CHAPTER 11

11.1 $u = \frac{\partial\phi}{\partial x} = V_\infty + \frac{2\pi\ (70)}{\sqrt{1-M_\infty^2}}\ e^{-2\pi\sqrt{1-M_\infty^2}y}\ \cos(2\pi x)$

$$v = \frac{\partial\phi}{\partial y} = -\frac{70}{\sqrt{1-M_\infty^2}}\left(2\pi\sqrt{1-M_\infty^2}\right)\ e^{-2\pi\sqrt{1-M_\infty^2}y}\ \sin(2\pi x)$$

$$= -140\ \pi\ e^{-2\pi\sqrt{1-M_\infty^2}y}\ \sin(2\pi x)$$

$$a_\infty = \sqrt{\gamma RT} = \sqrt{(1.4)(1716)(519)} = 1116.6 \text{ ft/sec}$$

$$M_\infty = \frac{V_\infty}{a_\infty} = \frac{700}{1116.6} = 0.6269$$

Thus, at (x,y) = (0.2, 0.2)

$$u = 700 + \frac{2\pi(70)}{0.779}\ e^{-2\pi(0.779)(0.2)}\ \cos[2\pi(.2)] = 765.6 \text{ ft/sec}$$

$$v = -140\ \pi\ e^{-2\pi(.779)(.2)}\ \sin[2\pi(.2)] = -157.2 \text{ ft/sec}$$

$$V = \sqrt{u^2 + v^2} = \sqrt{(765.6)^2 - (157.2)^2} = 781.6 \text{ ft/sec}$$

From Table A.1, for $M_\infty = 0.6269$, $\frac{T_o}{T_\infty} = 1.079$

$$T_o = 1.079\ T_\infty = 0.079\ (519) = 560°R$$

$$a_o = \sqrt{\gamma RT_o} = \sqrt{(1.4)(1716)(560)} = 1160 \text{ ft/sec}$$

$$a^2 = a_o^2 + \frac{\gamma-1}{2}(V^2) = 1.345 \times 10^6 - (.2)(781.6)^2 = 1.223 \times 10^6 \left(\frac{\text{ft}}{\text{sec}}\right)^2$$

$$a = 1106 \text{ ft/sec}$$

$$M = \frac{V}{a} = \frac{781.6}{1106} = \boxed{0.7067}$$

From Table A.1, for $M = 0.6269$: $\frac{p_o}{p_\infty} = 1.3065$, $\frac{T_o}{T_\infty} = 1.079$

For $M = 0.7067 = \frac{p_o}{p} = 1.400$, $\frac{T_o}{T} = 1.101$

$$p = \frac{p}{p_o}\frac{p_o}{p_\infty}p_\infty = \left(\frac{1}{1.4}\right)(1.3065)(1\text{ atm}) = \boxed{0.933\text{ atm}}$$

$$T = \frac{T}{T_o}\frac{T_o}{T_\infty}T_\infty = \left(\frac{1}{1.101}\right)(1.079)(519) = \boxed{508.6°R}$$

11.2 The results of Fig. 4.5 are for low-speed, incompressible flow. Hence, from Fig. 4.5, at $\alpha = 5°$, at $\alpha = 5°$,

$$c_{\ell_o} = 0.75$$

$$c_\ell = \frac{c_{\ell_o}}{\sqrt{1-M_\infty^2}} = \frac{0.75}{\sqrt{1-(0.6)^2}} = \boxed{0.938}$$

11.3 $C_p = \frac{C_{p_o}}{\sqrt{1-M_\infty^2}} = \frac{-0.54}{\sqrt{1-(.58)^2}} = \frac{-0.54}{0.8146} = \boxed{-0.663}$

$$\text{(b)}\ C_p = \frac{C_{p_o}}{\sqrt{1-M_\infty^2} + \left(\frac{M_\infty^2}{1+\sqrt{1-M_\infty^2}}\right)\frac{C_{p_o}}{2}} = \frac{-0.54}{0.8146 + \left[\frac{0.3364}{1+0.8146}\right]\frac{(-0.54)}{2}}$$

$$C_p = \boxed{-0.7063}$$

$$\text{(c)}\ C_p = \frac{C_{p_o}}{\sqrt{1-M_\infty^2} + \left[M_\infty^2\left(1+\frac{\gamma-1}{2}M_\infty^2\right)/2\sqrt{1-M_\infty^2}\right]C_{p_o}}$$

