CHAPTER 8

8.1 $a = \sqrt{\gamma RT} = \sqrt{(1.4)(287)(230)} = \boxed{304 \text{ m/sec}}$

8.2 $c_p T_o = c_p T_e + \dfrac{V_e^2}{2}$

$$T_e = T_o - \frac{V_e^2}{2c_p} = 519 - \frac{(1385)^2}{2(6006)} = 359.3\ ^\circ R$$

$$a_e = \sqrt{\gamma RT_e} = \sqrt{(1.4)(1716)(359.3)} = 929.1\ ^\circ R$$

$$M_e = \frac{V_e}{a_e} = \frac{1385}{929.1} = \boxed{1.49}$$

8.3 $a = \sqrt{\gamma RT_e} = \sqrt{(1.4)(287)(300)} = 347.2 \text{ m/sec}$

$$M = \frac{V}{a} = \frac{250}{347.2} = 0.72$$

From Tables: $\dfrac{T_o}{T} = 1.104$ and $\dfrac{p_o}{p} = 1.412$

$$T_o = 1.104\ T = 1.104\ (300) = \boxed{331.2\ ^\circ K}$$

$$p_o = 1.412\ p = 1.412\ (1.2) = \boxed{1.694 \text{ atm}}$$

$$\frac{p^*}{p} = \frac{p^*}{p_o}\frac{p_o}{p} = (0.528)(1.412) = 0.7455$$

$$p^* = 0.7455\ p = 0.455\ (1.2) = \boxed{0.8946 \text{ atm}}$$

$$\frac{T^*}{T} = \frac{T^*}{T_o}\frac{T_o}{T} = 0.8333\ (1.104) = 0.92$$

$$T^* = 0.92\ (300) = \boxed{276\ ^\circ K}$$

$$a^* = \sqrt{\gamma RT} = \sqrt{(1.4)(287)(276)} = 333 \text{ m/sec}$$

$$M^* = \frac{V}{a^*} = \frac{250}{333} = \boxed{0.75}$$

8.4 $a = \sqrt{\gamma RT} = \sqrt{(1.4)(1716)(700)} = 1297$ ft/sec

$$M = \frac{v}{a} = \frac{2983}{1297} = 2.3$$

From Tables: $\frac{T_o}{T} = 2.058$ and $\frac{p_o}{p} = 12.5$

$$T_o = 2.058\ T = 2.058\ (700) = \boxed{1441\ ^oR}$$

$$p_o = 12.5\ p = 12.5\ (1.6) = \boxed{20\ atm}$$

$$\frac{T^*}{T} = \frac{T^*}{T_o}\frac{T_o}{T} = (0.8333)\ (2.058) = 1.715$$

$$T^* = 1.715\ T = 1.715\ (700) = \boxed{1200\ ^oR}$$

$$\frac{p^*}{p} = \frac{p^*}{p_o}\frac{p_o}{p} = (0.528)(12.5) = 6.6$$

$$p^* = 6.6\ p = 6.6\ (1.6) = \boxed{10.56\ atm}$$

$$a^* = \sqrt{\gamma RT^*} = \sqrt{(1.4)(1716)(1200)} = 1698 \text{ ft/sec}$$

$$M^* = \frac{V}{a^*} = \frac{2983}{1698} = \boxed{1.757}$$

8.5 From Tables: $\frac{p_o}{p} = 7.824$ and $\frac{T_o}{T} = 1.8$

Hence, for the test section flow,

$$p_o = 7.824\ p = 7.824\ (1) = 7.824 \text{ atm}$$

$$T_o = 1.8\ T = 1.8\ (230) = 414\ ^oK$$

Since the flow is isentropic, both p_o and T_o are constant throughout the flow. Also, in the reservoir, $M \approx 0$. Hence, the reservoir pressure and temperature are

$$\boxed{\begin{array}{l} p_o = 7.824 \text{ atm} \\ T_o = 414\ ^\circ\text{K} \end{array}}$$

8.6 From the Standard Altitude Tables, at 10,000 ft.,

$p_\infty = 1455.6$ lb/ft^2 and $T_\infty = 483.04\ ^\circ$R

From Table A.1: For $M_\infty = 0.82$; $\frac{p_o}{p_\infty} = 1.555$, $\frac{T_o}{T_\infty} = 1.134$

