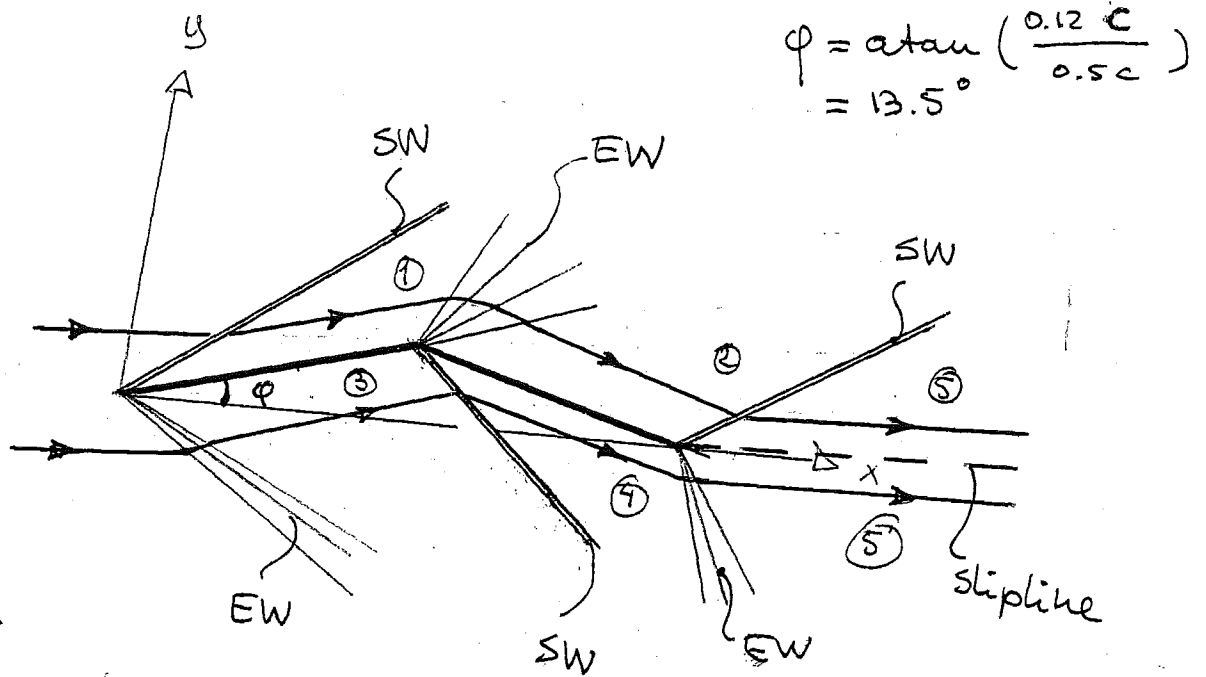


1a
ii)



(ii) Determine drag coefficient, cannot use linear theory since the angles are not small (especially between region ③ and ④).
Therefore use Shock - Expansion theory:

Upper side

0 → 1 Shock, deflection angle = $\phi - \alpha = 9.3^\circ = \delta_1$
 $\beta_1 = 31.2^\circ$, $M_1 = 2.1$, $\underline{P_1/P_0 = 1.79}$

1 → 2 Expansion, $\nu(M_2) = \nu(M_1) + 2 \cdot \phi = 29.1 + 27 = 56.1^\circ$
 $M_2 = 3.4$, $\underline{P_2/P_1 = 0.15}$

lower side

0 → 3 Expansion, $\nu(\pi_3) = \nu(\pi_0) + \varphi - \alpha$
 $= 39.1 + 9.3 = 48.4^\circ$

