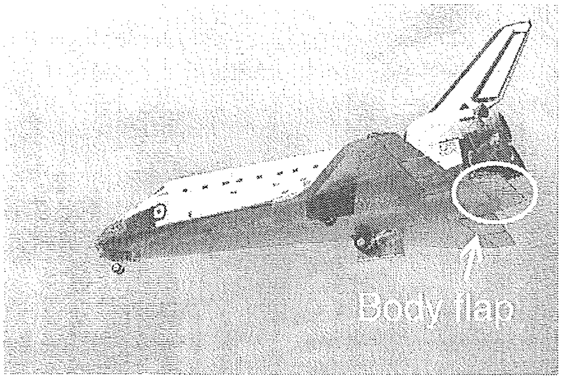
Delft University of Technology DEPARTMENT OF AEROSPACE ENGINEERING	
Course: Thermodynamics and compressible aerodynamics; Code AE2-125	Course year: 2
Date: Monday 19th January 2009	Time: 9 – 12

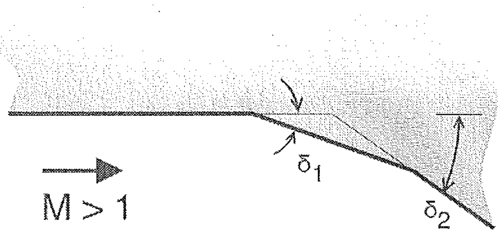
Problem 1-a

The Space Shuttle orbiter has a body flap in the rear-bottom part of the vehicle, used to trim the incidence during re-entry flight. Consider the geometry at zero degree angle of attack as schematically drawn in the figure on the right.

The flow around the flap is assumed to be steady two-dimensional and due to separation at the flap hinge two subsequent compression ramps are formed. Consider the following conditions: $\delta_1 = 10°$; $\delta_2 = 30°$; $M = 5.0$



Space Shuttle during re-entry



Simplified two-dimensional model of body flap

(i) Draw the pattern of shock waves, expansion waves and streamlines.

(ii) Compute the wave angles and determine the nature of the waves resulting from the shock waves interaction point (assume $\gamma = 1.4$).

(iii) Discuss a procedure to determine the direction of the slip-line emanating from the interaction point.

(iv) At the given Mach number the temperature on the body-flap will be so high that the perfect gas model is not accurate anymore. How does the value of γ change in these conditions? Justify your answer.

Problem 1-b

(i) Why can we regard as isentropic a shock wave at a Mach number close to 1? Discuss the entropy-Mach relation.

(ii) How low should be the Mach number in the air flow upstream of a normal shock wave such that it produces an entropy increase $\Delta s/R$ of less than 10%?

Problem 2-a

The nozzle of a supersonic wind tunnel has an exit-to-throat area ratio of 5.3. When the tunnel is running, a Pitot tube mounted in the test section measures 1.448 atm.

(i) What is the reservoir pressure for the tunnel?

(ii) What will be the reading of the Pitot tube if installed in the throat section for the conditions computed in (*i*)?

(iii) Determine the minimum value of the ratio between the diffuser throat area A_{t2} and the nozzle throat area A_{t1}.

(iv) What happens if the ratio is set below such minimum value?

Problem 2-b

Consider the airfoil in the figure below, immersed in a uniform supersonic stream at $M_\infty = 2.4$. The airfoil has a maximum thickness to cord ratio $h/c = 0.12$.

(i) Draw on two diagrams the pressure coefficient distribution along the airfoil on the upper and lower side.

(ii) Compute the lift and drag coefficient of the airfoil using shock expansion theory

(iii) What would be the lift coefficient computed from linearized theory?

Problem 3-a

(i) Describe schematically a Carnot heat engine.
(ii) Is this engine a closed or open system?
(iii) Describe each of the processes occurring in a Carnot cycle.
(iv) Draw the Carnot cycle in the *T-s* diagram and in the *p-v* diagram.
(v) Derive the expression of the thermal efficiency for a Carnot heat engine.

Problem 3-b

Consider a piston-cylinder device containing 0.4 kg of water vapor at 200 ˚C and 0.2 MPa (state 1).
A series of weights is progressively added on top of the piston in such a way that the water vapor follows a reversible polytropic compression (of exponent $n = 1.5$) until a pressure of 0.4 MPa and a temperature of 300 ˚C are reached (state 2). Then, the water vapor is cooled, maintaining the weights on the piston constant, until the system occupies a volume of 0.09 m^3 (state 3).
Assume that:

- during the transformation 1-2 the water vapor can be considered as a perfect gas ($C_p = 1952$ J/kg K and $R = 461$ J/kg K);
- during the transformation 2-3 the water properties correspond to those given in the tables (Use the tables to determine the water properties in state 2 when considering the transformation from state 2 to state 3).

