CHAPTER 6

6.1 $V_r = \frac{c}{r^2}$, $V_\theta = 0$, $V_\phi = 0$

$$\nabla \times \vec{V} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \vec{e}_r & r\,\vec{e}_\theta & (r\sin\theta)\,\vec{e}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ \frac{c}{r^2} & 0 & 0 \end{vmatrix}$$

$$= \frac{1}{r^2 \sin\theta} \{\vec{e}_r (0-0) - r\,\vec{e}_\theta \left(\frac{\partial c / r^2}{\partial \phi} - 0 \right) + r\sin\theta \;\; \vec{e}_\phi \left(0 - \frac{\partial c / r^2}{\partial \theta} \right) \}$$

$$= \frac{1}{r^2 \sin\theta} \;\; \{0 - 0 + 0\} = \boxed{0}$$

Flow is irrotational.

6.2 $V_r = \frac{c}{r^2}$, $V_\theta = 0$, $V_\phi = 0$

$$\nabla \cdot \vec{V} = \frac{1}{r^2} \frac{\partial}{\partial r} [r^2 (\frac{c}{r^2})] + \frac{1}{r\sin\theta} \frac{\partial}{\partial \theta} (0) + \frac{1}{r\sin\theta} \frac{\partial (0)}{\partial \phi}$$

$$\nabla \cdot \vec{V} = \frac{1}{r^2} \frac{\partial c}{\partial r} + 0 + 0 = 0 + 0 + 0 = \boxed{0}$$

The flow is a physical possible incompressible flow.

6.3

For the sphere: $(C_p) = 1 - \frac{9}{4} \sin^2\theta$

For the cylinder: $(C_p)_{cyl.} = 1 - 4\sin^2\theta$

At the top of the sphere: $\theta = \pi/2$, hence

$$(C_p)_{sphere} = -5/4 = -1.25$$

For no manometer deflection, $(C_p)_{sphere} = (C_p)_{cyl.}$

$$-1.25 = 1 - 4\sin^2\theta$$

$$\sin^2\theta = 0.5625$$

$$\sin\theta = 0.75$$

Hence:

$$\boxed{\theta = 48.6^\circ}$$

The pressure tap on the cylinder must be located at an angular position 48.6° above or below the stagnation point.
