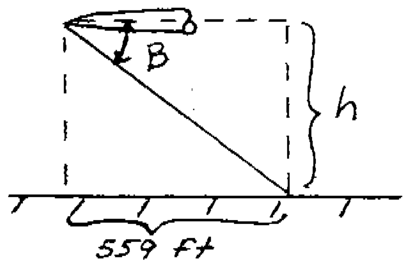
CHAPTER 9

9.1



$\beta = \mathrm{Sin}^{-1}\left(\frac{1}{1.5}\right) = 41.8°$

$h = 559 \tan\beta = 559 \tan 41.8°$

$\boxed{h = 500 \text{ ft}}$

9.2 $M_{n_1} = M_1 \sin\beta = (4.0)\sin 30° = 2$

From Table A.2, for $M_{n_1} = 2$: $\frac{p_2}{p_1} = 4.5$; $\frac{T_2}{T_1} = 1.687$, $\frac{p_{o_2}}{p_{o_1}} = 0.7209$, $M_{n_2} = 0.5774$

$$p_2 = \frac{p_2}{p_1} p_1 = (4.5)(2.65 \times 10^4) = \boxed{1.193 \times 10^5 \text{ N/m}^2}$$

$$T_2 = \frac{T_2}{T_1} T_1 = (1.687)(223.3) = \boxed{376.7°K}$$

From the θ-β-M diagram: $\theta = 17.7°$

$$M_2 = \frac{M_{n_2}}{\sin(\beta - \theta)} = \frac{0.5774}{\sin(30 - 17.7)} = \boxed{2.71}$$

From Table A.1, for $M_1 = 4$: $\frac{p_{o_1}}{p_1} = 151.8$, $\frac{T_{o_1}}{T_1} = 4.2$

$$p_{o_2} = \frac{p_{o_2}}{p_{o_1}} \frac{p_{o_1}}{p_1} p_1 = (0.7209)(151.8)(2.65 \times 10^4) = \boxed{2.9 \times 10^6 \text{ N/m}^2}$$

$$T_{o_2} = T_{o_1} = \frac{T_{o_1}}{T_1} T_1 = (4.2)(223.3) = \boxed{937.9°K}$$

$$s_2 - s_1 = -R \,\ell n \frac{p_{o_2}}{p_{o_1}} = -(287)\,\ell n\, 0.7209 = \boxed{93.9 \frac{\text{joule}}{\text{kgm}^\circ\text{K}}}$$

9.3 Consider an oblique shock. For such a case,

$$\frac{p_{o_2}}{p_{o_1}} = \underbrace{\left(\frac{p_{o_2}}{p_2}\right)}_{\substack{\text{Depends on actual Mach}\\ \text{number behind the shock}\\ M_2,\ \text{not } M_{n_2}}} \times \underbrace{\left(\frac{p_2}{p_1}\right)}_{\substack{\text{Depends on normal}\\ \text{Mach number upstream}\\ \text{of the shock, } M_{n_1}.}} \quad (1)$$

In the derivation of Eq. (8.80), we related M_2 directly to M_1 through Eq. (8.78). This holds only for a normal shock. If we wish to use Eq. (8.78) for an oblique shock, then both M_2 and M_1 in Eq. (8.78) are replaced by M_{n_2} and M_{n_1}. However, in Eq. (1) above, p_{o_2} / p_2 Depends on M_2, not M_{n_2}. Because Eq. (8.78) does not relate M_2 to M_1 for an oblique shock (it relates M_{n_2} to M_{n_1}), then Eq. (8.78) cannot be used for the derivation of p_{o_2} / p_1 for an oblique shock. Therefore, the derivation of Eq. (8.80) holds only for a normal shock. It can not be used for an oblique shock, even with M_1 replaced by M_{n_1}. On the other hand,

$$s_2 - s_1 = c_p \,\ell n \frac{p_2}{p_1} - R \,\ell n \frac{T_2}{T_1}$$

where p_2/p_1 and T_2/T_1 for an oblique shock depend only on M_{n_1}. Since $\frac{p_{o_2}}{p_{o_1}} = e^{-(s_2 - s_1)/R}$ then

clearly $\frac{p_{o_2}}{p_{o_1}}$ depends only on M_{n_1}. For these reasons, when using Table A.2 to determine

changes across an oblique shock, using M_{n_1}, the total pressure ratio $\frac{p_{o_2}}{p_{o_1}}$ is a valid column,

but the column giving $\frac{p_{o_2}}{p_1}$ is not valid.

9.4 To CORRECTLY calculate p_{o_2}:

$$M_{n_1} = M_1 \sin\beta = 3 \sin 36.87^\circ = 1.8$$

From Table A.2, for $M_{n_1} = 1.8$: $\frac{p_{o_2}}{p_{o_1}} = 0.8127$

From Table A.1, for $M_1 = 3$: $\frac{p_{o_2}}{p_1} = 36.73$

$$p_{o_2} \frac{p_{o_2}}{p_{o_1}} \frac{p_{o_1}}{p_1} p_1 = (0.8127)(36.73)(1) = \boxed{29.85 \text{ atm}}$$

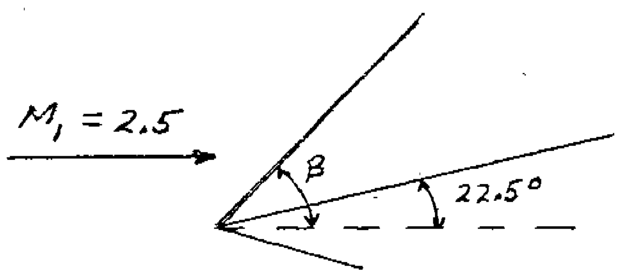
(b) The INCORRECT calculation of p_{o_2} would be as follows:

From Table A.2, for $M_{n_1} = 1.8$: $\frac{p_{o_2}}{p_1} = 4.67$

$$p_{o_2} \frac{p_{o_2}}{p_1} p_1 = 4.67 \text{ (1 atm)} = 4.67 \text{ atm.}$$ Totally WRONG

$$\% \text{ error} = \frac{29.85 - 4.67}{4.67} \times 100 = 539\%$$ -- a terribly large error.