Answer the following:

(i) Under which phase is the water in states 1, 2 and 3? Motivate your answer.
(ii) Draw the evolution of the transformation in the (p, v) plane.
(iii) Determine the amount of heat and work exchanged during the transformations 1-2 and 2-3. Which system is receiving and which one is delivering work during the transformations?
(iv) Consider the case in which the compression from state 1 to state 2 is adiabatic. Do you expect the work exchanged during the compression to be higher or lower than that calculated at point (*iii*)? Motivate you answer.

Table 1 – water properties as a function of pressure.

P [MPa]	$Tsat$ [K]	v_l [m^3/Mg]	v_g	h_l [kJ/kg]	h_g	s_l [kJ/kg K]	s_g
0,38	414,9	1,082	485,3	596,9	2736	1,758	6,914
0,4	416,8	1,084	462,4	604,8	2739	1,777	6,897
0,44	418,5	1,086	441,7	612,3	2741	1,795	6,880

Table 2 – water properties as a function of temperature.

T [K]	$Psat$ [MPa]	v_l [m^3/Mg]	v_g	h_l [kJ/kg]	h_g	s_l [kJ/kg K]	s_g
553,15	6,419	1,332	30,11	1237	2779	3,068	5,856
573,15	8,592	1,404	21,62	1346	2749	3,256	5,704
603,15	12,86	1,562	12,96	1528	2665	3,555	5,441

Table 3 – Superheated water properties at 300 ˚C.

$Tsat$ [˚C]	P [MPa]	v [m^3/Mg]	h [kJ/kg]	s [kJ/kg K]
133,5	0,3	875	3069	7,703
143,6	0,4	655	3067	7,567
151,8	0,5	522,6	3064	7,461

Note : 1 Mg = 10^6 g .

Appendix

Universal gas constant: $R_0 = 8314$ J/Kmol K, specific heat of air: $C_p = 1004$ J/Kg K result in 33 pts → 10 pts ⇒ Grade = $\frac{\text{result}}{33} \cdot 10$

Delft University of Technology **DEPARTMENT OF AEROSPACE ENGINEERING**	
Course: Thermodynamics and compressible aerodynamics; Code AE2-125	Course year: 2
Date: Thursday 26th March 2009	Time: 10 – 11:30

15 **Problem 1-a**

Consider a thermally insulated turbine, as depicted in fig. 1. A mass flow of 2kg/s of air is expanding through the turbine following a transformation from state 1 to 2, identified by the following state variables T1 = 700°C, p1 = 6bar, T2 = 400°C, p2 = 1bar.

Answer the following questions:

3 1) determine the power produced by the turbine;
3 2) verify that the transformation followed by the gas is irreversible;
3 3) represent the transformation 1-2 in the *T-s* plane;
3 4) determine the power that would have been produced by the turbine in the case of an isentropic expansion from the same initial state to the same final pressure;
3 5) calculate the isentropic efficiency of the turbine;

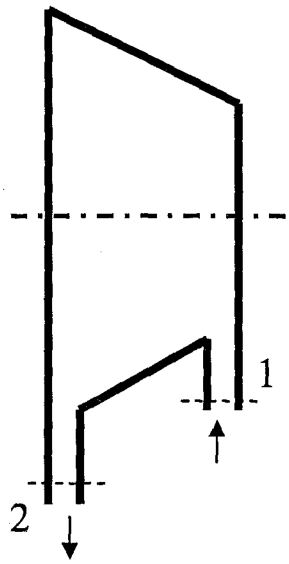


Fig. 1 –turbine schematic.

12 **Problem 1-b**

Consider a rigid and thermally insulated tank, divided into two parts, A and B, separated by a membrane as depicted in fig. 2. Each part has a volume of 1m^3. Part A initially contains 0.5kg of air at a temperature of 20°C, whereas part B is under vacuum conditions. The membrane is broken and after a certain time the air occupies the whole volume.

