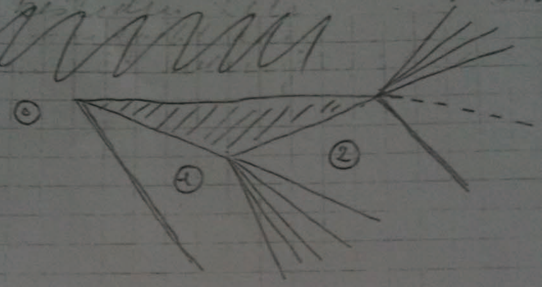
naam / name
studienummer / student number
vak / course
code / code
opleiding / program
aantal ingeleverde vellen / total number of sheets
datum / date
opgave nummer / question number

TU Delft
Technische Universiteit Delft

1a)

i)



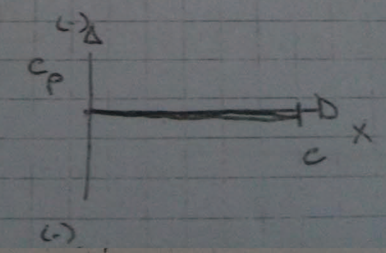
ii) linearized theory: $C_p = \frac{2\theta}{\sqrt{M_\infty^2 - 1}}$

$\theta = \text{atan}(0.24)$

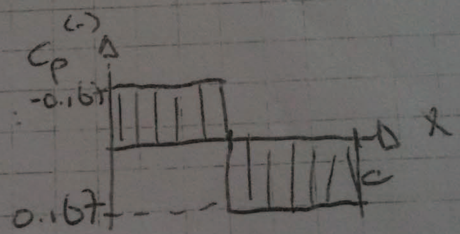
$$C_{p_1} = \frac{2\theta}{\sqrt{M_\infty^2 - 1}} = \frac{2 \cdot \text{atan}(0.24)}{\sqrt{8}} = 0.167$$

$$C_{p_2} = \frac{-2\theta}{\sqrt{M_\infty^2 - 1}} = \frac{-2 \cdot \text{atan}(0.24)}{\sqrt{8}} = -0.167$$

upper side:



lower side:



$C_l = 0$

NOT FOR PUBLICATION

iii) ⓪ → ① OSW

$M_0 = 3$ $\Bigg\{$ $\beta = 30.7$ $\quad \left(\begin{array}{l} M_{0_n} = 1.53 \\ M_{1_n} = 0.69 \end{array} \right)$

$\theta = \operatorname{atan}(0.24) = 13.5$

$\frac{p_1}{p_0} = 2.57$ $\quad \frac{p_1}{p_{0_1}} = 0.0763$ recipr 13.106

$M_1 = 2.33$

$\frac{p_2}{p_{0_2}} = 0.0095$ recipr. 105.2

① → ② PME

$M_1 = 2.33$ $\quad \nu_2 = \nu_1 + 2\theta = 62°$

$\nu_1 = 35°$ $\quad M_2 = 3.73$

$\frac{p_2}{p_1} = 0.125$

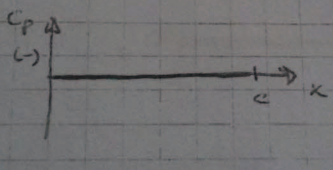
$\frac{p_2}{p_0} = \frac{p_2}{p_1} \cdot \frac{p_1}{p_0} = 0.125 \cdot 2.57 = 0.321$

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2} = \frac{2(p - p_\infty)}{\gamma p_\infty M_\infty^2} = \frac{2}{\gamma M_\infty^2}\left(\frac{p}{p_\infty} - 1\right)$$

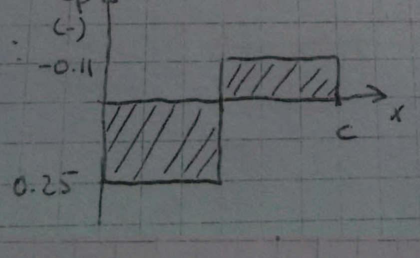
$$C_{p_1} = \frac{2}{\gamma \cdot M_\infty^2}\left(\frac{p_1}{p_0} - 1\right) = \frac{2}{1.4 \cdot 9}\left(2.57 - 1\right) = 0.25$$

$$C_{p_2} = \frac{2}{\gamma \cdot M_\infty^2}\left(\frac{p_1}{p_0} \cdot \frac{p_2}{p_1} - 1\right) = \frac{2}{1.4 \cdot 9}\left(2.57 \cdot 0.125 - 1\right) = -0.11$$

upper side:



lower side:



$$C_l = \frac{1}{2}\left(C_{p_1} + C_{p_2}\right) = \frac{1}{2}\left(0.25 - 0.11\right) = 0.07.$$

$$\rho_\infty V_\infty^2 = \frac{p_\infty}{RT_\infty} V_\infty^2 = \gamma p_\infty M_\infty^2$$

1b

(i) – P_t is constant, $p \uparrow$, T_t is constant, $T \uparrow$, $V \downarrow$

– $P_t \downarrow$, $p \uparrow$, T_t is constant, $T \uparrow$, $V \downarrow$

– P_t is constant, $p \downarrow$, T_t is constant, $T \downarrow$, $V \uparrow$

(ii) $h_0 = h + \frac{1}{2} u^2$

$T_0 = T + \frac{1}{2} \frac{u^2}{c_p}$

$T_0 = T + \frac{\gamma - 1}{2} M^2$

$\frac{P_0}{P} = \left(\frac{T_0}{T} \right)^{\gamma/\gamma - 1}$ } $\frac{P_0}{P} = \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{\gamma/\gamma - 1}$

