Delft University of Technology DEPARTMENT OF AEROSPACE ENGINEERING	
Course: Ae2-125;	Course year: 2
Date: Thursday 25th June 2009	Time: 14.00–17.00
Answers are expected in English	

33 **Problem 1**

17 a) Consider the supersonic air flow between two parallel walls at M_∞=2.2 with p_∞=45kPa and T_∞=182K. A positive deflection of 24° is imposed on one of the walls.

6 6 (i) Draw the waves produced by the deflection and the interaction with the opposite wall. On the same drawing also draw the flow streamlines.

4 5 (ii) Calculate the following flow properties past the reflected wave: Mach number, static and total pressure, total temperature, entropy (w.r.t. free stream conditions).

[illegible] 6 (iii) Calculate the maximum deflection angle beyond which a Mach reflection will occur instead of the regular reflection.

16 b) (8+8) [illegible] A supersonic flow at M_∞=3.2 expands around a convex corner. What is the maximum turning angle θ? What will be the velocity reached by the flow after such expansion? (Assume air with p_∞=50kPa and T_∞=380K)

34 **Problem 2**

18 a) [illegible] Air flows through a convergent-divergent channel with throat area A_t=350cm^2. The exit area is A_e=600cm^2. A pressurised reservoir with P_0=700kPa is connected to the nozzle. Determine:

6 (i) the Mach number at the exit M_e when the exit pressure is P_e=100kPa

6 (ii) the maximum static pressure at the exit below which the mass flow is constant

6 (iii) the range of static pressure at the exit in which oblique shocks emanate from the edge of the nozzle

16 b) (8+8) [illegible] Consider a diamond shaped symmetrical airfoil with thickness to cord ratio t/c=0.1 flying at Mach 3.0 and with zero angle of attack. Compare the drag coefficient calculated with shock-expansion theory and that obtained with linearized supersonic flow.

Appendix

Specific gas constant for air: R=287.04J/kg K

Specific heat ratio for air: γ=1.4

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Appendix

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33 **Problem 3**

12 a) Consider a thermally insulated compressor (fig. 1). The mass flow of air through the compressor is 1.5kg/s following a transformation from state 1 to 2, identified by the following state variables $T_1 = 25°C, p_1 = 1\text{bar}, T_2 = 300°C, p_2 = 5\text{bar}$.

3 i) determine the power required by the compressor;

3 ii) verify that the transformation followed by the gas is irreversible;

3 iii) represent the transformation 1-2 in the *T-s* plane;

3 iv) determine the power that would have been required by the compressor in the case of an isentropic compression from the same initial state to the same final pressure.

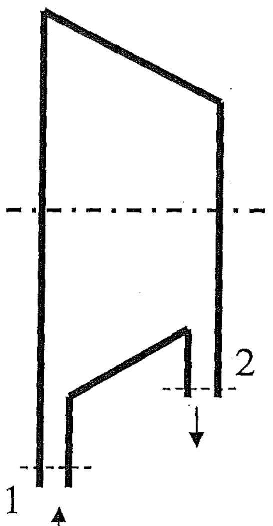

Fig. 1 – compressor schematics.

11 b) Consider a rigid and thermally insulated tank, divided into two parts, A and B, separated by a wall (fig. 2). Each part has a volume of $1m^3$. Part A and part B initially contain the same mass $m_{A,in}=m_{B,in}=1kg$ of air. The air initial temperature is 100°C in part A and 20°C in part B. Heat can be transferred through the wall but mass flow is allowed.

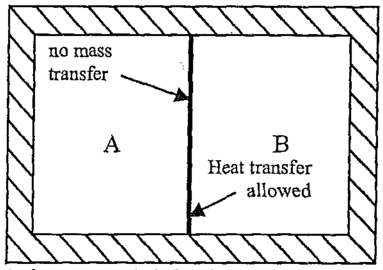

Fig. 2 – tank initial configuration

4 i) determine temperature and pressure in both parts A and B at the end of the transformation;

3 ii) is the transformation reversible? Motivate your answer;

4 iii) determine the air final temperature in both parts A and B in the case where $m_{A,in}=2 \cdot m_{B,in}$. What would have been the air final temperature, in A and B, in the case that $m_{A,in} >> m_{B,in}$?