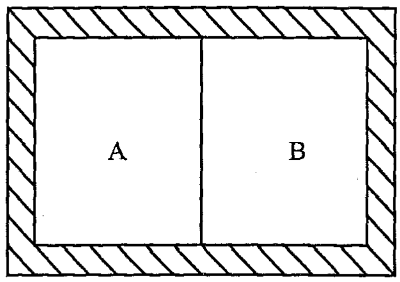


Fig. 2 – tank initial configuration.

Answer the following questions:

3 1) determine the final temperature of the gas;
3 2) determine the final pressure of the gas;
3 3) calculate the gas entropy variation. What can you conclude about the reversibility of the transformation?
3 4) Would the answer given for question 1 be still valid if instead of air in part A there would be a mixture of water under both the liquid and the vapor phase? Motivate your answer.

6 **Problem 1-c**

3 1) What is the definition of the compressibility factor for a gas?
3 2) Which choice of independent intensive variables can be made in order to come to a single compressibility diagram for more than one gas? Describe qualitatively the diagram obtained by the choice of such variables.

Appendix

Air properties: C_p = 1004 J/Kg K; γ = 1.4

Problem 1 a :

$V1 := 1m^3 \quad T1 := (20 + 273.15)K \quad mass := 0.5kg$

$p1 := \frac{mass \cdot Rair \cdot T1}{V1} \quad p1 = 4.205 \times 10^4 \, Pa$

$V2 := 2 \cdot V1$

Determination of T2 :

$u2 - u1 = 0$ leads to $Cv \cdot (T2 - T1) = 0$ i.e. $T2 := T1$

Determination of p2 :

$p1 \cdot V1 = mass \cdot Rair \cdot T1$

$p2 \cdot V2 = mass \cdot Rair \cdot T2 \quad p2 := \frac{p1}{2} \quad p2 = 2.102 \times 10^4 \, Pa$

Entropy increase :

Since ds>=dq/T and dq=0, for the transformation to be irreversible it must be Δs>0 as it is indeed the case :

$\Delta s = Cp \cdot \ln\left(\frac{T2}{T1}\right) - Rair \cdot \ln\left(\frac{p2}{p1}\right) = Rair \cdot \ln(2) \quad \Delta s = s2 - s1$

$\Delta s := -Rair \cdot \ln\left(\frac{p2}{p1}\right)$

$\Delta s = 198.834 \frac{m^2}{K \cdot s^2} \qquad Rair \cdot \ln(2) = 198.834 \frac{m^2}{K \cdot s^2}$

The transformation is indeed irreversible.

Problem 1 b :

$p1 := 6bar$ $\quad$ $p2 := 1bar$ $\quad$ $mdot := 2\frac{kg}{s}$ $\quad$ $Cp := 1004\frac{J}{kg \cdot K}$

$T1 := (700 + 273.15)K$ $\quad$ $T2 := (400 + 273.15)K$ $\quad$ $\gamma := 1.4$ $\quad$ $Cv := \frac{Cp}{\gamma}$

$Rair := Cp - Cv$

Work associated to the expansion transformation :

$W12 := Cp \cdot (T2 - T1)$ $\qquad$ $W12 = -3.012 \times 10^{5}\ Sv$

Power produced

$Wdot := -mdot \cdot W12$ $\qquad$ $Wdot = 6.024 \times 10^{5}\ W$

Entropy increase :

Since ds>=dq/T and dq=0, for the transformation to be irreversible it must be Δs>0 as it is indeed the case :

$\Delta s := Cp \cdot \ln\left(\frac{T2}{T1}\right) - Rair \cdot \ln\left(\frac{p2}{p1}\right)$ $\qquad$ $\Delta s = s2 - s1$

$\Delta s = 143.935\ \frac{m^{2}}{K \cdot s^{2}}$

Final temperature corresponding to an isentropic transformation.

$T2is := T1 \cdot \left(\frac{p2}{p1}\right)^{\frac{\gamma-1}{\gamma}}$ $\qquad$ $T2is = 583.245\ K$ $\quad$ $T2is - 273.15K = 310.095\ K$ $\quad$ Remark T2is<T2

Work associated to the isentropic expansion transformation :

$W12is := Cp \cdot (T2is - T1)$ $\quad$ $W12is = -3.915 \times 10^{5}\ Sv$

Power produced

$Wdot\_is := -mdot \cdot W12is$ $\quad$ $Wdot\_is = 7.829 \times 10^{5}\ W$

Isentropic efficiency

$\eta s := \frac{T2 - T1}{T2is - T1}$ $\qquad$ $\eta s = 0.769$

$ds = dh - v\,dp$

$\int dh = \int v\,dp$