2a

(1) A_1 : $M_1 = \frac{170.5}{\sqrt{1.4 \cdot 287 \cdot 289}} = 0.5$

$\frac{A_1}{A^*} = 1.34 > \frac{A_1}{A_2} = 1.32$ → Flow does not reach $M = 1$ in A_2

$\frac{A_4}{A^*} = \frac{A_4}{A_1} \cdot \frac{A_1}{A^*} = 0.785 \cdot 1.34 = 1.05$

$M_4 = 0.77$

$\frac{P_4}{P_0} = 0.675$

2a)

(i) $V_1 = 170.5$ m/s

$$M_1 = \frac{V_1}{\sqrt{\gamma R T_1}} = \frac{170.5}{340.8} = 0.5$$

$$T_0 = \frac{T_0}{T_1} \cdot T_1 = \frac{293}{0.95}$$

$$\frac{P_1}{P_{0_1}} = 0.84$$

$$\frac{A_1}{A^*} = 1.34 \rightarrow \frac{A_4}{A^*} = \frac{A_1}{A^*} \cdot \frac{A_4}{A_1} = 1.34 \cdot 0.785 = 1.05$$

$$M_4 = 0.77$$

$$T_4 = T_{0_1} \cdot \frac{T_4}{T_{0_4}} = 0.89$$

$$= 303 \cdot 0.89 = 27\ldots K$$

$$P_{0_1} = P_{0_4} = \frac{10^5}{0.84} = 1.19 e 10^5 \ N/m^2$$

$$V_4 = M_4 \sqrt{\gamma R T_4} = 254 \ m/s.$$

2a ii) For $\frac{A_2}{A_1} < \frac{A^*}{A_1} = 0.746$ the flow is sonic in A_2

(iii) $M_{3_A} = 1.2 \rightarrow \frac{A_3}{A^*_A} = 1.030 = \frac{A_3}{A_2}$ $\quad M_{3_B} = 0.84 \rightarrow \frac{A_3}{A^*_B} = 1.024$

3a=left side of shock
3b=right side of shock

$$\frac{P_{0_{3B}}}{P_{0_{3A}}} = 0.99 \qquad P_{0_4} = P_{0_{3B}} = 1.18 \cdot 10^5 \ N/m^2.$$

$$\frac{A_4}{A_3} = \frac{A_4}{A_1} \cdot \frac{A_1}{A_2} \cdot \frac{A_2}{A_3} = \frac{0.785}{0.746 \cdot 1.03} = 1.022.$$

$$\frac{A_4}{A^*_B} = \frac{A_4}{A_3} \cdot \frac{A_3}{A^*_B} = 1.022 \cdot 1.024 = 1.047$$

$$M_4 = 0.78$$

$$V_4 = 0.78 \sqrt{1.4 \cdot 287 \cdot 303 \cdot 0.89}$$

$$= 257 \ m/s.$$

i) $V = 2156 \text{ km/h} = 598.9 \text{ m/s}$

$$M = \frac{V}{a} = \frac{598.9}{\sqrt{1.4 \cdot 287 \cdot 213}} = 2.05$$

ii) $$T_{0_2} = T\left(1 + \frac{\gamma - 1}{2} M^2\right)$$

$$= 213\left(1 + 0.2 \cdot (2.05)^2\right) = 392 \text{ K}$$

$$P_{0_2} = \frac{P_{0_2}}{P_{0_1}} \cdot P_1 \left(1 + \frac{\gamma-1}{2} M^2\right)^{\gamma/\gamma - 1}$$

$$= 0.70 \cdot 2.27 \cdot 10^4 \cdot 8.46$$

$$= 1.34 \cdot 10^5 \text{ N/m}^2$$

3a)

i) Obtain specific volumes from Table 1:

$P_1 = 1 \text{ MPa}$, $T_1 = 400 \text{ °C}$ } $v_1 = 0.30661 \frac{m^3}{kg}$

$P_2 = 1 \text{ MPa}$, $T_2 = 250 \text{ °C}$ } $v_2 = 0.23275 \frac{m^3}{kg}$ → cylinder will not yet reach stops

Pressure is constant during process:

$W_b = m \cdot P (v_1 - v_2) = 22.16 \text{ kJ}$

ii) Volume of the cylinder is 60 % of initial volume

$W_b = mP(v_1 - 0.6 v_1) = 36.79 \text{ kJ}$ (0.184)

Temp decreases, piston drops, pressure remains 1 MPa → piston hits stops, temperature decreases further, therefore also pressure decreases.

iii) Temperature of the final state:

$P_2 = 0.5 \text{ MPa}$, $v_2 = 0.6 \cdot v_1$ } $T_2 = 151.8 \text{ °C}$ (Table 2) i)

$$w_{in} = c_p(T_2 - T_1) = 285 \text{ kJ/kg}$$

$T_2 = 300$, $T_1 = 20$

ii) Specific volume at inlet:

$10 \text{ L} = 10 \text{ dm}^3 = 10^{-2} \text{ m}^3$

$$v_1 = \frac{RT_1}{P_1} = 0.7008 \text{ m}^3/\text{kg}$$

$$\dot{m} = \frac{0.010}{0.7008} = 0.01427 \text{ kg/s}$$

$$\dot{W}_{in} = \dot{m}\, c_p (T_2 - T_1) = 4.068 \text{ kW}.$$