9.5



From the θ-β-M diagram: $\boxed{\beta = 46°}$

$$M_{n_1} = M_1 \sin\beta = 2.5 \sin 46° = 1.8$$

From Table A.2, for $M_{n_1} = 1.8$, $\frac{p_2}{p_1} = 3.613$, $\frac{T_2}{T_1} = 1.532$, $M_{n_2} = 0.6165$

$$p_2 = \frac{p_2}{p_1} p_1 = 3.613 \text{ (1 atm)} = \boxed{3.613 \text{ atm}}$$

$$T_2 = \frac{T_2}{T_1} T_1 = (1.532)(300) = \boxed{459.6°K}$$

$$M_2 = \frac{M_{n_2}}{\text{Sin}(\beta - \theta)} = \frac{0.6165}{\text{Sin}(46 - 22.5)} = \boxed{1.546}$$

9.6 From the θ-β-M diagram, shock detachment occurs when $\alpha > 28.7°$. When $\alpha = \theta = 28.7°$, $\beta = 64.5°$.

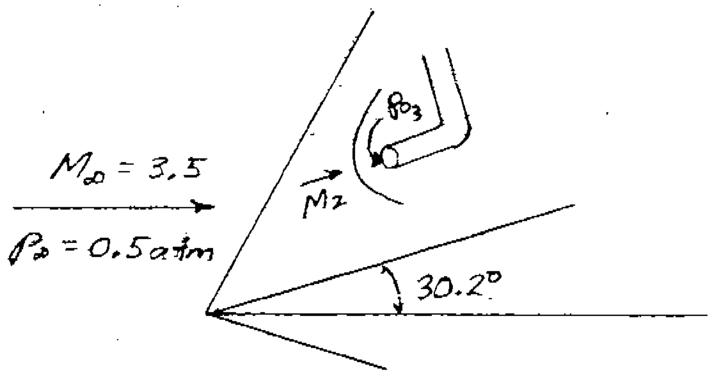
$$M_{n_1} = M_1 \sin \beta = 2.4 \ \sin 64.5° = 2.17$$

From Table A.2, for $M_{n_1} = 2.17$: $\frac{p_2}{p_1} = 5.327$

$$p_{max} = \frac{p_2}{p_1} p_1 = 5.327 \ (1 \text{ atm}) = \boxed{5.327 \text{ atm}}$$

and the maximum pressure occurs when $\alpha = \boxed{28.7°}$

9.7



From the θ-β-M diagram: $\beta = 48°$

$$M_{n_1} = M_1 \sin \beta = 3.5 \ \sin 48° = 2.60$$

From Table A.2: $\frac{p_{o_2}}{p_{o_\infty}} = 0.4601$, $M_{n_2} = 0.5039$,

$$M_2 = \frac{M_{n_2}}{\mathrm{Sin}(\beta - \theta)} = \frac{0.5039}{\mathrm{Sin}(48 - 30.2)} = 1.648$$

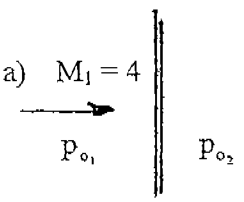
From Table A.2, for $M_2 = 1.648$; $\frac{p_{o_3}}{p_{o_2}} = 0.876$

From Table A.1, for $M = 3.5$: $\frac{p_{o_\infty}}{p_\infty} = 76.27$

$$p_{o_3} = \frac{p_{o_3}}{p_{o_2}} \frac{p_{o_2}}{p_{o_\infty}} \frac{p_{o_\infty}}{p_\infty} p_\infty = (0.876)(0.4601)(76.27)(0.5) = \boxed{15.37 \text{ atm}}$$

9.8 From Table A.1, for $M_1 = 4$, $\frac{p_{o_1}}{p_1} = 151.8$

Hence, $p_{o_1} = \frac{p_{o_1}}{p_1} p_1 = 151.8 \text{ (1 atm)} = 151.8 \text{ atm.}$

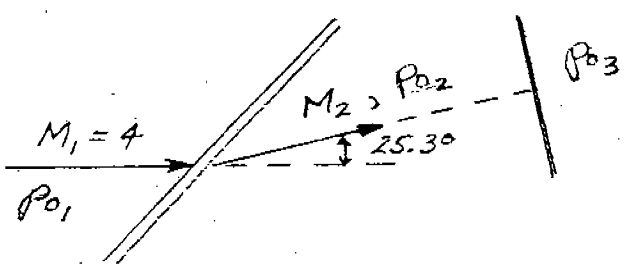
a) 

From Table A.2, for $M_1 = 4$: $\frac{p_{o_2}}{p_{o_1}} = 0.1388$

$$p_{o_2} = \frac{p_{o_2}}{p_{o_1}} p_{o_1} = 0.1388\ (151.8) = 21.07 \text{ atm}$$

Loss in total pressure = $p_{o_1} - p_{o_2} = 151.8 - 21.07 = \boxed{130.7 \text{ atm}}$

b)



From the θ-β-M diagram,

$$\beta = 38.7°$$

$$M_{n_1} = M_1 \sin\beta = 4 \sin 38.7° = 2.5$$

From Table A.2, for $M_{n_1} = 2.5$: $\frac{p_{o_2}}{p_{o_1}} = 0.499$, $M_{n_2} = 0.513$

