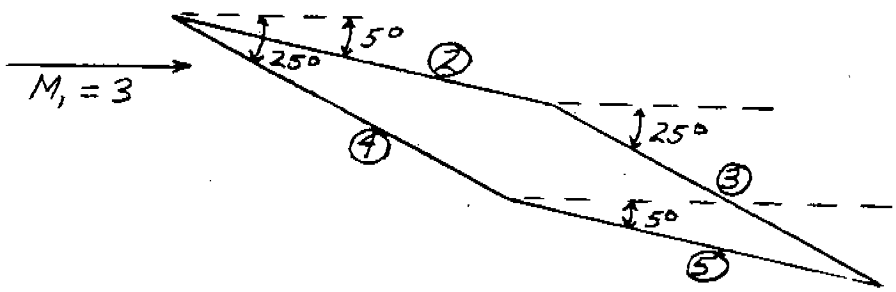
Conclusions: (1) Pressures predicted by linear theory rapidly become inaccurate as α increases. (2) Pressures predicted by linear theory are reasonable only at <u>low</u> values of α, say below 5°. (3) At each value of α, the % error is much greater for pressure than for lift and wave drag coefficients. (See Prob. 12.1). Hence, linear theory works better for c_ℓ and c_d than it does for p. What happens is that the inaccuracies in p on the top and bottom surfaces tend to compensate, yielding a more accurate aerodynamic force coefficient.

12.3 $\frac{p}{p_\infty} = \frac{\gamma\ M_\infty^2\ C_p}{2} + 1$ where $C_p = \pm \frac{2\theta}{\sqrt{M_\infty^2 - 1}}$

$$C_p = \pm \frac{2\theta}{\sqrt{(3)^2 - 1}} = \pm 0.7071\theta$$

$$\frac{p}{p_\infty} = \pm \frac{(1.4)(3)^2(0.7071)\theta}{2} + 1$$

$$\frac{p}{p_\infty} = \pm 4.455\theta + 1$$



Surface 2: $\theta = 5° = 0.08727$ rad.

$$\frac{p_2}{p_\infty} = -4.455\ (.08727) + 1 = 0.6112$$

Surface 3: $\theta = 25° = 0.4663$ rad

$$\frac{p_3}{p_\infty} = -4.455\ (.4363) + 1 = -0.9439$$

Note: Although a negative pressure is not physically possible, in order to calculate the net force, we must carry it as such.

Surface 4: $\theta = 25° = 0.4363$ rad

$$\frac{p_4}{p_\infty} = 4.455\ (.4363) + 1 = 2.944$$

Surface 5: $\theta = 5° = 0.08727$ rad

$$\frac{p_5}{p_\infty} = 4.455\,(.08727) + 1 = 1.3888$$

$$c_\ell = \frac{2}{\gamma\, M_1^2}\, \frac{\ell}{c}\left[\left(\frac{p_4}{p_\infty} - \frac{p_3}{p_\infty}\right)\cos 25° + \left(\frac{p_5}{p_\infty} - \frac{p_2}{p_\infty}\right)\cos 5°\right] \text{ (From Prob. 9.14)}$$

$$c_\ell = \frac{2}{(1.4)(3)^2}\, \frac{\ell}{c}\, [(2.944 + 0.9439)\cos 25° + (1.3888 - 0.6112)\cos 5°]$$

$$c_\ell = 0.682\, \frac{\ell}{c}. \text{ However, } \frac{\ell}{c} = 0.5077 \text{ (From Prob. 9.14)}$$

$$c_\ell = (0.682)(.5077) = \boxed{0.346}$$

$$c_d = \frac{2}{\gamma\, M_1^2}\, \frac{\ell}{c}\left[\left(\frac{p_4}{p_\infty} - \frac{p_3}{p_\infty}\right)\sin 25° + \left(\frac{p_5}{p_\infty} - \frac{p_2}{p_\infty}\right)\sin 5°\right]$$

$$c_d = \frac{2}{(1.4)(3)^2}\, (.5077)\, [(2.944 + 0.9439)\sin 25° + (1.3888 - 0.6112)\sin 5°]$$

$$c_d = \boxed{0.1089}$$

Comparison

	Exact (Prob. 9.14)	Linear Theory	% Error
c_ℓ	0.418	0.346	17.2%
c_d	0.169	0.1089	35.6%