$\pi_3 = 2.9$, $\frac{P_3}{P_0} = 0.52$

3 → 4 Shock, deflection angle = $2\varphi = 27^\circ$
 $\beta_4 = 48^\circ$, $\pi_4 = 1.5$, $\frac{P_4}{P_3} = 5.26$

$$D = \frac{P_1 \cdot c}{2 \cos \varphi} \cdot \sin(\varphi - \alpha) - \frac{P_3 \cdot c}{2 \cos \varphi} \cdot \sin(\varphi - \alpha) + \frac{P_4 \cdot c}{2 \cos \varphi} \cdot \sin(\varphi + \alpha) - \frac{P_2 \cdot c}{2 \cos \varphi} \cdot \sin(\varphi + \alpha)$$

$$C_d = \frac{D}{q_0 \cdot c} = \frac{2D}{\rho P_0 \pi_0^2 c}$$

$$C_d = \frac{1}{\rho \pi_0^2 \cos \varphi} \left[\left(\frac{P_1}{P_0} - \frac{P_3}{P_0} \right) \sin(\varphi - \alpha) + \left(\frac{P_4}{P_3} \cdot \frac{P_3}{P_0} - \frac{P_2}{P_1} \cdot \frac{P_1}{P_0} \right) \sin(\varphi + \alpha) \right]$$

$C_d = 0.11$

(iii) The flow behind the trailing edge must have the same direction and pressure.

Assume a flow direction in $\textcircled{5}$ and $\textcircled{5'}$ and compute P_5/P_2 and $P_{5'}/P_4$, now compute P_5/P_0 and $P_{5'}/P_0$.

When P_5/P_0 and $P_{5'}/P_0$ are not equal, assume a new deflection and repeat the process.

1b)

CO_2 : molar mass : $12 + 2 \cdot 16 = 44 \text{ g/mol}$

6i)

$$R_{\text{CO}_2} = \frac{R}{M_{\text{CO}_2}} = \frac{8314}{44} = 189 \text{ J/kg K}$$

$$f_{\text{CO}_2} = \frac{n_{\text{CO}_2} + 2}{n_{\text{CO}_2}} \quad n_{\text{CO}_2} \rightarrow \text{d.o.f. of } \text{CO}_2$$

If one assumes CO_2 is a linear molecule $n_{\text{CO}_2} = 5$

If one assumes CO_2 is fully 3D $n_{\text{CO}_2} = 6$

$$\rightarrow f = 1.4$$

$$\rightarrow f = 1.33$$

\rightarrow Actually $f_{\text{CO}_2} = 1.3$ (from tables)

$$n = \frac{V_0}{\sqrt{f k T_0}} = \frac{6 \cdot 10^3}{\sqrt{1.3 \cdot 189 \cdot 150}} = 31$$

$$T_t = T \left(1 + \frac{f-1}{2} n^2 \right) = 21.8 \cdot 10^3 \text{ K}$$

(ii) At high temperatures vibrational energy becomes important (next to translational and rotational). This increases the degrees of freedom and thus reduces f .

Furthermore the vibrational mode requires energy which reduces the total temperature.

2a)

$$A_e = 2.5 \text{ m}^2$$

$$P_o = 5.5 \times 10^5 \text{ Pa}$$

$$P_a = 1.013 \times 10^5 \text{ Pa}$$

$$T_o = 290 \text{ K}$$

(i) NSW @ exit.

$$\frac{P_a}{P_o} = \frac{P_2}{P_1} = \frac{P_2}{P_1} \cdot \frac{P_1}{P_{e1}} = \left(1 + \frac{2\gamma}{\gamma+1} (\pi_1^2 - 1)\right) \left[1 + \frac{\gamma-1}{2} \pi_1^2\right]^{\frac{-\gamma}{\gamma-1}}$$

$$\frac{P_a}{P_o} = 0.184$$

iterate: $\pi_1 = 3.5 = \pi_e$ (before shock).

$$\frac{A_e}{A^*} = 6.79$$

$$A^* = 0.37 \text{ m}^2$$

(ii) Mass flow

$$\dot{m} = \frac{P_o A^*}{\sqrt{T_o}} \sqrt{\frac{\gamma}{R} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} = 483 \text{ kg/s}$$