10 c) Answer the following questions on perpetual motion machines:

5 i) In order to achieve a more sustainable transport system, a variant of the *SUPERBUS* (electrically powered road vehicle) is imagined where a windmill is mounted on its roof. The energy captured by the windmill when the vehicle travels at a given speed is used to power its electrical engine. It is claimed that the vehicle can travel only powered by the windmill. Is this a perpetual motion machine? If yes, of I or II type? Motivate the answer.

5 ii) A pendulum of mass *M* is put initially in oscillatory motion. At each oscillation the pendulum hits a wheel keeping it in rotation. The wheel drives a dynamo, which powers a bulb light. It is claimed that once the pendulum is put in oscillation, this device will produce electricity for ever. Is this a perpetual motion machine? If yes, of I or II type? Motivate your answer.

Exam 25/06/2009 AE2-125

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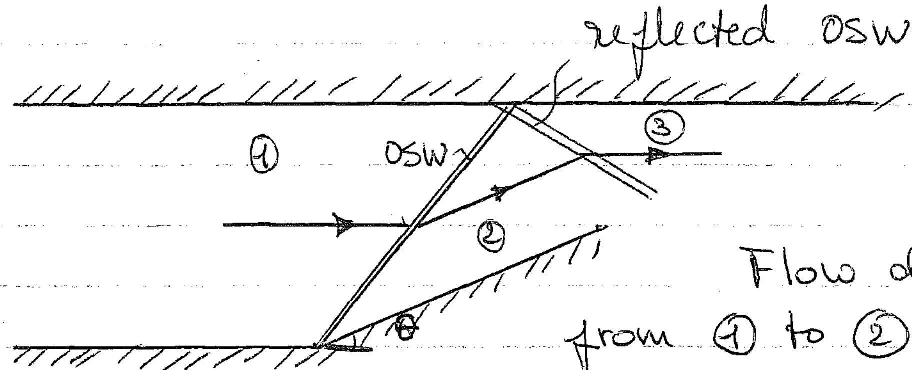
problem 1.

a)

(i)



Flow deflection from 1 to 2 is the same as 2 to 3

$\theta_{12} = \theta_{23}$

(ii) $M_1 = 2.2$

$\theta = 24° \rightarrow \beta = 55°$

$M_2 = 1.18$

$p_{0_2}/p_{0_1} = 0.81$

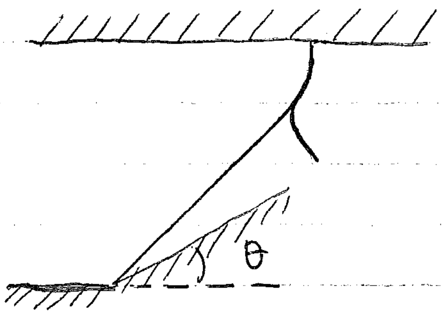
with these conditions a reflected shock is not possible → Mach reflection

$$T_{0_2} = T_{0_1} = T_\infty \left\{1 + \frac{\gamma-1}{2} M_1^2\right\} = 358 \text{ K}$$

$$p_{0_2} = \frac{p_{0_2}}{p_{0_1}} \cdot p_\infty \left\{1 + \frac{\gamma-1}{2} M_1^2\right\} = 389 \text{ kPa}$$

$$s_2 - s_1 = -R \ln\left\{\frac{p_{0_2}}{p_{0_1}}\right\} = 26.3 \frac{\text{J}}{\text{kg K}}$$

Mach reflection:



(iii) Mach reflection will occur when for a given $\theta_{12} = \theta_{23}$ there is not OSW possible at $\underline{M_2}$

Assume: $\theta_{12} = 10°$ $\quad \beta_2 = 36°$
$M_1 = 2.2$ $\quad \left.\begin{array}{l} M_2 = 1.82 \\ \theta_{23} = 10° \end{array}\right\} \beta_3 = 43.5°$
$\downarrow$
regular reflection

θ_{12} may be larger

Assume: $\theta_{12} = 18°$ $\quad \beta_2 = 45°$
$M_1 = 2.2$ $\quad \left.\begin{array}{l} M_2 = 1.5 \\ \theta_{23} = 10° \end{array}\right\} \beta_3$ not possible
$\downarrow$
possible Mach refl.

