# Basic Concepts

## 1 Flow types

If there is friction, thermal conduction or diffusion in a flow, it is termed **viscous**. If none of these things is present, the flow is **inviscid**. Inviscid flows do not appear in nature, but some flows are almost inviscid.

A flow in which the density $\rho$ is constant, is termed **incompressible**. If the density is variable, the flow is **compressible**.

The Mach number $M$ is defined as $V/a$, where $V$ is the airflow velocity and $a$ is the speed of sound. If $M < 1$, the flow is called **subsonic**. If $M = 1$, the flow is called **sonic**. If $M > 1$ the flow is called **supersonic**.

The **flow field variables** $p$, $\rho$, $T$ and $\mathbf{V} = u\mathbf{i} + v\mathbf{j} + w\mathbf{k}$ represent the **flow field**. All these variables are functions of $x$, $y$, $z$ and $t$ (they differ per position and time). However, for a **steady flow**, the flow field variables are constant in time. The flow is steady if

$$\frac{dp}{dt} = 0, \qquad \frac{d\rho}{dt} = 0, \qquad \frac{dT}{dt} = 0, \qquad \frac{du}{dt} = 0, \qquad \frac{dv}{dt} = 0 \quad \text{and} \quad \frac{dw}{dt} = 0. \tag{1.1}$$

Otherwise the flow is unsteady.

## 2 Gradient, divergence and curl

Consider the vector

$$\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}. \tag{2.1}$$

The **gradient** of a scalar field $p(x, y, z)$ is defined as

$$\nabla p = \frac{\partial p}{\partial x}\mathbf{i} + \frac{\partial p}{\partial y}\mathbf{j} + \frac{\partial p}{\partial z}\mathbf{k}. \tag{2.2}$$

The **divergence** of a vector field $\mathbf{A}(x, y, z) = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$ is defined as

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}. \tag{2.3}$$

The **curl** of a vector field $\mathbf{A}(x, y, z) = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$ is defined as

$$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\mathbf{i} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\mathbf{j} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\mathbf{k}. \tag{2.4}$$

Note that $\nabla p$ gives a vector field, $\nabla \cdot \mathbf{A}$ gives a scalar field and $\nabla \times \mathbf{A}$ gives a vector field.

These functions can also be derived for cylindrical coordinates (where $p = p(r, \theta, z)$ and $\mathbf{A} = \mathbf{A}(r, \theta, z)$) and for spherical coordinates (where $p = p(r, \theta, \phi)$ and $\mathbf{A} = \mathbf{A}(r, \theta, \phi)$), but those equations do not have to be known by heart.

## 3 Integrals

Given a closed curve $C$, the line integral is given by

$$\oint_C \mathbf{A} \cdot \mathbf{ds}. \tag{3.1}$$

where the counterclockwise direction around $C$ is considered positive.

Now consider a closed surface $S$, or a surface $S$ bounded by a closed curve $C$. The possible surface integrals that can be taken are

$$\iint_S p\,\mathbf{dS}, \qquad \iint_S \mathbf{A}\cdot\mathbf{dS} \quad \text{and} \quad \iint_S \mathbf{A}\times\mathbf{dS}. \tag{3.2}$$

where $\mathbf{dS} = \mathbf{n}\,dS$ with $\mathbf{n}$ being the unit normal vector. For closed surface $S$, $\mathbf{n}$ points outward.

Consider a volume $\nu$. Possible volume integrals are

$$\iiint_\nu p\,d\nu \quad \text{and} \quad \iiint_\nu \mathbf{A}\,d\nu. \tag{3.3}$$

# 4 Integral Theorems

There are several theorems using the integral described in the previous paragraph. If $S$ is the surface bounded by the closed curve $C$, **Stokes' theorem** states that

$$\oint_C \mathbf{A}\cdot\mathbf{ds} = \iint_S (\nabla\times\mathbf{A})\cdot\mathbf{dS}. \tag{4.1}$$

If $\nu$ is the volume closed by the closed surface $S$, **Gauss' divergence theorem** states that

$$\iint_S \mathbf{A}\cdot\mathbf{dS} = \iiint_\nu (\nabla\cdot\mathbf{A})d\nu. \tag{4.2}$$

Analogous to this equation is the **gradient theorem**, which states that

$$\iint_S p\,\mathbf{dS} = \iiint_\nu \nabla p\,d\nu. \tag{4.3}$$