Drag and 3D wings

1 Definitions

$D_{profile}$ = Profile drag (N)
$D_{friction}$ = Friction drag(N)
$D_{pressure}$ = Pressure (N)
$c_{d,profile}$ = Profile drag coefficient for unit length (dimensionless)
$c_{d,f}$ = Friction drag coefficient for unit length (dimensionless)
$c_{d,p}$ = Pressure drag coefficient for unit length (dimensionless)
D_{wave} = Wave drag(N)
D = Total drag (N)
$c_{d,w}$ = Wave drag coefficient for unit length (dimensionless)
c_d = Drag coefficient for unit length (dimensionless)
α = Angle of attack (deg)
α_{eff} = Effective angle of attack (deg)
α_i = Induced angle of attack (rad)
D_i = Induced drag (N)
L = Lift (N)
C_L = Lift coefficient (dimensionless)
A = Aspect ratio (dimensionless)
b = Wing span (m)
c = Wing chord length (m)
S = Wing area (m^2)
q_∞ = Dynamic pressure in free-stream ($Pa = N/m^2$)
e = Span effectiveness ratio (sometimes also called Oswald factor) (dimensionless)

2 Drag types for 2D airfoils

There are three important types of drag in aerodynamics. Skin friction drag has already been discussed in a previous chapter, and so does pressure drag due to flow separation. Together these two types of drag form the profile drag. In formula:

$$D_{profile} = D_{friction} + D_{pressure} \tag{2.1}$$

$$c_{d,profile} = c_{d,f} + c_{d,p} \tag{2.2}$$

But there is another type of drag, called wave drag. This is caused by shock waves, which are caused by supersonic velocities. So the total drag is:

$$D = D_{wave} + D_{profile} = D_{wave} + D_{friction} + D_{pressure} \tag{2.3}$$

$$c_d = c_{d,w} + c_{d,f} + c_{d,p} \tag{2.4}$$

3 Induced Drag

Induced drag doesn't occur in 2-dimensional airfoils. In 3-dimensional airfoils it does appear. And since airplanes have 3-dimensional airfoils, it plays an important role. It usually occurs that the local flow direction of the air differs from the relative wind. Therefore the effective angle of attack α_{eff} is smaller than the geometric angle of attack α. Their difference is α_i, the induced angle of attack. In formula:

$$\alpha_i = \frac{\pi}{180}(\alpha - \alpha_{eff}) \tag{3.1}$$

Note that a conversion factor is necessary. This is because α_i is in radians (this is necessary for equation 3.3), while the normal angle of attack is in degrees.

Geometrically it can be shown that:

$$D_i = L \sin \alpha_i = L\alpha_i \tag{3.2}$$

The latter part of the equation is an approximation, since α_i is very small, and therefore $\sin \alpha_i \approx \alpha_i$. For elliptical lift distribution, which is often approximately the case for airplanes, the following formula is true for incompressible flows:

$$\alpha_i = \frac{C_L}{\pi A} \tag{3.3}$$

Where the aspect ratio A is equal to the 'slenderness' of the wing $\frac{b}{c}$. However, c is not constant along the wing, so aerodynamicists therefore have defined the aspect ratio as:

$$A = \frac{b^2}{S} \tag{3.4}$$

Combining previous equations results in:

$$D_i = L\frac{C_L}{\pi A} = q_\infty S \frac{C_L^2}{\pi A} \tag{3.5}$$

So now the induced drag coefficient can be found:

$$C_{D,i} = \frac{C_L^2}{\pi A} \tag{3.6}$$

However, elliptical lift distributions aren't always the case. Therefore, the span efficiency factor e (also sometimes called Oswald factor) has been defined, such that:

$$C_{D,i} = \frac{C_L^2}{\pi A e} \tag{3.7}$$

Now let's calculate the total drag coefficient for the wing. We don't know the induced drag for supersonic speeds, so for (low) subsonic speeds, the following equation holds:

$$C_D = c_{d,profile} + \frac{C_L^2}{\pi e A} = c_{d,f} + c_{d,p} + \frac{C_L^2}{\pi e A} \tag{3.8}$$

4 Slope of the (C_L - α) curve

For 2D wings, the slope of the (C_L - α) curve is $\frac{dC_L}{d\alpha} = a_0$. This is not the case for 3D wings. Now the relation $\frac{dC_L}{\alpha_{eff}} = a_0$ holds. Using equations 3.1 and 3.3 (the latter with a new span effectiveness factor e_1), and solving it for C_L gives the following relation:

$$a = \frac{a_0}{1 + \frac{57.3 a_0}{\pi e_1 A}} \tag{4.1}$$

e_1 is in theory a different factor than e, but in practice they are approximately equal. The factor 57.3 is in fact $\frac{180}{\pi}$, a conversion factor that was used to convert α_i to radians in the derivation of this formula.