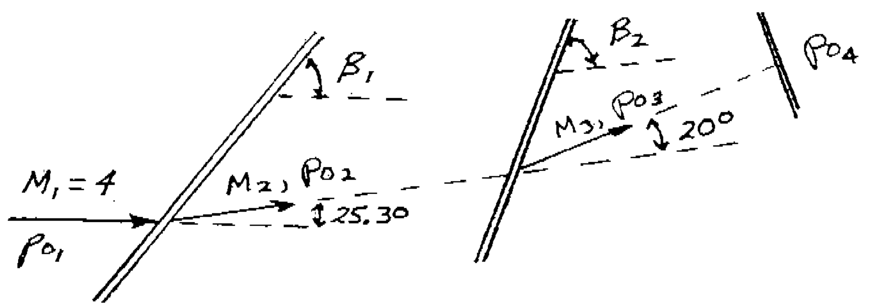
$$M_2 = \frac{M_{n_2}}{\sin(\beta - \theta)} = \frac{0.513}{\sin(38.7 - 25.3)} = 2.21$$

From Table A.2, for $M_2 = 2.21$: $\frac{p_{o_3}}{p_{o_2}} = 0.6236$

$$p_{o_3} = \frac{p_{o_3}}{p_{o_1}} \frac{p_{o_2}}{p_{o_1}} \frac{p_{o_1}}{p_1} p_1 = (0.6236)(0.499)(151.8)(1 \text{ atm}) = 47.24 \text{ atm}$$

Loss in total pressure = $p_{o_1} - p_{o_3} = 151.8 - 47.24 =$ $\boxed{104.6 \text{ atm}}$

c)



From part (b) above, $M_2 = 2.21$, $\frac{p_{o_2}}{p_{o_1}} = 0.499$.

From the β-θ-M diagram: $\beta_2 = 47.3°$

For the second shock: $M_{n_2} = M_2 \sin \beta_2 = 2.21 \sin 47.3° = 1.624$

From Table A.2, for $M_{n_2} = 1.624$: $\frac{p_{o_3}}{p_{o_2}} = 0.8877$, $M_{n_3} = 0.6625$

$$M_3 = \frac{M_{n_3}}{\sin(\beta_2 - \theta_2)} = \frac{0.6625}{\sin(47.3 - 20)} = 1.444$$

From Table A.2, for $M_3 = 1.444$: $\frac{p_{o_4}}{p_{o_3}}$ - 0.947

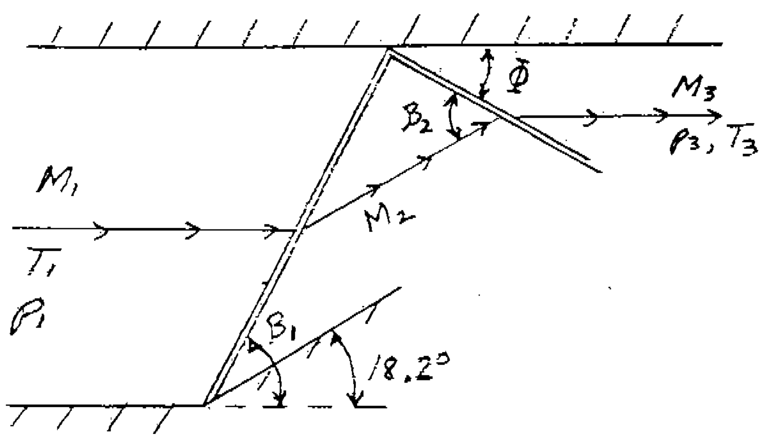
$$p_{o_4} = \frac{p_{o_4}}{p_{o_3}} \frac{p_{o_3}}{p_{o_2}} \frac{p_{o_2}}{p_{o_1}} \frac{p_{o_1}}{p_1} p_1 = (0.947)(0.8877)(0.499)(151.8)$$

$$p_{o_4} = 63.68 \text{ atm}$$

Loss in total pressure = $p_{o_1} - p_{o_4} = 151.8 - 63.68 = \boxed{88.1 \text{ atm}}$

CONCLUSION: To decrease a supersonic flow to subsonic speeds via a shock system, a series of oblique shocks followed by a normal shock yields a smaller total pressure loss than a normal shock by itself. Hence, a system of oblique shocks, followed by a normal shock is a more efficient means of slowing a supersonic flow to subsonic speeds than a single normal shock itself.

9.9



From the θ-β-M diagram, $\beta_1 = 34.2°$

$$M_{n_1} = M_1 \sin \beta_1$$

$$= (3.2) \sin 34.2° = 1.8$$

From Table A.2; for $M_{n_1} = 1.8$: $\frac{p_2}{p_1} = 3.613$, $\frac{T_2}{T_1} = 1.532$,

$$M_{n_2} = 0.6165$$

$$M_2 = \frac{M_{n_2}}{\sin(\beta_1 - \theta_1)} = \frac{0.6165}{\sin(34.2 - 8.2)} = 2.24$$

For the <u>Reflected Shock</u>:

From the θ-β-M diagram, for $M_2 = 2.24$ and $\theta = 18.2°$: $\beta_2 = 44°$

$$M_{n_2} = M_2 \sin \beta_2 = 2.24 \sin 44° = 1.56$$

From Table A.2, for $M_{n_2} = 1.56$: $\frac{p_3}{p_2} = 2.673$, $\frac{T_3}{T_2} = 1.361$, $M_{n_3} = 0.6809$

$$M_3 = \frac{M_{n_3}}{\sin(\beta_2 - \theta)} = \frac{0.6809}{\sin(44 - 18.2)} = \boxed{1.56}$$