$$C_p = \frac{-0.54}{0.8146 + [0.3364(1.067)/1.6292](-0.54)}$$

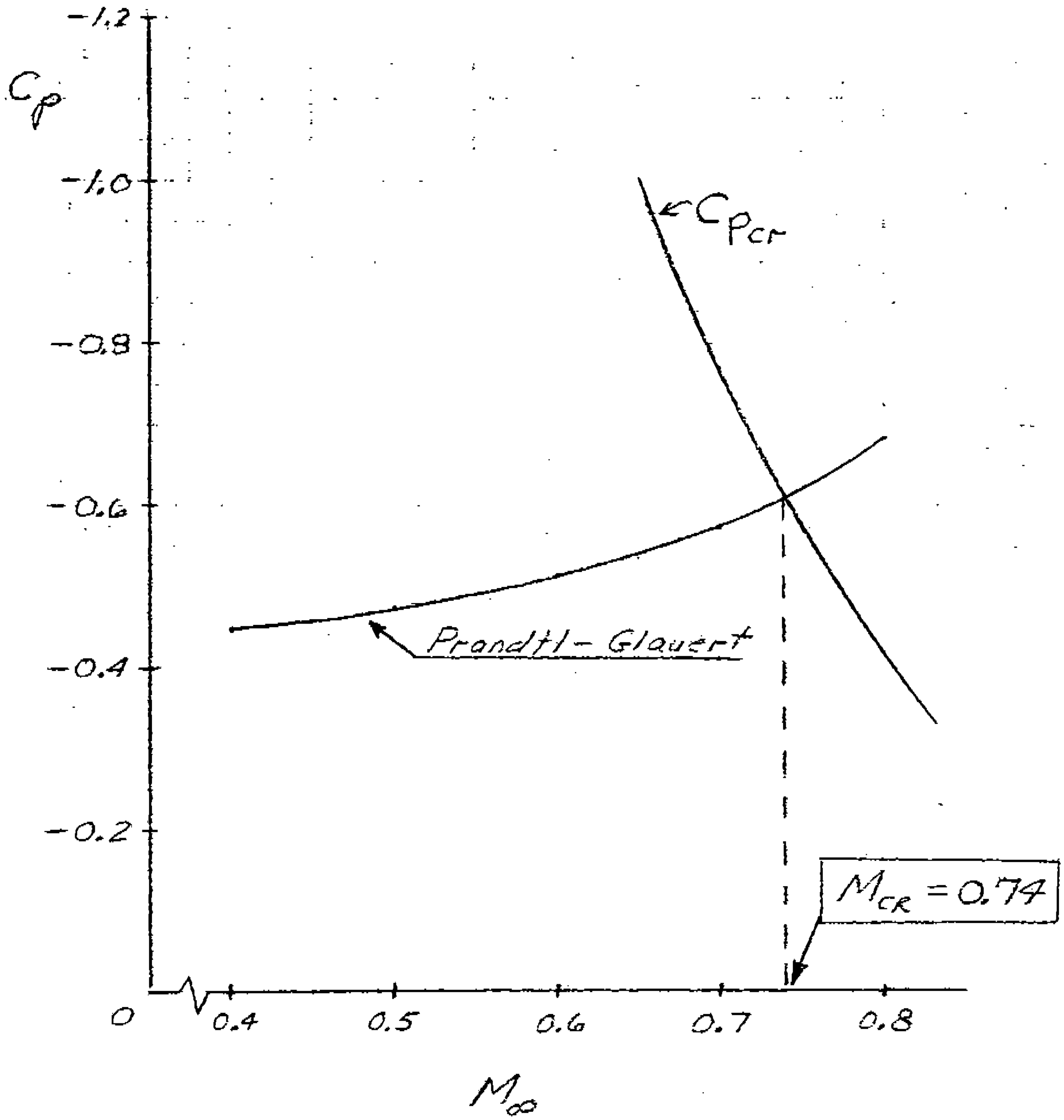
$$C_p = \boxed{-0.7763}$$

Note the differences: There is a 17% discrepancy between the three compressibility corrections. Of the three, experience has shown the Karman-Tsien rule to be more accurate.

11.4 For the pressure coefficient on the airfoil:

$$C_p = \frac{C_{p_o}}{\sqrt{1-M_\infty^2}} = \frac{-0.41}{\sqrt{1-M_\infty^2}}$$

M_∞	0.3	0.4	0.5	0.6	0.7	0.8
C_p	-0.43	-0.447	-0.473	-0.513	-0.574	-0.683



C_p
-1.2
-1.0
-0.8
-0.6
-0.4
-0.2
0
0.4
0.5
0.6
0.7
0.8
M_∞
$C_{p_{cr}}$
Prandtl-Glauert
$M_{CR} = 0.74$

