

Note:

The compression from point 1 to point 2 is ~~is~~ said to be a polytropic transformation. No information is given about the reversibility of said transformation.

As a consequence two interpretations should be allowed ~~for~~ for the solution of the exercise (NOTE that this effects only ~~the~~ the answer to point 1)

First case

The transformation is considered reversible. As a consequence, ~~the~~ the condition $ds > \frac{\delta Q}{T}$ should be satisfied during the transformation.

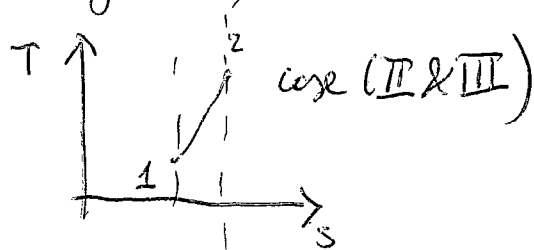
No information is given about the heat exchanged during the compression. The fluid could be cooled, heated or ~~no~~ exchange no heat with the surrounding.

Hence, the entropy can ~~be~~ increase during the transformation ~~(cases II and III)~~ (cases II and III).

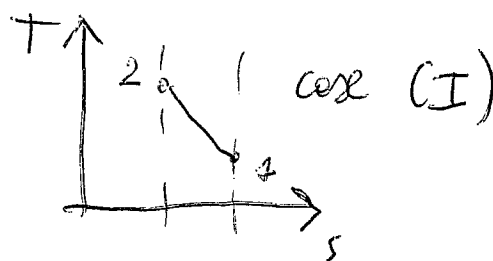
However, it can also decrease (case I) although it is not obliged to ~~be~~.

The sign ($>$) applying.

As a consequence these two graphs should be considered ~~as~~, provided an explanation is given, allowable.



To be given as good by default



To be accepted provided

Second case

The assumption of a reversible transformation is made.

Only in this case, the relation:

$$ds = \frac{dq}{T}$$

applies and should be therefore satisfied during the transformation.

A direct consequence of such a relation is that, for a thermally and elastically perfect gas, $\int ds$ is given by:

$$\int_1^2 ds = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \quad (1)$$

or in the infinitesimal form $ds = c_p \frac{dT}{T} - \frac{RdP}{P}$

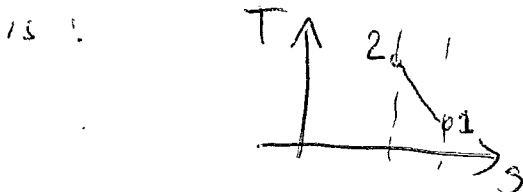
From (1) we conclude that if the transf. is reversible and polytropic of exponent n , the entropy variation is given by:

$$\int_1^2 ds = c_p \ln \left[\frac{P_2}{P_1} \right]^{\frac{n-1}{n}} - R \ln \frac{P_2}{P_1}$$

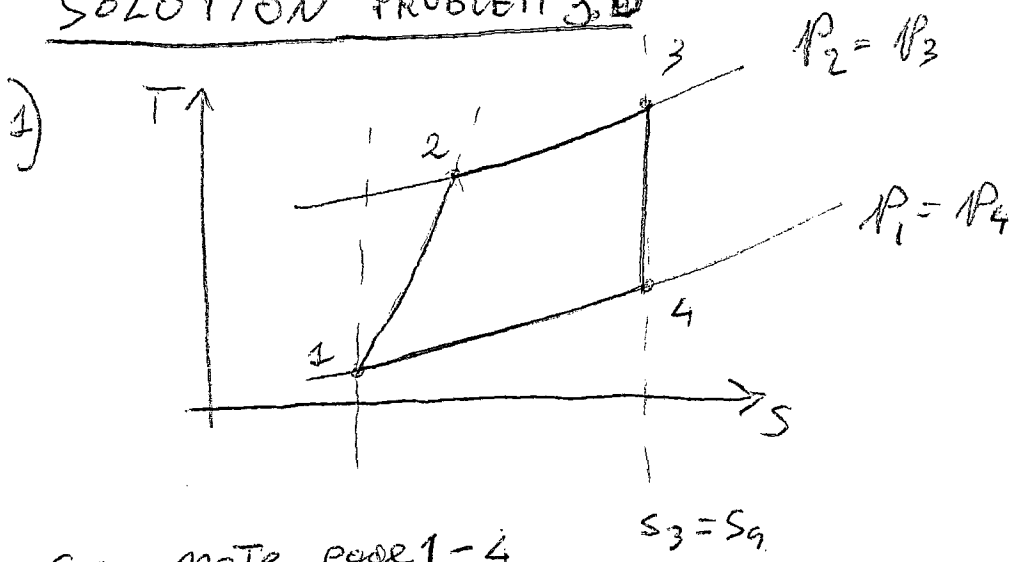
$$s_2 - s_1 = c_p \ln \left(\frac{P_2}{P_1} \right) \left[\frac{n-1}{n} - \frac{\gamma-1}{\gamma} \right] \begin{cases} > 0 \text{ if } n > \gamma \\ < 0 \text{ if } n < \gamma \end{cases}$$

if $n < \gamma$, as in the present case, $s_2 < s_1$ which means that the compression is not isentropic and moreover the fluid is cooled ($sa < 0$).