For $M = 1$; $\frac{p_o}{p} = 1.893$, $\frac{T_o}{T} = 1.2$

Since the flow is isentropic, p_o = constant and T_o = constant.

$$p = \frac{p}{p_o}\frac{p_o}{p_\infty}p_\infty = \frac{1}{1.893}(1.555)(1455.6) = \boxed{1196 \text{ lb/ft}^2}$$

$$T = \frac{T}{T_o}\frac{T_o}{T_\infty}T_\infty = \frac{1}{1.2}(1.134)(483.04) = \boxed{456.5\ ^\circ\text{R}}$$

8.7 From Table A.2: $\frac{p_2}{p_1} = 7.72$, $\frac{\rho_2}{\rho_1} = 3.449$, $\frac{T_2}{T_1} = 2.238$,

$\frac{p_{o_2}}{p_1} = 9.181$, $\boxed{M_2 = 0.5039}$, $\frac{p_{o_2}}{p_{o_1}} = 0.4601$

Hence,

$$p_2 = \frac{p_2}{p_1}p_1 = 7.72\ (1) = \boxed{7.72 \text{ atm}}$$

$$T_2 = \frac{T_2}{T_1}T_1 = 2.238\ (288) = \boxed{644.5\ ^\circ\text{K}}$$

$$\rho_1 = \frac{p_1}{RT_1} = \frac{(1)(1.01 \times 10^5)}{(287)(288)} = 1.222 \text{ kg/m}^3$$

$$\rho_2 = \frac{\rho_2}{\rho_1}\rho_1 = 3.449\,(1.222) = \boxed{4.21 \text{ kg/m}^3}$$

$$p_{o_2} = \frac{p_{o_2}}{p_1} p_1 = 9.181\,(1) = \boxed{9.181 \text{ atm}}$$

$$T_{o_2} = T_{o_1} = \frac{T_{o_1}}{T_1} T_1 = (2.352)(288) = \boxed{677.4\,^\circ\text{K}} \text{ (using Table A.1)}$$

$$s_2 = s_1 = -R\,\ell n\,\frac{p_{o_2}}{p_{o_1}} = (287)\,\ell n\,0.4601 = 222.8\,\frac{\text{joule}}{\text{kg }^\circ\text{K}}$$

8.8 $\frac{\rho_2}{\rho_1} = 10.33$. From Table A.2, $\boxed{M_1 = 3.0}$, $\frac{T_2}{T_1} = 2.679$, $\frac{p_{o_2}}{p_1} = 12.06$

Thus,

$$T_1 = \frac{T_1}{T_2} T_2 = \frac{1}{2.679}(1390) = \boxed{518.9^\circ\text{R}}$$

From Table A.1, for $M_1 = 3.0$, $\frac{T_{o_1}}{T_1} = 2.8$

$$T_{o\,2} = T_{o_1} = \frac{T_{o_1}}{T_1} T_1 = 2.8\,(518.9) = \boxed{1453^\circ\text{R}}$$

$$p_{o\,2} = \frac{p_{o_2}}{p_1} p_1 = (12.06)\,(1) = \boxed{12.06 \text{ atm}}$$

8.9 $\frac{p_{o_2}}{p_{o_1}} = e^{-(s_2 - s_1)/R} = e^{-(199.5)/287} = 0.499$

From Table A.2: $\boxed{M_1 = 2.5}$

8.10 From Table A.2: $\frac{T_2}{T_1} = 2.799$ and $M_2 = 0.4695$

Hence,

$$T_2 = \frac{T_2}{T_1} T_1 = 2.799\,(480) = 1343.5^{\circ}R$$

$$a_2 = \sqrt{(1.4)(1716)(1343.5)} = 1796.6 \text{ ft/sec}$$

$$V_2 = M_2\, a_2 = (0.4695)(1796.6) = \boxed{843.5 \text{ ft/sec}}$$

From Table A.1, for $M_2 = 0.4695$, $\frac{T_{o_2}}{T_2} = 1.044$

$$T_2{}^* = \frac{T_2{}^*}{T_{o_2}} \frac{T_{o_2}}{T_2} T_2 = (0.8333)(1.044)(1343.5) = 1169^{\circ}R$$

