Aerodynamics C - Exam June 2008 Problems

1 A Supersonic Airfoil

1. Consider a thin airfoil, immersed in a uniform supersonic stream at $M_\infty = 3.0$ at an angle of attack of $\alpha = 2.5\,\text{deg}$. The airfoil has a thickness of $2h$ and has a parabolic upper profile given by:

$$\frac{y}{h} = 4\frac{x}{c}\left(1 - \frac{x}{c}\right). \tag{1.1}$$

   The bottom profile is made out of two straight parts. The half-thickness to cord ratio is $h/c = 0.087$.

   (a) Draw on two diagrams the pressure coefficient distribution along the airfoil on the upper and lower side.

   (b) Compute the lift and drag coefficient of the airfoil using linearized theory.

   (c) Evaluate the pressure coefficient on the lower side by shock-expansion theory and compare the results with those obtained by linearized theory.

2. Demonstrate that the total temperature remains constant across a stationary normal shock wave. Also demonstrate that the total pressure decreases across the normal shock wave.

2 Designing a Supersonic Wind Tunnel

1. Consider the flow of air through a convergent-divergent nozzle with a throat cross-section $A_t = 0.4m^2$ and an exit-to-throat area ratio of 1.616. Assuming that the reservoir conditions are $p_0 = 5.5 \cdot 10^5 Pa$ and $T_0 = 550°C$, determine the exit Mach number M_e and the mass flow $\dot{m}$ through the nozzle when the pressure outside the reservoir p_a attains the following values:

   (a) $p_a = 5.0 \cdot 10^5 Pa$.

   (b) $p_a = 2.2 \cdot 10^5 Pa$.

   (c) $p_a = 0.4 \cdot 10^5 Pa$.

   (d) Draw the pattern of shocks and expansion waves at the nozzle exit for the last case.

2. In low-speed incompressible flow, the peak pressure coefficient (at the minimum pressure point) is $C_p = -0.48$. Estimate the critical Mach number for the given airfoil, using the Prandtl-Glauet rule.

3 Analyzing a Power Plant

Consider the power plant below. The working fluid is air.

The following assumptions are made.

1. The transformations hosted by the compressor and the turbines are adiabatic and the isentropic efficiency is 0.82 for the compressor and 0.87 for both turbines.

2. The power delivered by the first turbine (operating between point 4 and point 5) directly equals that used by the compressor.

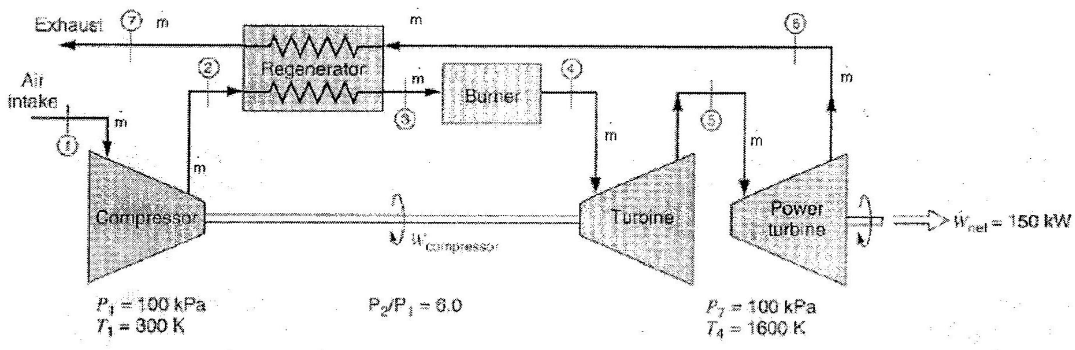

Figure 1: An overview of the power plant.

3. The regenerator is an open system with two inlets and two outlets hosting an isobaric ($p_3 = p_2$ and $p_7 = p_6$) heat exchange process.

4. The gas expands in the second turbine (operating between points 5 and 6) delivering a power output of $150kW$.

5. The transformation in the burner takes place at constant pressure.

6. $p_1 = 100kPa$, $T_1 = 300K$, $p_2/p_1 = 6$, $p_7 = 100kPa$, $T_4 = 1600K$.

Answer the following questions:

1. Draw the evolution of the cycle in the $T - s$ plane and later verify the correspondence between the graphical representation and the obtained numerical values of the state variables.

2. Determine the temperature of the air T_2 at the exit of the compressor.

3. Determine the temperature of the air T_5 at the first turbine. (Hint: apply the first principle to both the compressor and the first turbine and then make use of the assumption that the power delivered by the fist turbine is equal to the power used by the compressor.

4. Determine the value of the pressure p_5 after the first turbine.

5. Determine the temperature of the air T_6 at the exit of the power turbine.

6. Determine the net work per unit mass produced by the power plant.

7. Explain the effect of the regenerator on the thermal efficiency of the power plant.