As a consequence, the only drawing coherent with such an assumption [reversible transformation]



SOLUTION PROBLEM 3b



See note page 1-4

2) Net work $\stackrel{\text{def.}}{\Rightarrow}$ | Work produced by The turbine - Work received by The compressor

Turbine: ${}_3W_4 + {}_3W_4 = h_4 - h_3$

isentropic efficiency $\Rightarrow \eta_s = 1 \Rightarrow s_4 = s_3$
 $ds \geq \frac{\delta q}{T} \Rightarrow \frac{\delta q}{T} \leq 0$
 The transformation is isentropic and hence adiabatic and reversible $\Rightarrow \frac{\delta q}{T} = 0 \Rightarrow \delta q = 0$
 The transformation is adiabatic

${}_3W_4 = h_4 - h_3$

compressor: ${}_1W_2 + {}_1W_2 = h_2 - h_1$

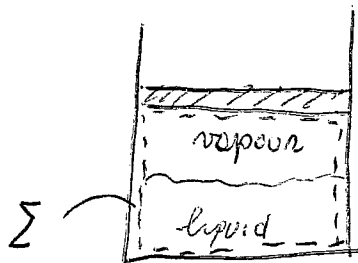
No information is explicitly given in the text about the heat exchange processes taking place during the compression. However in the class we've always considered adiabatic compression within cycles like the one proposed herein.

it is therefore ~~not~~ easy for the student to make this assumption. ~~if~~ if such an assumption is not made, the evaluation of the answer should be based on the reasoning proposed

SOLUTION PROBLEM 3.9

Jan 2014

Piston - cylinder device



1: INITIAL STATE

$m_1 = m = \text{constant}$ during the process, since the system is closed.

V_1 is known

p_1 is known

i)

Two independent state variables are needed to uniquely identify (determine) the state of Σ .

$$p_1 \quad \text{and} \quad n_1 = \frac{V_1}{m_1} = \frac{0,1 \text{ m}^3}{0,5 \text{ kg}} = 0,2 \frac{\text{m}^3}{\text{kg}}$$

NOTE: $n_1 = 0,2 \frac{\text{m}^3}{\text{kg}} = 0,2 \frac{\text{m}^3}{10^{-3} \text{ Mg}} = 0,2 \cdot 10^3 \frac{\text{m}^3}{\text{Mg}} = 200 \frac{\text{m}^3}{\text{Mg}}$

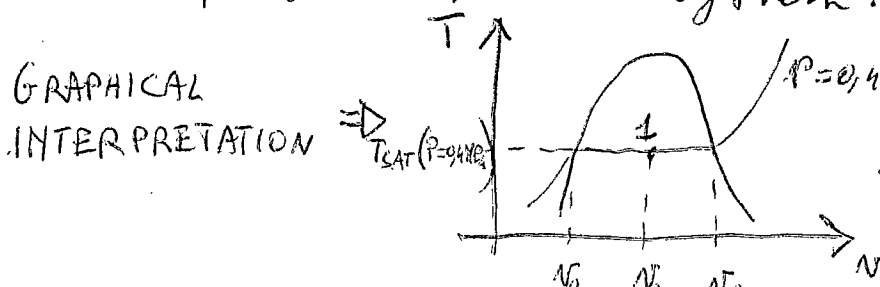
$$\Rightarrow n_1 = 200 \frac{\text{m}^3}{\text{Mg}}$$

From Table 1: $p = 0,4 \text{ MPa} \Rightarrow n_g = 1,084 \text{ m}^3/\text{Mg}$

$$n_f = 462,4 \text{ m}^3/\text{Mg}$$

$\Rightarrow n_g < n_1 < n_f \Rightarrow$ The Σ is a mixture!

\Rightarrow Both liquid and vapour phase are present in the system.



$$\Rightarrow T_1 = T_{\text{SAT}}(p = 0,4 \text{ MPa}) = 416,8 \text{ K} = 143,6^\circ\text{C}$$

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From the definition of work we have that the work done by the system Σ to expand during the transformation is:

$$dw = -p dV$$

$${}_1W_2 = \int_1^2 dw = - \int_1^2 p dV = -p_1 \int_1^2 dV$$

$$= -p_1 (V_2 - V_1) = -p_2 V_2 + p_1 V_1$$

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since $p_1 = p_2$

Hence,

$${}_1Q_2 + {}_1W_2 = U_2 - U_1$$

$${}_1Q_2 - p_2 V_2 + p_1 V_1 = U_2 - U_1$$

$${}_1Q_2 = U_2 + p_2 V_2 - (U_1 + p_1 V_1)$$

$${}_1Q_2 = H_2 - H_1 \quad \text{dividing by the system mass } m$$

$$q_2 = h_2 - h_1$$

where $q_2 = {}_1Q_2/m$ and $\frac{H_{1,2}}{m} = h_{1,2}$

So the amount of heat transferred to Σ is

$${}_1Q_2 = m q_2 = m \cdot (h_2 - h_1)$$

both values of h_2 and h_1 are available from the tables.

Ja 4/5

$$N = \frac{R_u}{M_{mol}} \frac{T}{P}$$

$$R_u = 8,314 \frac{J}{mol \cdot K}$$

$$M_{mol} = 18 \frac{kg}{mol}$$

so N_2^{PGL} can be computed as:

$$N_2 = \frac{R_u}{M_{mol H_2O}} \frac{T_2}{P_2}$$

$$= \frac{8,314 \frac{J}{mol \cdot K}}{18 \frac{kg}{mol}} \frac{573,15 K}{0,4 \cdot 10^6 \frac{N}{m^2}}$$

$$N_2^{PGL} = 662 \frac{m^3}{kg}$$

where $1N = 1kg \frac{m}{s^2}$
 $1J = 1N \cdot m$

therefore the accuracy of the perfect gas model can be computed as:

$$\frac{\|N_2^{PGL} - N_2\|}{N_2^{PGL}} \sim 0,45\%$$

~~OPTIONAL DISCUSSION
As far as the compressibility factors concern~~

$$Z = \frac{N_2}{N_2^{PGL}} = 1,6$$

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