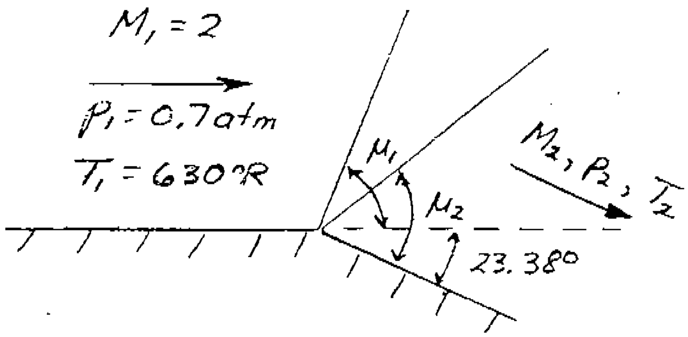
<u>Note</u>: The fact that M_3 and M_{n_2} are equal is just a coincidence.

$$\Phi = \beta_2 - \theta = 44 - 18.2 = \boxed{25.8°}$$

$$p_3 = \frac{p_3}{p_2}\frac{p_2}{p_1} p_1 = (2.673)(3.613)(1 \text{ atm}) = \boxed{9.66 \text{ atm}}$$

$$T_3 = \frac{T_3}{T_2}\frac{T_2}{T_1} T_1 = (1.361)(1.532)(520) = \boxed{1084°R}$$

9.10



From Table A.3: For $M_1 = 2$, $\nu_1 = 26.38°$

$$\nu_2 = \theta + \nu_1 = 23.38° + 26.38° = 49.76°$$

Hence,

$$\boxed{M_2 = 3.0}$$

From Table A.1, for $M_1 = 2$: $\dfrac{p_{o_1}}{p_1} = 7.824$, $\dfrac{T_{o_1}}{T_1} == 1.8$

For $M_2 = 3$: $\dfrac{p_{o_2}}{p_2} = 36.73$, $\dfrac{T_{o_2}}{T_2} = 2.8$

However: $p_{o_1} = p_{o_2}$ and $T_{o_1} = T_{o_2}$. Thus

$$p_2 = \frac{p_2}{p_{o_2}}\frac{p_{o_1}}{p_1}\,p_1 = \left(\frac{1}{36.73}\right)(7.824)(0.7) = \boxed{0.149 \text{ atm}}$$

$$T_2 = \frac{T_2}{T_{o_2}}\frac{T_{o_1}}{T_1}\,T_1 = \left(\frac{1}{2.8}\right)(1.8)(630) = \boxed{405°R}$$

$$\rho_2 = \frac{p_2}{RT_2} = \frac{(0.149)(2116)}{(1716)(405)} = \boxed{4.537 \times 10^{-4} \text{ slug/ft}^3}$$

$$p_{o_2} = p_{o_1} = \frac{p_{o_1}}{p_1}\,p_1 = (7.824)(0.7) = \boxed{5.477 \text{ atm}}$$

$$T_{o_2} = T_{o_1} = \frac{T_{o_1}}{T_1} T_1 = (1.8)(630) = \boxed{1134°R}$$

From Table A.3: for $M_1 = 2$, $\mu_1 = 30°$

For $M_2 = 3$, $\mu_2 = 19.47$

Referenced to the upstream direction:

Angle of forward Mach line $= \mu_1 = \boxed{30°}$

Angle of rearward Mach line $= \mu_2 - \theta = 19.47 - 23.38° = \boxed{-3.91°}$

Note: The rearward Mach line is <u>below</u> the upstream direction for this problem.

9.11 From Table A.1, for $M_1 = 1.58$: $\frac{p_{o_1}}{p_1} = 4.127$

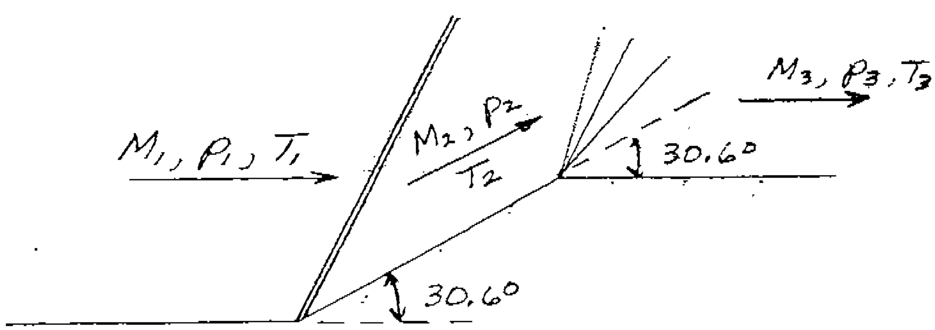
$$\frac{p_{o_2}}{p_2} = \frac{p_{o_1}}{p_2} = \frac{p_{o_1}}{p_1}\frac{p_1}{p_2} = (4.127)\left(\frac{1}{0.1306}\right) = 31.6$$

From Table A.1, for $\frac{p_{o_2}}{p_2} = 31.6$, $M_2 = 2.9$

From Table A.3, for $M_1 = 1.58$; $\nu_1 = 14.27$, for $M_2 = 2.9$: $\nu_2 = 47.79$

$$\theta = \nu_2 - \nu_1 = 47.79 - 14.27 = \boxed{33.52°}$$

9.12



From the θ-β-M diagram:

For $M_1 = 3$ and $\theta = 30.6°$, $\beta = 53.1°$

$$M_{n_1} = M_1 \sin\beta = 3 \sin 53.1 = 2.4$$

From Table A.2, for $M_{n_1} = 2.4$: $\frac{p_2}{p_1} = 6.553$, $\frac{T_2}{T_1} = 2.04$, $\frac{p_{o_2}}{p_{o_1}} = 0.541$, $M_{n_2} = 0.531$

$$M_2 = \frac{M_{n_2}}{\sin(\beta - \theta)} = \frac{0.5231}{\sin(53.1 - 30.6)} = 1.37$$

From Table A.3: For $M_2 = 1.37$, $\nu_2 = 8.128$

$$\nu_3 = 8.128 + 30.6 = 38.73°$$

From Table A.3: For $\nu_3 = 38.73°$, $M_3 = \boxed{2.48}$

From Table A.1: For $M_1 = 3$, $\frac{p_{o_1}}{p_1} = 36.73$, $\frac{T_{o_1}}{T_1} = 2.8$

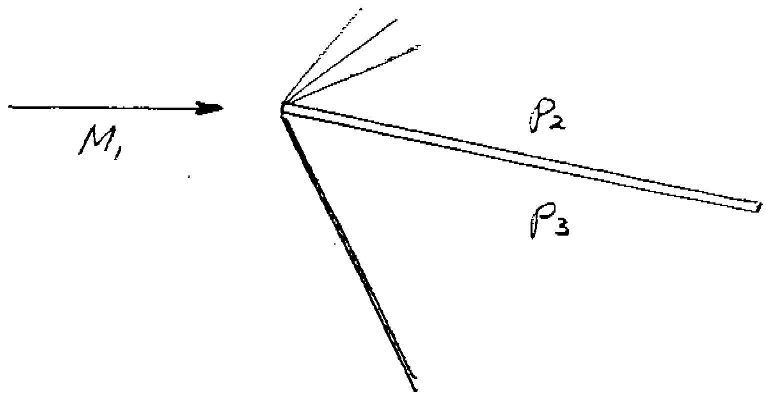
For $M_3 = 2.48$, : $\frac{p_{o_3}}{p_3} = 16.56$, $\frac{T_{o_3}}{T_3} = 2.23$

$$p_3 = \frac{p_3}{p_{o_3}} \frac{p_{o_3}}{p_{o_2}} \frac{p_{o_2}}{p_{o_1}} \frac{p_{o_1}}{p_1} p_1 = \left(\frac{1}{16.56}\right) (1)(0.5401)(36.73)(1 \text{ atm}) = \boxed{120 \text{ atm}}$$

$$T_3 = \frac{T_3}{T_{o_3}} \frac{T_{o_3}}{T_{o_2}} \frac{T_{o_2}}{T_{o_1}} \frac{T_{o_1}}{T_1} T_1 = \left(\frac{1}{2.23}\right) (1)(1)(2.8)(285) = \boxed{357.8°K}$$

Clearly, $p_3 \neq p_1$, $T_3 \neq T_1$, and $M_3 \neq M$. Why? Because there is an entropy increase across the shock wave, which permanently alters the thermodynamic state of the original flow, even after it is brought back to its original direction.

9.13



(a) For $M_1 = 2.6$ and $\theta = 5°$, $\beta = 26.5°$

$$M_{n_1} = M_1 \sin\beta = 2.6 \sin 26.5° = 1.16$$

From Table A.2: $\frac{p_3}{p_1} = 1.403$

From Table A.1, for $M_1 = 2.6$: $\frac{p_{o_1}}{p_1} = 19.95$

From Table A.3, for $M_1 = 2.6$: $\nu_1 = 41.41°$

$$\nu_2 = \nu_1 + \theta = 41.41 + 5° = 46.41° \rightarrow M_2 = 2.83$$

From Table A.1, for $M_2 = 2.83$: $\frac{p_{o_2}}{p_2} = 28.4$

$$\frac{p_2}{p_1} = \frac{p_2}{p_{o_2}} \frac{p_{o_2}}{p_{o_1}} \frac{p_{o_1}}{p_1} = (0.0352)(1)(19.95) = 0.7022$$

$$c_\ell = \frac{L'}{q_\infty S} = \frac{(p_3 - p_2)c \cos\alpha}{q_\infty\ c\ (1)} = \frac{(p_3 - p_2)}{q_\infty} \cos\alpha$$

$$q_\infty = q_1 = \tfrac{1}{2} \rho_1 V_1^2 = \frac{\gamma\ p_1 \rho_1 V_1^2}{2\ \gamma\ p_1} = \frac{\gamma\ p_1 V_1^2}{2\ a_1^2} = \frac{\gamma\ p_1 M_1^2}{2}$$

$$c_\ell = \frac{2(p_3 - p_2)}{\gamma\ p_1\ M_1^2} \cos\alpha = \frac{2}{\gamma\ M_1^2}\left(\frac{p_3}{p_1} - \frac{p_2}{p_1}\right) \cos\alpha$$

$$c_\ell = \frac{2}{(1.4)(2.6)^2}\ (1.403 - 0.7022) \cos 5° = \boxed{0.148}$$

$$c_d = \frac{2}{\gamma\ M_1^2}\left(\frac{p_3}{p_2} - \frac{p_2}{p_1}\right) \sin\alpha = c_\ell\ \frac{\sin\alpha}{\cos\alpha} = 0.148\ \frac{\sin 5°}{\cos 5°} = \boxed{0.0129}$$