$$A_2{}^* = \sqrt{\gamma R T_2^*} = \sqrt{(1.4)(1716)(1169)} = 1676 \text{ ft/sec}$$

$$M_2^* = \frac{V_2}{a_2^*} = \frac{843.5}{1676} = \boxed{0.503}$$

8.11 Is the flow subsonic or supersonic? For sonic flow, $\frac{p_o}{p} = \frac{1}{0.528} = 1.893$, which is higher than 1.555. Hence, the flow is <u>subsonic</u>. From Table A.1, for

$$\frac{p_o}{p} = 1.555, \; M = 0.82.$$

$$a = \sqrt{\gamma RT} = \sqrt{(1.4)(287)(288)} = 340.2 \text{ m/sec}$$

$$V = Ma = (0.82)(340.2) = 278.9 \text{ m/sec}$$

8.12 The ratio $\frac{7712.8}{2116} = 3.645$ is larger than 1.893. Hence, the flow is <u>supersonic</u>. This means that a normal shock wave exists in front of the nose of the Pitot tube. From Table A.2, for

$$\frac{p_{o_2}}{p_1} = \frac{7712.8}{2116} = 3.645,\ M_1 = 1.56$$

$$a_1 = \sqrt{\gamma R T_1} = \sqrt{(1.4)(1716)(519)} = 1116.6 \text{ ft/sec}$$

$$V_1 = M_1\ a_1 = (1.56)(1116.6) = \boxed{1742 \text{ ft/sec}}$$

8.13 (a) $\rho = \dfrac{p}{RT} = \dfrac{1.01 \times 10^5}{(287)(288)} = 1.22 \text{ kg/m}^3$

$$V = \sqrt{\frac{2(p_o - p)}{\rho}} = \sqrt{\frac{2(1.555 - 1.0)(1.01 \times 10^5)}{1.22}} = 303 \text{ m/sec} \quad \underline{\text{INCORRECT}}$$

$$\% \text{ error} = \frac{303 - 278.9}{278.9} = \boxed{8.69\%}$$

(b) $\rho = \dfrac{p}{RT} = \dfrac{2116}{(1716)(519)} = 0.002376 \text{ slug/ft}^3$

$$V = \sqrt{\frac{2(p_o - p)}{\rho}} = \sqrt{\frac{2(7712.8 - 2116)}{0.002376}} = 2170.5 \text{ ft/sec} \quad \underline{\text{INCORRECT}}$$

$$\% \text{ error} = \frac{2170.5 - 1742}{1742} = \boxed{24.6\%}$$

8.14 $\dfrac{p_2}{p_1} = 1 + \dfrac{2\gamma}{\gamma+1}\left(M_1^2 - 1\right) = \dfrac{\gamma + 1 + 2\gamma\ M_1^2 - 2\gamma}{\gamma+1} = \dfrac{1 - \gamma + 2\gamma\ M_1^2}{\gamma+1}$ (1)

$$\frac{p_{o_2}}{p_2} = \left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{\frac{\gamma}{\gamma-1}} \qquad (2)$$

$$M_2^2 = \frac{1 + [(\gamma-1)/2]M_1^2}{\gamma\ M_1^2 - (\gamma-1)/2)}$$

Working with the expression inside the parenthesis of Eq. (2):

$$1+\frac{\gamma-1}{2}M_2^2=1+\frac{\gamma-1}{2}\left[\frac{1+\left(\frac{\gamma-1}{2}\right)M_1^2}{\gamma\ M_1^2-(\gamma-1)/2}\right]=1+(\gamma-1)\left[\frac{1+\left(\frac{\gamma-1}{2}\right)M_1^2}{2\gamma\ M_1^2-(\gamma-1)}\right]$$

$$=1+(\gamma-1)\left[\frac{2+(\gamma-1)\ M_1^2}{4\gamma\ M_1^2-2(\gamma-1)}\right]=\frac{4\gamma\ M_1^2-2(\gamma-1)+2(\gamma-1)+(\gamma-1)^2M_1^2}{4\gamma\ M_1^2-2\ (\gamma-1)}$$