11.5 When $M = M_{cr}$, then p at the minimum pressure point is clearly p_{cr}.

$$\frac{p}{p_\infty} = \frac{p_{cr}}{p_\infty} = \underbrace{\left(\frac{p_{cr}}{p_o}\right)}_{\text{Evaluated at } M=1}\underbrace{\left(\frac{p_o}{p_\infty}\right)}_{\text{Evaluated at } M=0.8} = (0.528)(1.524) = \boxed{0.805}$$

11.6 From Appendix A:

For $M_\infty = 0.5$, $\frac{p_o}{p_\infty} = 1.186$

For $M = 0.86$, $\frac{p_o}{p} = 1.621$

$$C_p = \frac{p - p_\infty}{q_\infty} = \frac{p - p_\infty}{\frac{\gamma}{2} p_\infty M_\infty^2} = \frac{2}{\gamma\, M_\infty^2}\left(\frac{p}{p_\infty} - 1\right)$$

$$\frac{p}{p_\infty} = \frac{p_o / p_\infty}{p_o / p} = \frac{1.186}{1.621} = 0.7316$$

$$C_p = \frac{2}{(1.4)(0.5)^2} = (0.7316 - 1) = \boxed{-1.53}$$

Check: Using Eq. (11.58)

$$C_p = \frac{2}{\gamma\, M_\infty^2}\left[\left(\frac{1 + \frac{\gamma - 1}{2} M_\infty^2}{1 + \frac{\gamma - 1}{2} M^2}\right)^{\frac{\gamma}{\gamma - 1}} - 1\right]$$

$$= \frac{2}{(1.4)(0.5)^2}\left[\left(\frac{1 + 0.2(0.5)^2}{1 + 0.2(0.86)^2}\right)^{3.5} - 1\right] = \boxed{-1.53}$$

It checks!

11.7 First, calculate $C_{p,o}$ at point A from the information in Figure 11.5(a). The actual pressure coefficient is

$$C_{p,A} = \frac{2}{\gamma M_\infty^2}\left(\frac{p_A}{p_\infty} - 1\right)$$

where

$$\frac{p_A}{p_\infty} = \frac{p_A}{p_o}\,\frac{p_o}{p_\infty}$$

From Appendix A (interpolating between entries for more accuracy for this problem),

$$\text{For } M_\infty = 0.3: \qquad \frac{p_o}{p_\infty} = 1.064$$

$$\text{For } M_A = 0.435: \qquad \frac{p_o}{p_A} = 1.139$$

Thus,

$$C_{p,A} = \frac{2}{(1.4)(0.3)^2}\left(\frac{1.064}{1.139} - 1\right) = -\,1.045$$

From the Prandtl-Glauert rule,

$$C_{p,o} = C_{p,A}\,\sqrt{1 - M_\infty^2} = (-1.045)\sqrt{1-(0.3)^2} = -\,0.9969$$

For the case of part (c) where $M_\infty = 0.61$, again using the Prandtl-Glauert rule,

$$C_{p,A} = \frac{C_{p,o}}{\sqrt{1 - M_\infty^2}} = \frac{-0.9969}{\sqrt{1-(0.61)^2}} = -\,1.258$$

To find the local Mach number, M_a, which corresponds to this value of $C_{p,A}$, note that

$$C_{p,A} = \frac{2}{\gamma M_\infty^2}\left(\frac{p_A}{p_\infty} - 1\right)$$

or,

$$\frac{p_A}{p_\infty} = \frac{\gamma\, M_\infty^2 C_{p,A}}{2} + 1 = \frac{(1.4)(0.61)^2(-1.258)}{2} + 1 = 0.6723$$

However,

$$\frac{p_A}{p_\infty} = \frac{p_A}{p_o}\,\frac{p_o}{p_\infty} \text{ where } \frac{p_o}{p_\infty} \text{ for } M_\infty = 0.61 \text{ is } 1.286$$

Thus,

$$\frac{p_A}{p_o} = \frac{p_A / p_\infty}{p_o / p_\infty} = \frac{0.6723}{1.286} = 0.523$$

Hence,

$$\frac{p_o}{p_A} = 1.912.$$

From Appendix A, for $\frac{p_o}{p_A} = 1.912$, $M_A = \boxed{1.01}$

This is close enough. Hence, given the numbers in Figure 11.5(a), the numbers in Figure 11.15(c) are consistent with the laws of physics.

11.8 There is a three-dimensional relieving effect for the flow over a sphere. The flow over a cylinder is two-dimensional – in order to get out of the way of the cylinder, the flow can move only upwards or downwards. This means it must greatly accelerate to get out of the way of the cylinder. In contrast, the flow over a sphere is three-dimensional – it can move not only upward or downward but also sideways. This extra degree of freedom means that the flow does not have to speed up so much in flowing over the sphere. Hence, the freestream Mach number of the sphere is higher in order to achieve sonic flow on the sphere – i.e., the critical Mach number is higher.