(iii) $\frac{P_a}{P_o} = \frac{1.013}{1.5} = 0.675 < \frac{P_{e3}}{P_o} \rightarrow$ normal shock somewhere in the nozzle.

$$\pi_e = \frac{1}{\gamma-1} \left\{ \sqrt{1 + 2(\gamma-1) \left[\frac{2}{\gamma+1}\right]^{\frac{\gamma+1}{\gamma-1}} \cdot \left(\frac{A^*}{A_e} \cdot \frac{P_o}{P_e}\right)^2} - 1 \right\}$$

\downarrow
 $\frac{1}{6.79}$

\downarrow
 $\frac{1.5}{1.013}$

2b) See chapter 11 and 12 of Anderson.

The fact that the assumption of an irreversible transformation implies that a condition like $pV^m = \text{constant}$ cannot hold for each point of the transformation, since ~~the~~ the states ~~are~~ under which the system is to be found during the transformation are not equilibrium states. However, it is common practice to consider the equation pV^m valid for the initial and final states and it is ~~therefore~~ therefore used to link ~~the~~ such states. ~~Therefore~~.
 In other words an irreversible polytropic transformation is a transformation for which

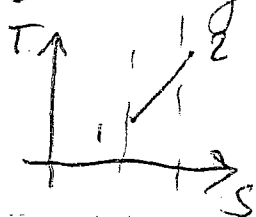
$$P_{IN} V_{IN}^m = P_{FIN} V_{FIN}^m$$

although such relation cannot be written for an intermediate point of the ~~transformation~~ transformation.
 Such transformation is usually named the equivalent polytropic transformation.

6

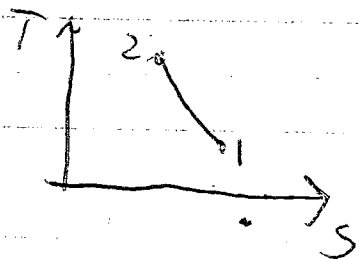
SUMMARY:

1) I would give full points to such graph



because this is the common way in which the ~~is~~ equivalent polytropic transformation is used to describe a compression.

2) I would also give full points to such graph



provided that a justification is given by the student.

The assumption made should however be coherent with the answer given to part (i) [see the note on page 1-4].

$$q_2 = 0 \Rightarrow w_2 = h_2 - h_1$$

$$\Rightarrow w_{net} = w_2 + w_4 \quad (\text{The sign adjusts the final result})$$
$$= h_2 - h_1 + h_4 - h_3$$

$$w_{net} = c_p (T_2 - T_1 + T_4 - T_3)$$

We therefore need T_2 and T_4 .

T_2 : Polytropic Transformation

$$\left. \begin{aligned} P_2 V_2^m &= P_1 V_1^m \\ \frac{P_2 V_2}{T_2} &= \frac{P_1 V_1}{T_1} \end{aligned} \right\} T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{m-1}{m}} = 426,9 \text{ K}$$

T_4 : Isentropic Transformation

$$T_4 = T_3 \left(\frac{P_4}{P_3} \right)^{\frac{\gamma-1}{\gamma}} \quad (P_1 = P_4, P_2 = P_3) \Rightarrow T_4 = 1211,1 \text{ K}$$

$$w_{NET} = 473,9 \frac{\text{kJ}}{\text{kg}}$$

Bi) Carnot efficiency $\eta_{th,C} = 1 - \frac{T_C}{T_H} = 0,83$

$$\eta_{th} = \frac{||w_{net}||}{||q_{in}||} = \frac{|w_2 + w_4|}{|q_3|} = \frac{|w_{NET}|}{c_p (T_3 - T_2)} = 0,34$$

2-3: $q_2 = 0, q_3 = h_3 - h_2 = c_p (T_3 - T_2)$

The Transformation corresponds to an isobaric transformation ($P = \text{const}$)