$$=\frac{4\gamma\ M_1^2+(\gamma^2-2\gamma+1)\ M_1^2}{4\gamma\ M_1^2-2\ (\gamma-1)}=\frac{(\gamma^2+2\gamma+1)\ M_1^2}{4\gamma\ M_1^2-2\ (\gamma-1)}$$

$$=\frac{(\gamma+1)^2M_1^2}{4\gamma\ M_1^2-2\ (\gamma-1)} \qquad (4)$$

Combining Eqs. (4), (2), and (1), we have:

$$\frac{p_{o_2}}{p_1}=\frac{p_{o_2}}{p_2}\frac{p_2}{p_1}=\left[\frac{(\gamma+1)^2M_1^2}{4\gamma\ M_1^2-2\ (\gamma-1)}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{1-\gamma+2\gamma\ M_1^2}{\gamma+1}\right]$$ which is Eq. (8.80)

8.15 At 80,000 ft., $T_\infty = 389.99°R$

$$V_\infty = 2112\left(\frac{88}{60}\right) = 3097.6 \text{ ft/sec}$$

$$a_\infty = \sqrt{\gamma RT} = \sqrt{(1.4)(1716)(389.99)} = 967.9 \text{ ft/sec}$$

$$M_\infty = \frac{3097.6}{967.9} = 3.2$$

From Appendix A:

For $M_\infty = 3.2$, $\frac{T_o}{T_\infty} = 3.048$

$T_0 = 3.048\ T_\infty = 3.048\ (389.99) = 1188.7\ °R$

Since $0^\circ F = 460^\circ R$, the

$$\boxed{T_o = 728.7^\circ F}$$

8.16 $\frac{p_{o_2}}{p_1} = \frac{1.13}{0.1} = 11.3.$

From Appendix B, $M_\infty = \boxed{2.9}$

8.17 The temperature at the stagnation point is the total temperature in the freestream, because the total temperature is constant across the normal shock. From Eq. (8.40),

$$\frac{T_o}{T_\infty} = 1 + \frac{\gamma - 1}{2} M_\infty^2 = 1 + \frac{1.4 - 1}{2}(36)^2 = 260.2$$

Since $T_\infty = 300$ K, we have

$$T_o = (260.2)(300) = \boxed{78{,}060 K}$$

This is an ungodly high temperature. It is also incorrect, because long before the air would reach this temperature, it would chemically dissociate and ionize. In such a chemically reacting gas, the specific heats are not constant, which means that Eq. (8.40) is not valid for such a chemically reacting flow. In reality, the temperature at the stagnation point on the Apollo was close to 11,000 K, much lower than our estimate above, but still plenty high. Air at 11,000 K is a partially ionized plasma. For the analysis of high temperature, chemically reacting flows, techniques much different than those discussed in this book must be used. See for example Anderson, Modern Compressible Flow, 2nd ed., McGraw-Hill, 1990, or Anderson, Hypersonic and High Temperature Gas Dynamics, McGraw-Hill, 1989, reprinted by the American Institute of Aeronautics and Astronautics, 2000.

8.18 Use Eq. (8.40)

$$\frac{T_o}{T_\infty} = 1 + \frac{\gamma - 1}{2} M_\infty^2$$

For $T_o = 11{,}000$ K, $T_\infty = 300$ K, and $M_\infty = 36$, this equation becomes:

$$\frac{11{,}000}{300} = 1 + \frac{\gamma - 1}{2}(36)^2$$

$$35.67 = 648\,\gamma - 648$$

or,

$$\gamma = \frac{683.7}{648} = \boxed{1.055}$$

In order to use Eq. (8.40) to estimate a reasonably valid stagnation temperature for the Apollo, we have to use an "effective gamma" of 1.055. To double check this, return to Eq. (8.40), insert $\gamma = 1.055$, and calculate T_o.

$$\frac{T_o}{T_\infty} = 1 + \frac{\gamma - 1}{2} M_\infty^2 = 1 + \frac{1.055 - 1}{2} (36)^2 = 36.64$$

or,

$$T_o = 36.64\ T_\infty = 36.64\ (300) = \boxed{11{,}000\text{ K}}$$
