CHAPTER 7

7.1 $p = \rho RT$

$$\rho = \frac{p}{RT} = \frac{(7.8)(2116)}{(1716)(934)} = \boxed{0.0103 \text{ slug/ft}^3}$$

7.2 (a)

$$c_p = \frac{\gamma R}{\gamma - 1} = \frac{(1.4)(1716)}{0.4} = \boxed{6006 \frac{\text{ft lb}}{\text{slug }^\circ\text{R}}}$$

$$c_v = \frac{R}{\gamma - 1} = \frac{1716}{0.4} = \boxed{4290 \frac{\text{ft lb}}{\text{slug }^\circ\text{R}}}$$

$$e = c_v T = 4290\ (934) = \boxed{4.007 \times 10^6 \frac{\text{ft lb}}{\text{slug}}}$$

$$h = c_p T = 6006\ (934) = \boxed{5.610 \times 10^6 \frac{\text{ft lb}}{\text{slug}}}$$

(b) For a calorically perfect gas, c_p and c_v are constants, independent of temperature. Hence, we have again

$$\boxed{\begin{aligned} c_p &= 6006 \frac{\text{ft lb}}{\text{slug }^\circ\text{R}} \\ c_v &= 4290 \frac{\text{ft lb}}{\text{slug }^\circ\text{R}} \end{aligned}}$$

Also, at standard sea level, $R = 519^\circ R$. Hence

$$E = 4290\ (519) = 2.227 \times 10^6 \frac{\text{ft lb}}{\text{slug}}$$

$$h = 6006\ (519) = \boxed{3.117 \times 10^6 \frac{\text{ft lb}}{\text{slug}}}$$

7.3 $c_p = \frac{R}{\gamma - 1} = \frac{(1.4)(287)}{0.4} = 1004.5 \ \frac{\text{joule}}{\text{kg} \ ^\circ\text{K}}$

$$c_v = \frac{R}{\gamma - 1} = \frac{287}{0.4} = 717.5 \ \frac{\text{joule}}{\text{kg} \ ^\circ\text{K}}$$

$$h_2 - h_1 = c_p (T_2 - T_1) = (1004.5)(690\text{-}288) = \boxed{4.038 \times 10^5 \ \frac{\text{joule}}{\text{kg}}}$$

$$e_2 - e_1 = c_v (T_2 - T_1) = (718.5)(690\text{-}288) = \boxed{2.884 \times 10^5 \ \frac{\text{joule}}{\text{kg}}}$$

$$s_2 - s_1 = c_p \ \ell n \frac{T_2}{T_1} - R \ \ell n \frac{p_2}{p_1} = (1004.5) \ \ell n \frac{690}{288} - (287) \ \ell n \ 8.656 = \boxed{258.2 \ \frac{\text{joule}}{\text{kg} \ ^\circ\text{K}}}$$

7.4 $\rho_\infty = \frac{\rho_\infty}{RT_\infty} = \frac{4.35 \times 10^4}{(287)(245)} = 0.6186 \text{ kg/m}^3$

$$\frac{\rho}{\rho_\infty} = \left(\frac{p}{p_\infty}\right)^{1/\gamma}$$

$$\rho = \rho_\infty \left(\frac{p}{p_\infty}\right)^{1/\gamma} = 0.6186 \left(\frac{3.6 \times 10^4}{4.35 \times 10^4}\right)^{1/1.4} = \boxed{0.5404 \ \frac{\text{kg}}{\text{m}^3}}$$

7.5 $\frac{p}{p_o} = \left(\frac{T}{T_o}\right)^{\frac{\gamma}{\gamma-1}}$

$$T = T_o \left(\frac{p}{p_o}\right)^{\frac{\gamma}{\gamma-1}} = 500\left(\frac{1}{10}\right)^{0.2857} = \boxed{259^\circ\text{K}}$$

$$\rho = \frac{\rho}{RT} = \frac{1.01 \times 10^5}{(287)(259)} = \boxed{1.359 \text{ kg/m}^3}$$

7.6 $pv = RT, \text{ hence } v = \frac{RT}{p}$

$$\left(\frac{\partial v}{\partial p}\right)_T = -\frac{RT}{p^2} = -\frac{v}{p}$$

$$\tau_T = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T = \frac{1}{p}$$

<u>Note:</u> 1 atm = 1.01 x 10^5 N/m^2

$$\tau_T = \frac{1}{p} = \frac{1}{(0.2)(1.01 \text{ x } 10^5)} = \boxed{4.95 \text{ x } 10^{-5} \frac{m^2}{N}}$$

For an isentropic process: $\frac{p_1}{p_2} = \left(\frac{\rho_1}{\rho_2}\right)^\gamma = \left(\frac{v_2}{v_1}\right)^\gamma$

I.e., $p_1 v_1^\gamma = p_2 v_2^\gamma$ or pv^γ = constant = c_1

$$v = \left(\frac{c_1}{p}\right)^{1/\gamma}$$

$$\left(\frac{\partial v}{\partial p}\right)_s = \frac{1}{\gamma}(c_1)^{1/\gamma}(p)^{-(1/\gamma)-1} = -\frac{1}{\gamma}\left(pv^\gamma\right)^{1/\gamma}(p)^{(-1-\gamma)/\gamma} = -\frac{1}{\gamma} v\, p^{-1}$$

$$\tau_s = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_s = -\frac{1}{v}\left(-\frac{v}{\gamma p}\right) = \frac{1}{\gamma p}$$

$$\tau_s = \frac{1}{(1.4)(0.2)(1.01 \text{ x } 10^5)} = \boxed{3.536 \text{ x } 10^{-5} \frac{m^2}{N}}$$