(b) For $M_1 = 2.6$ and $\theta = 15°$, $\beta = 35.9°$

$M_{n_1} = M_1 \sin\beta = 2.6 \sin 35.9° = 1.525$

From Table A.2: $\frac{p_3}{p_1} = 2.529$

From Table A.1, for $M_1 = 2.6$: $\frac{p_{o_1}}{p_1} = 19.95$

From Table A.3, for $M_1 = 2.6$: $\nu_1 = 41.41°$

$\nu_2 = \nu_1 + \theta = 41.41 + 15 = 56.41° \rightarrow M_2 = 3.37$

From Table A.1, for $M_2 = 3.37$: $\frac{p_{o_2}}{p_2} = 63.33$

$$\frac{p_2}{p_1} = \frac{p_2}{p_{o_2}}\ \frac{p_{o_2}}{p_{o_1}}\ \frac{p_{o_1}}{p_1} = \left(\frac{1}{63.33}\right)(1)(19.95) = 0.315$$

$$c_\ell = \frac{2}{\gamma\ M_1^2}\left(\frac{p_3}{p_1} - \frac{p_2}{p_1}\right) \cos\alpha = \frac{2}{(1.4)(2.6)^2}\ (2.529 - 0.315) \cos 15° = \boxed{0.452}$$

$$c_d = c_\ell\ \frac{\sin\alpha}{\cos\alpha} = 0.452\ \frac{\sin 15°}{\cos 15°} = \boxed{0.121}$$

(c) For $M_1 = 2.6$ and $\theta = 30°$, $\beta = 59.3°$

$M_{n_1} = M_1 \sin\beta = 2.6 \sin 59.3° = 2.24$

$$\frac{p_3}{p_1} = 5.687,\ \frac{p_{o_1}}{p_1} = 19.95,\ \nu_1 = 41.41°$$

$$\nu_2 = \nu_1 + \theta = 41.41 + 30 = 71.41° \rightarrow M_2 = 4.46$$

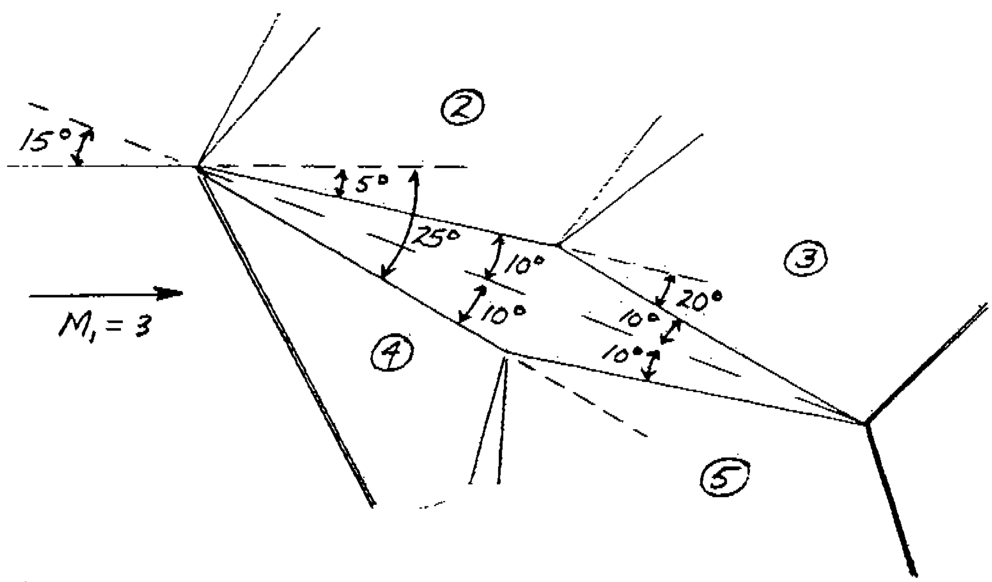
$$\frac{p_{o_2}}{p_2} = 275.25$$

$$\frac{p_2}{p_1} = \frac{p_2}{p_{o_2}} \frac{p_{o_2}}{p_{o_1}} \frac{p_{o_1}}{p_1} = \left(\frac{1}{275.25}\right)(1)(19.95) = 0.0725$$

$$c_\ell = \frac{2}{\gamma M_1^2}\left(\frac{p_3}{p_1} - \frac{p_2}{p_1}\right)\cos\alpha = \frac{2}{(1.4)(2.6)^2}\ (5.687 - 0.0725) = \boxed{1.19}$$

$$c_d = 1.19\ \frac{\sin 30°}{\cos 30°} = \boxed{0.687}$$

9.14



For region 2:

$\nu_1 = 49.76°$

$\nu_2 = \nu_1 + \theta = 49.76° + 5° = 54.76° \rightarrow M_2 = 3.27$

For $M_1 = 3$: $\dfrac{p_{o_1}}{p_1} = 36.73$:

For $M_2 = 3.27$, $\dfrac{p_{o_2}}{p_2} = 54.76$

For region 3:

$\nu_3 = \nu_2 + \theta = 54.76° + 20° = 74.76° \rightarrow M_3 = 4.78$

For $M_3 = 4.78$: $\dfrac{p_{o_3}}{p_3} = 407.83$

For region 4:

$M_1 = 3$ and $\theta = 25° \rightarrow \beta = 44°$

$M_{n_1} = M_1 \sin\beta = 3 \sin 44 = 2.08$

$\dfrac{p_4}{p_1} = 4.881$, $M_{n_4} = 0.5643$, and $\dfrac{p_{o_4}}{p_{o_1}} = 0.6835$

$$M_4 = \frac{M_{n_4}}{\sin(\beta - \theta)} = \frac{0.5643}{\sin(44 - 25)} = 1.733.$$

Thus,

$\nu_5 = 18.69$, $\dfrac{p_{o_4}}{p_4} = 5.165$

For region 5:

$\nu_5 = \nu_4 + \theta = 18.69° + 20° = 38.69° \rightarrow M_5 = 2.48$

$$\frac{p_{o_5}}{p_5} = 16.56$$

<u>Pressure ratios</u>

$$\frac{p_2}{p_1} = \frac{p_2}{p_{o_2}} \frac{p_{o_2}}{p_{o_1}} \frac{p_{o_1}}{p_1} = \left(\frac{1}{54.76}\right)(1)(36.73) = 0.6707$$

$$\frac{p_3}{p_1} = \frac{p_2}{p_1}\frac{p_3}{p_2} = \frac{p_2}{p_1}\frac{p_3}{p_{o_3}}\frac{p_{o_3}}{p_{o_2}}\frac{p_{o_2}}{p_2} = (0.6707)\left(\frac{1}{407.83}\right)(1)(54.76) = 0.09$$

$$\frac{p_4}{p_1} = 4.881$$

$$\frac{p_5}{p_1} = \frac{p_5}{p_{o_5}}\frac{p_{o_5}}{p_{o_4}}\frac{p_{o_4}}{p_{o_1}}\frac{p_{o_1}}{p_1} = \left(\frac{1}{16.56}\right)(1)(0.6835)(36.73) = 1.516$$

Let ℓ = length of each face of the diamond wedge.

$$L' = p_4\ \ell\ \cos 25^\circ + p_5\ \ell\ \cos 5^\circ - p_2\ \ell\ \cos 5^\circ - p_3 \cos 25^\circ$$

$$L' = (p_4 - p_3)\ \ell\ \cos 25^\circ + (p_5 - p_2)\ \ell\ \cos 5^\circ$$

$$c_\ell = \frac{L'}{q_\infty S} = \frac{L'}{\frac{\gamma}{2} p_1 M_1^2 c} = \frac{2}{\gamma\, M_1^2}\frac{\ell}{c}\left[\left(\frac{p_4}{p_1} - \frac{p_3}{p_1}\right)\cos 25^\circ + \left(\frac{p_5}{p_1} - \frac{p_2}{p_1}\right)\cos 5^\circ\right]$$

$$c_\ell = \frac{2}{(1.4)(3)^2}\ \frac{\ell}{c}\ [(4.881 - 0.09)\cos 25^\circ + (1.516 - 0.6707)\cos 5^\circ]$$

$$c_\ell = 0.823\ \frac{\ell}{c}.$$

However,

$$\frac{c/2}{\ell} = \cos 10^\circ \quad \frac{\ell}{c} = \frac{1}{2\ \cos\ 10^\circ} = 0.5077$$

$$c_\ell = (0.823)(0.5077) = \boxed{0.418}$$

$D' = p_4 \, \ell \, \sin 25° + p_5 \, \ell \, \sin 5° - p_2 \, \ell \, \sin 5° - p_3 \, \ell \, \sin 25°$

$D' = (p_4 - p_3) \, \ell \, \sin 25° + (p_5 - p_2) \, \ell \, \sin 5°$

$$c_d = \frac{D'}{q_\infty S} = \frac{D'}{\frac{\gamma}{2} p_1 M_1^2 c} = \frac{2}{\gamma M_1^2} \frac{\ell}{c} \left[\left(\frac{p_4}{p_1} - \frac{p_3}{p_1} \right) \sin 25° + \left(\frac{p_5}{p_1} - \frac{p_2}{p_1} \right) \sin 5° \right]$$

$$c_d = \frac{2}{(1.4)(3)^2} \frac{\ell}{c} [(4.881 - 0.09) \sin 25° + (1.516 - 0.6707) \sin 5°]$$

$$c_d = 0.333 \frac{\ell}{c} = 0.333\ (0.5077) = \boxed{0.169}$$

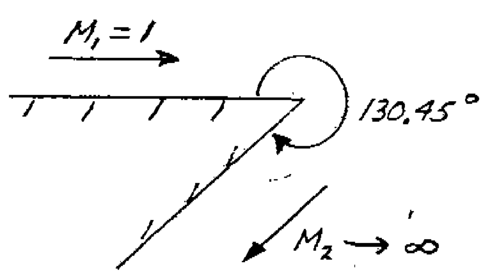
9.15 The maximum expansion would correspond to $M_2 \to \infty$. From Eq. (9.42) in the text,

$$\lim_{M_2 \to \infty} \nu_2 = \lim_{M_2 \to \infty} \left\{ \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \sqrt{\frac{\gamma-1}{\gamma+1}(M_2^2 - 1)} - \tan^{-1} \sqrt{M^2 - 1} \right\}$$

$$= \sqrt{\frac{\gamma+1}{\gamma-1}} \frac{\pi}{2} - \frac{\pi}{2} - \left(\sqrt{\frac{\gamma+1}{\gamma-1}} - 1 \right) \frac{\pi}{2} = 2.277 \text{ rad} = 130.45°$$

Since, for $M_1 = 1$, $\nu_1 = 0$, then

$$\theta_{max} = \nu_2 - \nu_1 = 130.45 - 0 = \boxed{130.45°}$$



9.16 For the cylinder, with c_d based on frontal area,

$$(D')_{cyl} = q_\infty S c_d = q_\infty d(1)/(4/3) = \frac{4}{3} (d) q_\infty$$

For the dimensional wedge airfoil, referring to Figure 9.27.

$$(D')_w = (p_2 - p_3) t$$

Hence,

$$\frac{(D')_{cyl}}{(D')_w} = \frac{\frac{4}{3}(d) q_\infty}{(p_2 - p_3) t}$$

However, $t = d$ and $q_\infty = \frac{\gamma}{2} p_1 M_1^2$

Thus,

$$\frac{(D')_{cyl}}{(D')_w} = \frac{\frac{4}{3}\left(\frac{\gamma}{2}\right)M_1^2}{\left(\frac{p_2}{p_1} - \frac{p_3}{p_1}\right)} = \frac{\frac{2}{3} \gamma M_1^2}{\left(\frac{p_2}{p_1} - \frac{p_3}{p_1}\right)}$$