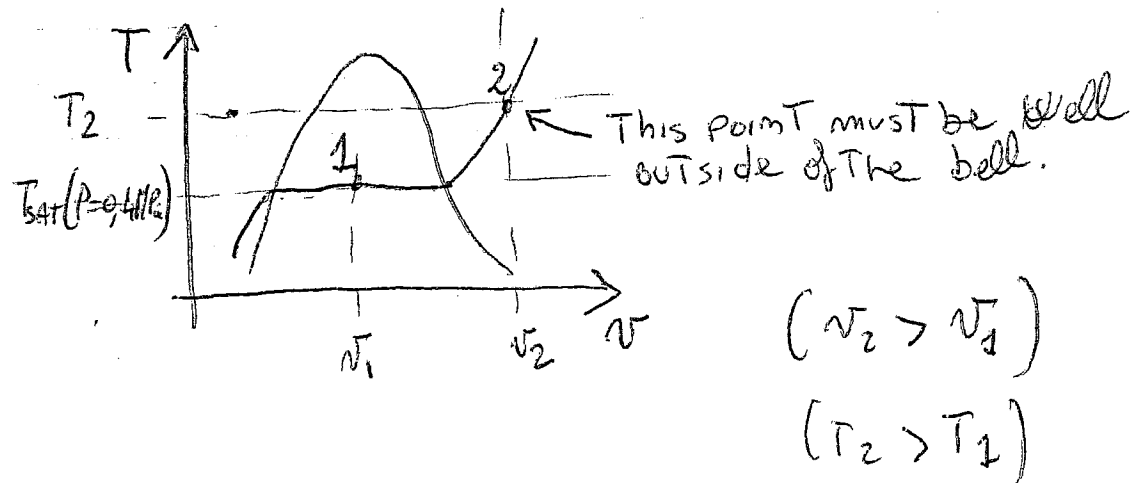
$\Rightarrow P_2 = P_1 = 0,4 \text{ MPa}$
 $\Rightarrow T_2 = 300^\circ\text{C}$

These are the 2 necessary and sufficient state variables needed

From Table 2 and 3 we notice that the SAT. Temperature corresponding to $0,4 \text{ MPa}$ is $143,6^\circ\text{C}$ (or $416,8 \text{ K}$). Since $T_2 = 300^\circ\text{C} > T_{\text{SAT}}(P = 0,4 \text{ MPa})$ we can be sure that the system is under a superheated vapor state.

To determine the final state of the system Σ

ii) Evolution of the transformation in the (T, v) plane



iii) First principle for CLOSED SYSTEMS

per unit mass
 or else
 how can we compute w_2 ?

$$Q_2 + w_2 = U_2 - U_1 + (\text{negligible variation of kinetic and potential energy})$$

From Table 1.

$$P = 0,4 \text{ MPa}$$

$$v_2 = 1,084 \frac{\text{m}^3}{\text{kg}}$$

$$v_g = 462,4 \frac{\text{m}^3}{\text{kg}}$$

$$h_e = 604,8 \frac{\text{kJ}}{\text{kg K}}$$

$$h_g = 2739 \frac{\text{kJ}}{\text{kg}}$$

quality:

$$x_1 = \frac{v_2 - v_e}{v_g - v_e} = 0,431$$

$$\Rightarrow h_1 = h_e + x(h_g - h_e) = 1525 \frac{\text{kJ}}{\text{kg}}$$

From Table 3

$$P_2 = 0,4 \text{ MPa} \Rightarrow v_2 = 655 \frac{\text{m}^3}{\text{kg}} \text{ \& } h_2 = 3067 \frac{\text{kJ}}{\text{kg K}}$$

$$\Rightarrow Q_2 = m(h_2 - h_1) = 771 \text{ kJ}$$

(ii)

From the superheated vapor table we get:

$$v_2 = 655 \frac{\text{m}^3}{\text{kg}} = 0,655 \frac{\text{m}^3}{\text{kg}}$$

Using The P.G.L. ∴

$$P V = m R T \text{ or equivalently}$$

$$P v = \frac{R_u}{M_{mol}} T \text{ we have:}$$