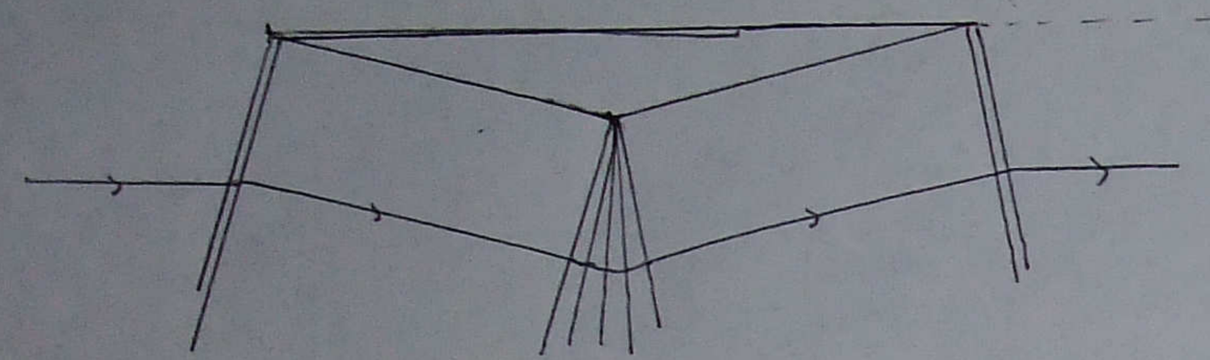
1a) i



ii b) $\theta = \operatorname{atan} \frac{h}{c/2}$

$= 13.5°$

$C_p = \frac{2\theta}{\sqrt{M^2-1}}$

$C_{pm} = 0$

$C_{p,l_1} = \frac{2\theta}{\sqrt{3^2-1}} = 0.167$

$C_{p,l_2} = \frac{-2\theta}{\sqrt{3^2-1}} = -0.167$

$C_L = \cancel{C_n -} \frac{4\theta}{\sqrt{M_\infty^2-1}} \quad \frac{2}{\gamma M_\infty^2}\left(\frac{p_1}{p_\infty} + \frac{p_2}{p_\infty}\right)$

$C_p = \frac{p-p_\infty}{q_\infty} = \frac{2}{\gamma M_\infty^2}\left(\frac{p_1}{p_\infty} - 1\right)$

$= \frac{4}{\gamma M_\infty^2} + C_{p,l_1} + C_{p,l_2}$

$= 0.317$

iii $\theta = 13.5°$, $\beta = 31°$ at $M_\infty = 3$

$M_n = 3 \sin 31$

$= 1.545$

$\frac{p_1}{p_\infty} = 2.63 \qquad M_1 = \frac{0.684}{\sin(31-13.5)} = 2.27 \qquad \nu_1 = 33.65°$

$C_p = \frac{p-p_\infty}{q_\infty} = \frac{2}{\gamma M_\infty^2}\left(\frac{p_1}{p_\infty} - 1\right)$

$C_{p,l_1} = \frac{2}{1.4 \times 3^2}(2.63-1)$

$= 0.259$

$\nu_2 = \nu_1 + 2\theta = 33.65 + 2(13.5) = 60.65°$ $\qquad M_2 = 3.64 \qquad \frac{p_2}{p_1} = \left(\frac{2+(\gamma-1)M_1^2}{2+(\gamma-1)M_2^2}\right)^{\frac{\gamma}{\gamma-1}} = 0.128$

$\frac{p_2}{p_\infty} = \frac{p_2}{p_1}\frac{p_1}{p_\infty} = 0.128 \times 2.63 = 0.337$

$C_{p,l_2} = \frac{2}{1.4 \times 3^2}(0.337-1) = -0.105$

$C_L = \cancel{0.154 \; C_p \ldots} \; \frac{4}{\gamma M_\infty^2} + C_{p,l_1} + C_{p,l_2}$

$= \cancel{0.154} \quad 0.471$

iv Difference in C_p is ~~58%~~ 55% and 37%, difference in C_L is 31%.

The errors are due to the assumptions that linearized theory is applied for thin airfoils at small angles of attack, which is not a valid assumption here.

1b) i - Supersonic flow without shockwaves in convergent channel

No shock waves hence isentropic flow, entropy constant. Thus, total temperature and pressure remains constant, based on 2nd law of thermodynamics

By area-velocity relation, $\frac{dA}{A} = (M^2-1)\frac{du}{u}$. For supersonic flow, velocity decreases as area decreases; and hence static pressure and temperature increase based on isentropic flow relations

- Uniform supersonic flow turned into itself

Shockwave occurs. Adiabatic, thus total ~~temperature~~ enthalpy and hence total temperature remains constant. From $\frac{p_{t2}}{p_{t1}} = e^{-\frac{s_2-s_1}{R}}$, total pressure decreases. From Prandtl relation, $M_2 < M_1$. As $v = M \cdot a$, flow velocity decreases; and static temperature and pressure increase.