To calculate p_2/p_1, we have, for $M_1 = 5$ and $\theta = 5°$, $\beta = 15.1°$.

$$M_{n,1} = M_1 \sin \beta = 5 \sin (15.1°) = 1.303$$

From Appendix B, for $M_{n,1} = 1.302$, $\frac{p_2}{p_1} = 1.805$. Also,

$$M_2 = \frac{M_{n,2}}{\sin(\beta - \theta)} = \frac{0.786}{\sin(15.1 - 5)} = 4.48.$$

To calculate $\frac{p_3}{p_1}$, the flow is expanded through an angle of 10°. From Table C, for $M_2 =$ 4.48, $\nu_2 = 71.83$ (nearest entry).

$$\nu_3 = \nu_2 + \theta = 71.83 + 10 = 81.38°$$

Hence, $M_3 = 5.6$ (nearest entry)

From Appendix A: For $M_1 = 5$, $\frac{p_{o_1}}{p_3} = 529.1$

For $M_3 = 5.6$, $\frac{p_{o_3}}{p_3} = 1037$

From Appendix B: For $M_{n_1} = 1.303$, $\frac{p_{o_2}}{p_{o_1}} = 0.9794$

Thus,

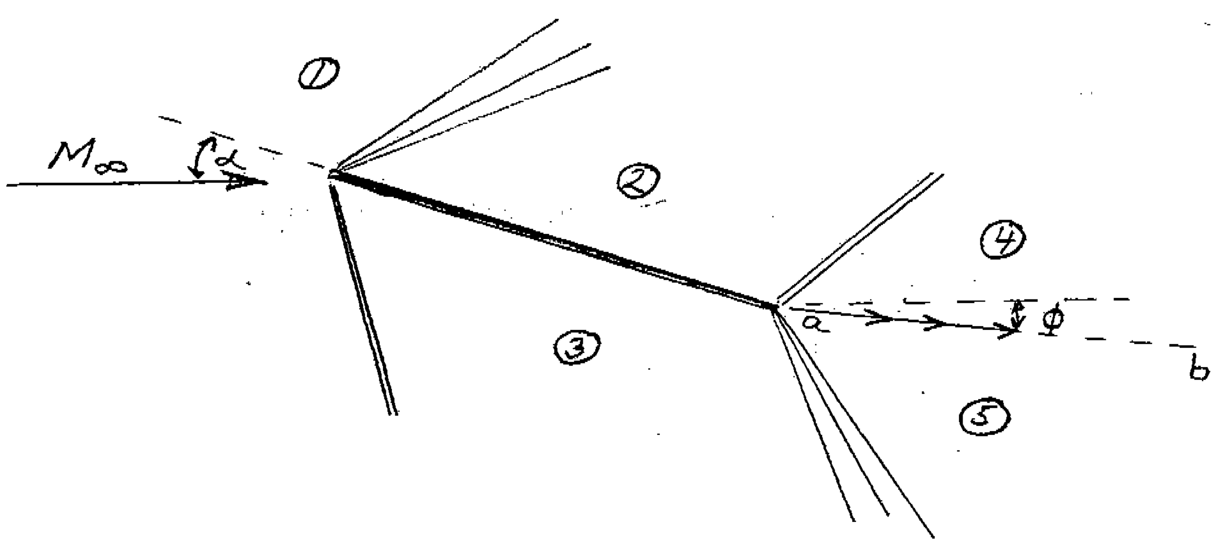
$$\frac{p_3}{p_1} = \frac{p_3}{p_{o_3}}\frac{p_{o_3}}{p_{o_2}}\frac{p_{o_2}}{p_{o_1}}\frac{p_{o_1}}{p_1} = \left(\frac{1}{1037}\right)(1)(0.9794)(529.1) = 0.5$$

Hence,

$$\frac{(D')_{cyl}}{(D')_w} = \frac{\frac{2}{3}\gamma\, M_1^2}{\left(\frac{p_2}{p_1} - \frac{p_3}{p_1}\right)} = \frac{\frac{2}{3}(1.4)(5)^2}{(1.805 - 0.5)} = \boxed{17.9}$$

<u>Note</u>: This is why we try to avoid blunt leading edges on supersonic vehicles. (However, at hypersonic speeds, blunt leading edges are necessary to reduce the aerodynamic heating.)

9.17 The supersonic flow over a flat plate at a given angle of attack in a freestream with a given Mach number, M_∞, is sketched below.



The flow direction downstream of the leading edge is given by line ab. The flow direction is below the horizontal (below the direction of M_∞) because lift is produced on the flat plate, and due to overall momentum considerations, the downstream flow must be inclined slightly downward. Also, line ab is a slip line; the entropy in region 4 is different than in region 5 because the flows over the top and bottom of the plate have gone through shock waves of different strengths. The boundary condition that must hold across the slip line is constant pressure, i.e., $p_4 = p_5$. It is this boundary condition that fixes the strengths of the expansion wave and the shock wave at the trailing edge.

To calculate the trailing edge shock and expansion waves, and the flow direction downstream, use the following iterative approach:

1. Assume a value for ϕ.
2. Calculate the strength of the trailing edge shock for the local deflection angle (α-ϕ). This gives, among other quantities, a value of p_4.
3. Calculate the strength of the trailing edge expansion wave for a local expansion angle of (α-ϕ). This gives a value for p_5.
4. Compare p_4 and p_5 from steps 3 and 4. If they are different, assume a new value of ϕ.
5. Repeat steps 2-4 until $p_4 = p_5$. When this condition is satisfied, the iteration has converged, and the trailing edge flow is now determined.