7.7 $$c_p = \frac{\gamma R}{\gamma - 1} = \frac{(1.4)(1716)}{(0.4)} = 6006 \frac{\text{ft lb}}{\text{slug °R}}$$

$$h_o = h + \frac{V^2}{2} = c_p T + \frac{V^2}{2} = (6006)(480) + \frac{(1300)^2}{2} = \boxed{3.728 \text{ x } 10^6 \frac{\text{ft lb}}{\text{slug}}}$$

7.8 Let $(h_o)_{res}$ = total enthalpy of the reservoir = $c_p\,(T_o)_{res}$

$(h_o)_e$ = total enthalpy at the exit = $c_p\, T_e + \frac{V_e^2}{2}$

For an adiabatic flow, h_o = constant. Hence

$$(h_o)_{res} = (h_o)_e$$

$$c_p (T_o)_{res} = c_p\, T_e + \frac{V_e^2}{2}$$

$$V_e = \sqrt{2\ c_p \left[(T_o)_{res} - T_e\right]} = \sqrt{2(1004.5)(1000-600)} = \boxed{896.4 \text{ m/sec}}$$

7.9 $\quad T_\infty = \dfrac{p_\infty}{\rho_\infty R} = \dfrac{(0.61)(1.01\ \text{x}\ 10^5)}{(0.819)(287)} = 262.1\ ^\circ K$

$$\frac{T}{T_\infty} = \left(\frac{p}{p_\infty}\right)^{(\gamma-1)/\gamma}\ ;\ T = T_\infty \left(\frac{p}{p_\infty}\right)^{(\gamma-1)/\gamma} = 262.1 \left(\frac{0.5}{0.61}\right)^{0.2857} = 247.6\ ^\circ K$$

Since the flow is isentropic, it is also adiabatic. Hence, h_o = constant

$$h_\infty + \frac{V_\infty^2}{2} = h + \frac{V^2}{2}$$

$$V = \sqrt{2(h_\infty - h) + V_\infty^2} = \sqrt{2\ c_p (T_\infty - T) + V_\infty^2} = \sqrt{2(1004.5)(262.1-247.6)+(300)^2}$$

$$= \boxed{345 \text{ m/sec}}$$

7.10 $\quad p_\infty + \rho \dfrac{V_\infty^2}{2} = p + \rho \dfrac{V^2}{2}$

$$V = \sqrt{\frac{2(p_\infty - p)}{\rho} + V_\infty^{\ 2}} = \sqrt{\frac{2(1.01\ \text{x}\ 10^5)(0.61-0.5)}{0.819} + (300)^2} = 342.2\ \text{m/sec}$$

$$\%\ \text{error} = \left(\frac{345-342.2}{345}\right) \text{x}\ 100 = \boxed{0.81\%}$$

7.11 $T = T_\infty \left(\frac{p}{p_\infty}\right)^{(\gamma-1)/\gamma} = 262.1\left(\frac{0.3}{0.61}\right)^{0.2857} = 214\ ^\circ K$

$$V = \sqrt{2(1004.5)(262.1-214)+(300)^2} = 432 \text{ m/sec}$$

7.12 $V = \sqrt{\frac{2(1.01 \times 10^5)(0.61-0.3)}{0.819} + (300)^2} = 408 \text{ m/sec}$

$$\% \text{ error} = \left(\frac{432-408.7}{432}\right) \times 100 = \boxed{5.55\%}$$

7.13 From Eq. (7.53)

$$h + \frac{V^2}{2} = \text{constant}$$

From Eqs. (7.6b) and (7.9),

$$h = c_p T = \frac{\gamma RT}{\gamma - 1} \qquad (1)$$

From the equation of state,

$$RT = p/\rho \qquad (2)$$

Combining Eqs. (1) and (2),

$$h = \frac{\gamma}{\gamma-1}\left(\frac{p}{\rho}\right) \qquad (3)$$

Hence, Eq. (7.53) can be written as

$$\frac{\gamma}{\gamma-1}\left(\frac{p}{\rho}\right) + \frac{V^2}{2} = \text{constant} \qquad (4)$$

In the limit of $\gamma \to \infty$, Eq. (4) becomes

$$\frac{p}{\rho} + \frac{V^2}{2} = \text{constant}$$

or,

$$p + \tfrac{1}{2} \rho V^2 = \text{constant}$$

which is Bernoulli's equation. Hence, the energy equation for compressible flow can be reduced to Bernoulli's equation for the case of $\gamma \rightarrow \infty$. Hence, the ratio of specific heats for incompressible flow is infinite, which of course does not exist in nature. This is just another example of the special inconsistencies associated with the assumption of incompressible flow, i.e., constant density flow, which of course does not exist in nature. This is why we have stated earlier in this book that incompressible flow is a myth.

As to the question whether Bernoulli's equation is a statement of Newton's second law or an energy equation, we now see that it is both. For an incompressible flow, the application of the fundamental principles of Newton's second law and the conservation of energy are redundant, both leading to the same equation, namely Bernoulli's equation. However, philosophically this author feels strongly that Bernoulli's equation is fundamentally a statement of Newton's second law – it is a mechanical equation. This is how we derived Bernoulli's equation in a very straightforward manner in Chapter 3. For the study of inviscid incompressible flow, we need only to apply the fundamental principles of mass conservation and Newton's second law. The principle of conservation of energy is redundant and is not needed.