θ_{12} must be smaller

Assume: $\theta_{12} = 15°$ $\quad \beta_2 = 41.3°$
$M_1 = 2.2$ $\quad \left.\begin{array}{l} M_2 = 1.62 \\ \theta_{23} = 15° \end{array}\right\} \beta_3 = 63°$
$\downarrow$
regular reflection.

θ_{12} may be a bit larger

Assume: $\theta_{12} = 16°$ $\quad \beta_2 = 42.5°$
$M_1 = 2.2$ $\quad \left.\begin{array}{l} M_2 = 1.58 \\ \theta_{23} = 16° \end{array}\right\} \beta_3$ not possible

$\rightarrow$ Mach reflection occurs somewhere between $\theta = 15°$ and $\theta = 16°$ degrees

Problem 1
b)

Maximum turning angle when $M \to \infty$.

$$\lim_{M\to\infty} \nu(M) = \lim_{M\to\infty} \sqrt{\frac{\gamma+1}{\gamma-1}} \cdot \operatorname{atan}\sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)} - \operatorname{atan}(\sqrt{M^2-1})$$

$$\nu_{max} = \frac{\pi}{2}\left(\sqrt{\frac{\gamma+1}{\gamma-1}} - 1\right)$$

$\gamma = 1.4$, $\nu_{max} = 130.5°$

Or take maximum index in table $\nu(M = 50) = 124.7°$

$\nu(M = 3.2) = 53.5°$

Max turning angle : $\theta = \overset{(124.7°)}{130.5°} - 53.5° = \overset{(71.2°)}{77°}$

Maximum velocity :

$$T_t = T_\infty \left(1 + \frac{\gamma-1}{2} M_\infty^2\right) = 1158 \text{ K}.$$

$$V_{max} = \sqrt{2 c_p T_t} = 1525 \text{ m/s}. \qquad \left[c_p T + \frac{V^2}{2} = c_p T_t \right]$$

Problem 2

a)

(i) Fully subsonic flow: I

$$\left(\frac{P}{P_0}\right)_{I} = 0.912.$$

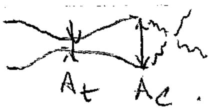
Fully supersonic flow: II

$$\left(\frac{P}{P_0}\right)_{II} = 0.124$$

Normal shock at the exit III

$$\left(\frac{P}{P_0}\right)_{III} = \left(\frac{P}{P_0}\right)_{II} \cdot \frac{P_2}{P_1} = 0.124 \cdot 4.59 = 0.56$$

In this case: $\frac{P}{P_0} = \frac{1}{7} \Rightarrow \left(\frac{P}{P_0}\right)_{II} < \frac{P}{P_0} < \left(\frac{P}{P_0}\right)_{III}$

$\Rightarrow$ Oblique shocks at the exit 

Therefore $M_e = 2$

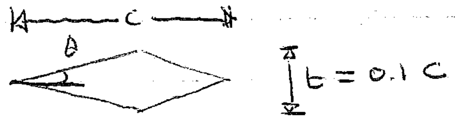
(ii) Chocked flow: $\frac{P}{P_0} < \left(\frac{P}{P_0}\right)_{I}$

$\Rightarrow P < 0.912 \cdot 700 = 638$ kPa.

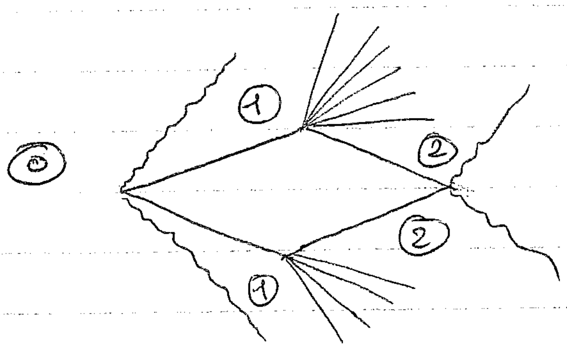
iii) Oblique shocks when: $\left(\frac{P}{P_0}\right)_{II} < \frac{P}{P_0} < \left(\frac{P}{P_0}\right)_{III}$

Problem 2

b)



$\theta = \operatorname{atan} 0.1 = 5.7°$



OSW

$$M_0 = 3 \qquad \beta_{01} = 23.7°$$
$$\theta_{01} = 5.7° \qquad M_1 = 2.7$$
$$P_1/P_0 = 1.53$$

PM EXP.