- Uniform supersonic flow turned away from itself

Expansion wave occurs. From Prandtl-Meyer function, one can observe that $M_2 > M_1$ thus flow velocity increases, while static pressure and temperature decrease. Isentropic flow thus total temp and pressure remains constant.

ii $h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$

$h_t = \text{constant}, \quad h = c_p T$

$c_p T_t = c_p T + \frac{u^2}{2} \qquad \text{sub } \frac{u^2}{2} = \frac{M^2 a^2}{2} = \frac{M^2 \gamma R T}{2}$

$T_t = T + \frac{M^2 \gamma R T}{2 c_p}$

$\frac{T_t}{T} = 1 + \frac{M^2 \gamma R}{2 c_p} \qquad \text{sub } c_p = R\frac{\gamma}{\gamma-1}$

$\frac{T_t}{T} = 1 + \frac{\gamma-1}{2}M^2 \qquad \text{sub } \frac{T_t}{T} = \left(\frac{p_t}{p}\right)^{\frac{\gamma-1}{\gamma}}$

$\frac{p_t}{p} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}$

2a) i $M_1 = \frac{v_1}{a_1} = \frac{v_1}{\sqrt{\gamma R T_1}}$

$= \frac{170.5}{\sqrt{1.4\times287\times289}}$

$= 0.500$

$\frac{P_0}{P_1} = 1.186$ $\quad \frac{T_0}{T_1} = 1.05$ $\quad T_0 = 1.05\times289 = 303.45K$ $\quad P_0 = 1.186\times10^5 N/m^2$

$\frac{A_1}{A^*} = 1.34$

$\frac{A_2}{A^*} = \frac{A_2}{A_1}\times\frac{A_1}{A^*} = 0.76\times1.34 = 1.0184$

$A_2 > A^*$ Subsonic at A_2

No shockwave, isentropic

$P_{0_4} = P_0 = 1.186\times10^5 N/m^2$

$\frac{A_4}{A^*} = \frac{A_4}{A_1}\times\frac{A_1}{A^*} = 0.785\times1.34 = 1.052$

$M_4 = 0.77$ $\quad \frac{T_0}{T_4} = 1.12$ $\quad T_4 = \frac{1}{1.12}\times303.45 = \cancel{339.56K}$ $270.94K$

$v_4 = M_4\sqrt{\gamma R T_4} = 0.77\sqrt{1.4\times287\times\cancel{339.56}\ 270.94}$

$= \cancel{284.4\ m/s}$ $254.1\ m/s$

ii $M_2 = 1$ $\quad \frac{A_2}{A^*} = 1$ $\quad \frac{A_2}{A_1} = \frac{A_2}{A^*}\cdot\frac{A^*}{A_1} = \frac{1}{1.34} = 0.746$

iii Let $M_{3'}$ denote Mach no. directly upstream of shock, M_3 directly downstream of shock

$M_{3'} = 1.2$ $\quad \cancel{\frac{A_3}{A^*} = 1.03}$ $\quad \frac{A_3}{A_2} = 1.03$ $\quad \cancel{\frac{P_{03'}}{P_3}}$

$M_3 = 0.8422$ $\quad \frac{A_3}{A^*} = 1.024$ $\quad \frac{P_{03}}{P_{03'}} = \frac{P_{03}}{P_{02}} = 0.9928$

$\frac{A_4}{A^*} = \frac{A_4}{A_1}\cdot\frac{A_1}{A_2}\cdot\frac{A_2}{A_3}\cdot\frac{A_3}{A^*} = 0.785\times\frac{1}{0.746}\times\frac{1}{1.03}\times1.024$

$= 1.046$

$M_4 = 0.78$ $\quad \frac{T_0}{T_4} = 1.122$ $\quad \frac{P_{04}}{P_4} = 1.495$

$P_4 = \frac{P_4}{P_{04}}\cdot\frac{P_{04}}{P_{03}}\cdot\frac{P_{03}}{P_{02}}\cdot\frac{P_{02}}{P_{01}}\cdot\frac{P_{01}}{P_1}\cdot P_1$

$= \frac{1}{1.495}\times1\times0.9928\times1\times1.186\times10^5 = 0.7876\times10^5 N/m^2$

$P_{04} = \frac{P_{04}}{P_4}\times P_4$

$= 1.177\times10^5 N/m^2$

$T_4 = \frac{T_4}{T_0}\times T_0 = \frac{1}{1.122}\times303.45$

$= 270.45 K$

$v_4 = M_4\sqrt{\gamma R T_4} = 0.78\sqrt{1.4\times287\times270.45}$

$= 257.1\ m/s$

2b) i $M = \frac{v}{a} = \frac{v}{\sqrt{\gamma R T}}$

$= \frac{2156\times1000/60^2}{\sqrt{1.4\times287\times(273-60)}}$

$= 2.046$

ii $M_1 = 2.046$, $\quad M_2 = 0.569$ $\quad \frac{P_{01}}{P_1} = 8.458$ $\quad \frac{T_{01}}{T_1} = 1.84$

$\frac{P_{02}}{P_2} = 1.247$ $\quad \frac{P_2}{P_1} = 4.736$

$P_{02} = \frac{P_{02}}{P_2}\cdot\frac{P_2}{P_1}\cdot P_1$ valid for normal shocks

$= 1.247\times4.736\times2.27\times10^4$

$= 1.34\times10^5 N/m^2$

$T_{02} = T_{01} = \frac{T_{01}}{T_1}$

$= 1.84\times(-60+273.15)$

$= 392.2 K$