$$M_1 = 2.7 \qquad \nu_2 = \nu_1 + \theta_{12} = 43.6 + 11.4 = 55$$
$$\theta_{12} = 2 \cdot 5.7° = 11.4° \qquad M_2 = 3.3$$
$$P_2/P_1 = 0.41 \qquad \frac{P_2}{P_0} = 0.41 \cdot 1.53 = 0.63$$

$$\underline{C_D} = \left(\frac{P_1}{P_0} - \frac{P_2}{P_0}\right) \frac{2 \cdot t}{\gamma M_0^2 \cdot c} = \underline{0.0143}$$

<u>Linearized theory</u>

$$C_p = \frac{2\theta}{\sqrt{M_\infty^2 - 1}} \qquad C_{p_1} = 0.070 \qquad C_{p_2} = -0.070$$

$$\underline{C_D} = 2 \cdot \frac{1}{2} \cdot C_{p_1} \cdot \frac{t}{c} - 2 \cdot \frac{1}{2} \cdot C_{p_2} \cdot \frac{t}{c} = \underline{0.0141}$$

Problem 3a:

$p1 := 1\,bar$ $\quad$ $p2 := 5\,bar$ $\quad$ $mdot := 1.5\,\frac{kg}{s}$ $\quad$ $Cp := 1004\,\frac{J}{kg \cdot K}$

$T1 := (25 + 273.15)K$ $\quad$ $T2 := (300 + 273.15)K$ $\quad$ $\gamma := 1.4$ $\quad$ $Cv := \frac{Cp}{\gamma}$

$Rair := Cp - Cv$

Work associated to the expansion transformation :

$W12 := Cp \cdot (T2 - T1)$ $\qquad$ $W12 = 2.761 \times 10^{5}\ Sv$

Power required

$Wdot := mdot \cdot W12$ $\qquad$ $Wdot = 4.141 \times 10^{5}\ W$

Entropy increase :

Since ds>=dq/T and dq=0, for the transformation to be irreversible it must be $\Delta s>0$ as it is indeed the case :

$$\Delta s := Cp \cdot \ln\left(\frac{T2}{T1}\right) - Rair \ln\left(\frac{p2}{p1}\right) \qquad \Delta s = s2 - s1$$

$$\Delta s = 194.486\,\frac{m^2}{K \cdot s^2}$$

Final temperature corresponding to an isentropic transformation.

$$T2is := T1 \cdot \left(\frac{p2}{p1}\right)^{\frac{\gamma-1}{\gamma}} \qquad T2is = 472.216K \qquad T2is - 273.15K = 199.066K$$

Remark T2is<T2

Work associated to the isentropic expansion transformation :

$W12is := Cp \cdot (T2is - T1)$ $\qquad$ $W12is = 1.748 \times 10^{5}\ Sv$

Power required

$Wdot\_is := mdot \cdot W12is$ $\qquad$ $Wdot\_is = 2.621 \times 10^{5}\ W$

Isentropic efficiency

$$\eta s := \frac{T2is - T1}{T2 - T1} \qquad \eta s = 0.633$$

Problem 3b:

$VA := 1m^3 \quad TAin := (100 + 273.15)K \quad mAin := 1kg \quad mAfin := mAin$

$VB := 1m^3 \quad TBin := (20 + 273.15)K \quad mBin := 1kg \quad mBfin := mBin$

$$pAin := \frac{mAin \cdot Rair \cdot TAin}{VA} \qquad \frac{pAin}{1000} = 107.041 Pa$$

$$pBin := \frac{mBin \cdot Rair \cdot TBin}{VB} \qquad \frac{pBin}{1000} = 84.092 Pa$$

Determination of the air density :

The densities remain constant since there is no mass transfer through the membrane, hence

$\rho Ain = \rho Afin$ where $\rho Ain := \frac{mAin}{VA} \quad \rho Ain = 1 \frac{kg}{m^3} \quad \rho Afin := \rho Ain$

$\rho Bin = \rho Bfin$ where $\rho Bin := \frac{mBin}{VB} \quad \rho Bin = 1 \frac{kg}{m^3} \quad \rho Bfin := \rho Bin$

Determination of Tfin:

$Ufin - Uin = 0$ leads to $(mAfin \cdot uAfin + mBfin \cdot uBfin) - (mAin \cdot uAin + mBin \cdot uBin) = 0$

observing that the heat transfer ends when the air in both parts reaches the same temperatures, i.e. TAfin=TBfin=Tfin, and that for air at the specified temperatures u=Cp*(T-Tref) we have :

$(mAfin + mBfin) \cdot Tfin = mAin \cdot TAin + mBin \cdot TBin$

hence

$$Tfin = \frac{(mAin \cdot TAin + mBin \cdot TBin)}{(mAfin + mBfin)}$$

which in the present case is equivalent to

$$Tfin = \frac{1}{2} \cdot (TAin + TBin)$$

$$Tfin := \frac{mAin \cdot TAin + mBin \cdot TBin}{mAfin + mBfin} \qquad Tfin = 333.15 K$$

$$Tfin - 273.15K = 60 K$$

Determination of the air final pressure :

$pAfin := \rho Afin \cdot Rair \cdot Tfin \qquad \frac{pAfin}{1000} = 95.566 Pa \qquad pAfin - pAin = -1.147 \times 10^4 Pa$

note : pAfin<pAin

$pBfin := \rho Bfin \cdot Rair \cdot Tfin \qquad \frac{pBfin}{1000} = 95.566 Pa \qquad pBfin - pBin = 1.147 \times 10^4 Pa$

note : pAfin<pAin

Entropy increase :

Considering the system comprising both parts A and B, since dS>=dQ/T and dQ=0, for the transformation to be irreversible it must be $\Delta S>0$ as it is indeed the case :

$$\Delta S = Sfin - Sin$$

$$Sfin = mAfin\cdot sfin + mBfin\cdot sBfin \qquad Sin = mAin\cdot sin + mBin\cdot sBin$$

$$\Delta S = mAin\cdot(sAfin - sAin) + mBin\cdot(sBfin - sBin) = mAin\cdot\Delta sA + mBin\cdot\Delta sB$$

$$\Delta sA := Cp\cdot\ln\left(\frac{Tfin}{TAin}\right) - Rair\cdot\ln\left(\frac{pAfin}{pAin}\right)$$

$$\Delta sB := Cp\cdot\ln\left(\frac{Tfin}{TBin}\right) - Rair\cdot\ln\left(\frac{pBfin}{pBin}\right)$$

$$\Delta S := mAin\cdot\Delta sA + mBin\cdot\Delta sB$$

$$\Delta S = 10.413\,\frac{m^2\cdot kg}{K\cdot s^2}$$

The transformation is indeed irreversible.

In the case mAin=2*mBin :

$$Tfin1 := \frac{2mBin\cdot TAin + mBin\cdot TBin}{2mBfin + mBfin}$$

$$Tfin1 = 346.483K$$

$$Tfin1 - 273.15K = 73.333K \qquad Tfin1 - Tfin = 13.333K$$

$$\frac{Tfin}{TAin}\cdot 100 = 89.28$$

$$\frac{Tfin1}{TAin}\cdot 100 = 92.854$$

In the case mAin>>mBin :

$$Tfin2 := \frac{100mBin\cdot TAin + mBin\cdot TBin}{100mBfin + mBfin}$$

$$Tfin2 = 372.358K$$

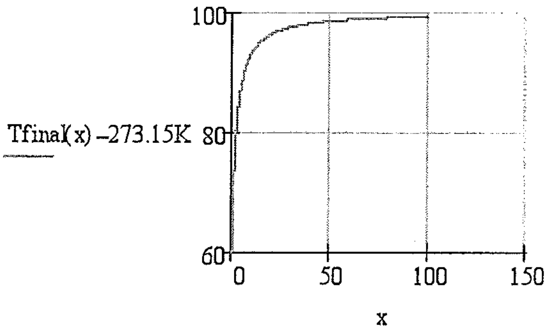
$$Tfin2 - 273.15K = 99.208K \quad Tfin2 - Tfin = 39.208K$$

$$\frac{Tfin2}{TAin}\cdot 100 = 99.788$$

Just for fun :

$$Tfinal(\xi) := \frac{\xi \cdot TAin + TBin}{\xi + 1} \qquad \xi = \frac{mA}{mB}$$

$$x := 1, 1 + \frac{1}{100} .. 100$$



We could even ask them, which is the value of the ratio mA/mB for whih the final temperature equals 95% of the initial temperature of the gas in the part A. Stai ridendo?